Revised Edition.

FELTER'S
INTERMEDIATE

J. L. Giles del.

ARITHMETIC.

NEW-YORK
Charles Scribner & Co.

Bluffton
Ind.

NATURAL SERIES.

THE

ANALYSIS

OF

WRITTEN ARITHMETIC.

BOOK FIRST,

DESIGNED FOR

PUBLIC AND PRIVATE SCHOOLS,

CONTAINING

Mental, Slate, and Blackboard Exercises.

BY

S. A. FELTER, A. M.,

GRADUATE OF THE STATE NORMAL SCHOOL, AND LATE OF THE BROOKLYN COLLEGIATE AND POLYTECHNIC INSTITUTE, AND AUTHOR OF A SERIES OF SCHOOL ARITHMETICS, ETC.

REVISED EDITION.

NEW YORK:
PUBLISHED BY CHARLES SCRIBNER & CO.
HADLEY BROTHERS, CHICAGO, ILL.
INGHAM AND BRAGG CLEVELAND, OHIO.

Entered according to Act of Congress, in the year 1862, by
S. A. FELTER,
In the Clerk's Office of the District Court for the District of New Jersey.

Entered according to Act of Congress, in the year 1866, by
S. A. FELTER,
In the Clerk's Office of the District Court of the United States, for the Southern District of New York.

PREFACE.

In the published text books on the subject of Arithmetic, it has generally been the object to give under each rule a number of examples sufficient for illustration only. Experience proves, however, that this is not enough; for the rule occupies the attention of the pupil for a time so short that the impression made upon his memory is soon effaced.

The want of a larger number of progressive exercises for practice, particularly in the elementary rules, is almost universally felt by teachers, and especially by those in the public schools, whose arduous duties prevent them from supplying the deficiency by original examples. To such, the remedy has been either to require from the pupil the unpleasant task of performing the same examples a number of times, or else to seek a supply from books other than the one used.

Another source of great inconvenience and embarrassment to the teacher of Arithmetic arises from the personal explanation required by his pupils. This is caused, to a great extent, by the number of examples being too small and not systematically progressive, while the principles comprehended by them are too numerous and complex. Before one principle, with its applications, is thoroughly understood by the pupil, another is necessarily thrust upon his attention, thus often producing discouragement and a lack of interest.

Inaccuracy in the fundamental operations of arithmetic often causes both teacher and pupil much trouble and vexation. There is nothing, perhaps, that interferes so much with a rapid advancement in the latter part of an arithmetical course. Much valuable time is consumed in correcting mistakes in Addition, Multiplication and Division, which ought never to have been made. This difficulty can be traced, almost directly, to the need of a sufficient number of abstract examples in the fundamental rules to give the pupils skill and accuracy in simple computation.

On the introduction of Mental Arithmetic into our schools, a great revolution took place in the solution of arithmetical problems. Now, instead of following apparently arbitrary rules, analytical forms are given, and the pupil is taught to reason from cause to effect, rather than follow what appeared to him blind direction. This application of them, however, is made to mental questions only, while the analyses of the text books on Written Arithmetic seem better adapted to the teacher than the pupil. In consequence of this, while the pupil may be quick to perceive the logical relations of numbers in mental questions, he is unable to distinguish the same relations when the numbers are so large as to require the use of the slate.

This work is an attempt to meet these difficulties. It was, for the most part, prepared for the Author's own use, and is, consequently, not the development of a theory, but the result of practical experience. In giving it its present form, it has been his constant aim to adapt it to the wants of both teacher and pupil;—to the teacher, by furnishing examples so systematically progressive,* that the knowledge and discipline acquired by the pupil in what he has accomplished will enable him to surmount the difficulties before him with but little external aid;—to the pupil, by furnishing so large a number of examples that his progress must necessarily be thorough, without having recourse to the irksome task of repeatedly performing the same examples. While the examples are copious, they are so varied that they at once exercise the ingenuity and discrimination, and strengthen the judgment, and thereby excite interest; for every child delights to do what he is conscious of doing well.

Much attention has been given to analysis, believing that it is not only the best means of disciplining the mind, but that it is the key to arithmetical science. Analytical forms are to problems, what *demonstrations* are to *theorems.* For each abstract process, and for each peculiar combination of *elementary questions* in the practical examples, a model analytical form has been given; not for the teacher alone, but for the daily use of the pupil.

The review questions appended to each lesson form a very important feature of this work; for every teacher will readily acknowledge that the most difficult part of his labor is to cause his pupils to retain what they have once learned. By repeating these questions, and not confining them to the subject under consideration, it becomes necessary for the pupils to recall continually the facts and principles of that portion of the work over which they have passed; thus constituting a thorough daily review.

---

* See Introduction.

Particular attention is called to the "Test Questions for Examination." It is hoped that these will aid the teacher in securing thoroughness on the part of the pupils, in the facts and principles of each subject before leaving it.

While most of the examples have been composed expressly for this work, a few have been carefully compiled from other arithmetics. The Author wishes to acknowledge his indebtedness to Greenleaf's and Robinson's series for valuable information and suggestions.

He also wishes to acknowledge his obligations to S. A. FARRAND, A. M., *Principal* of the NEWARK ACADEMY, who has rendered valuable assistance while the work has been in press.

It has been the Author's earnest desire in the preparation of this work, to make it emphatically a book for the *class room*;—to make it analytic, progressive and practical, and copious without being monotonous. How well he has succeeded he will leave for his fellow teachers to decide.

COLLEGIATE AND POLYTECHNIC INSTITUTE,
BROOKLYN, N. Y., Oct., 1862.

INTRODUCTION.

SUGGESTIONS TO TEACHERS.

1. **Giving Assistance.**—Giving assistance is a habit to which the sympathetic teacher often yields, to the injury of his pupils. Business men often complain, and with justice, that when our best scholars enter business, many of them are incompetent on account of their inaccuracy and want of self-reliance. If the pupil can not perform the work given him without answers * or assistance, the work is too difficult, and something should be given him that he can do, for the discipline received from the thorough and accurate accomplishment of a work, is far greater than that resulting from attempts to do what can only be done with assistance, or with approximate accuracy.

2. **Thoroughness.**—In arithmetic, as in all other exact sciences, each step is so intimately connected with, and dependent upon, that which precedes, that no principle, however trifling, can be slighted without injury to the pupil's ultimate advancement.

3. **Definitions.**—By very young pupils, the definitions may be omitted until some skill shall have been attained in the abstract operations of the fundamental rules, except what may be taught them orally. Older scholars, when able to understand them, should commit them to memory. It is not

---

* The Author at first intended the work to be without answers; but finally concluded to insert them at the close, for the use of such teachers and private pupils as may desire to consult them. He sincerely hopes, however, that those teachers who wish their pupils to secure the full benefit of the course laid down in it, before placing the book in the hands of their pupils, will remove the answers from it entirely, or seal the leaves containing them. Increased interest and self-reliance will result from this course.

to be expected that many pupils, at the commencement, will understand the full scope and meaning of all the definitions, yet it is *important, as well as convenient, to have certain operations and relations associated with precise terms*, and besides, youth is, emphatically, the time for memorizing. It is not desirable to attempt to explain terms or definitions which are so difficult, that in the end pupils will have but a vague conception of them; for by this means they are often led to believe that they really understand things of which they are comparatively ignorant, and, hence, become satisfied with superficial knowledge.

4. **Analysis.**—The analysis of the abstract operations of Addition, Subtraction, Multiplication, and Division, may be omitted until a review of these rules; but the analysis of the "Practical Problems" under each rule should be rigidly required, and no recitation should be considered perfect without it.

Some teachers may object to repeating the *analytical formulas* on the ground that the pupil does not, or will not, understand them; that they are to him a mere repetition of words. It may be so in some cases; but by constant repetition and the exercise of judgment in their application, the pupil will intuitively form a habit of methodically investigating the relations of numbers, which cannot be acquired with equal facility by any other means.

The practical examples throughout the work are constructed on the principle that every complex problem may be separated into a certain number of elementary questions; and that the difficulty experienced in its solution depends upon the number and peculiar combinations of those questions. For illustration, take the following problem:

"Mr. Place purchased 15 cows; he sold 9 of them for $35 apiece, and the remainder for $32 apiece, when he found that he had lost $123; how much did he pay apiece for his cows?"

Analytical Steps.—**1.** *Find how many cows Mr. P. had after selling* 9.

Elementary Questions.—(*a*) If Mr. Place had 15 cows, and sold 9, how many did he have left?

2. *Find how much he received for the* 9 *cows.*

Question.—(*b*) If he received $35 for 1 cow, how much did he receive for 9 cows?

3. *Find how much he received for the remaining* 6 *cows.*

Question.—(*c*) If he received $32 for 1 cow, how much did he receive for 6 cows?

4. *Find how much he received for the whole* 15 *cows.*

Question.—(*d*) If he received $315 for 9 cows, and $192 for 6 cows, how much did he receive for the 15 cows?

5. *Find the cost of the* 15 *cows.*

Question.—(*e*) He received $507, which was $123 less than he gave for them; how much did the cows cost?

6. *Find the cost of* 1 *cow.*

Question,—(*f*) 15 cows cost $630; how much did 1 cow cost?

The above problem was copied from an Elementary Arithmetic, and is found under the "Applications" of the *Fundamental Rules.* It is an example which is seldom, if ever, performed by pupils without assistance, yet very little difficulty would be experienced were they able to discover the *elementary questions* composing it.

One of the particular objects of this work is to render the pupil skilful in this exercise. To accomplish this, the examples are strictly progressive, and so constructed that in Addition no problem is found containing more than *one* of these *questions.* In Subtraction, Multiplication, and Division, no problem contains more than *two*, and in United States Money, with the exception of Bills, no problem contains more than *three*, &c.; thus, as the pupil progresses in his course, difficulties are gradually and systematically placed before him in such a manner as to excite interest instead of creating discouragement.

For the purpose of exercising the judgment and discrimination, numbers and conditions which do not affect the result, are often introduced into the problems.

5. **Review.**—"Line upon line, and precept upon precept," should be the motto in the school-room. Hence the large number of examples under each rule; but should the teacher find that his pupils do not need the exer-

cise thus given, a portion of each lesson may be omitted without the least trouble or inconvenience. The fault of performing *too few*, rather than *too many*, is however to be particularly guarded against.

The "*Review*" *appended to each lesson ought by no means to be omitted.* It must not be supposed that it is meant to be exhaustive; on the contrary, it only forms a basis for thorough questioning, and is intended merely to be suggestive of a convenient and effective method of daily review.

CONTENTS.

SECTION I.

PAGE.
DEFINITIONS, ... 1.
NOTATION, ... 2.
NUMERATION, ... 5.

SECTION II.

ADDITION, ... 13.

SECTION III.

SUBTRACTION, ... 49.

SECTION IV.

MULTIPLICATION, ... 77.

SECTION V.

DIVISION, ... 96.

SECTION VI.

UNITED STATES MONEY, ... 130.
REDUCTION, ... 132.
Addition of U. S. Money, ... 134.
Subtraction of U. S. Money, ... 141.
Multiplication of U. S. Money, ... 145.
Division of U. S. Money, ... 147.
Bills, ... 155.

SECTION VII.

COMPOUND DENOMINATE NUMBERS, ... 174.
English Money, ... 174.
Troy Weight, ... 179.
Avoirdupois Weight, ... 185.
Apothecaries' Weight, ... 190.
Apothecaries' Fluid Measure, ... 190.
Long Measure, ... 193.
Cloth Measure, ... 193.
Surveyors' Measure, ... 199.
Square Measure, ... 200.
Surveyors' Square Measure, ... 209.
Cubic Measure, ... 210.
Liquid Measure, ... 216.
Dry Measure, ... 219.
Measure of Time, ... 222.
Circular or Angular Measure, ... 224.
Miscellaneous Tables, ... 229.
Addition of Compound Numbers, ... 229.
Subtraction of Compound Numbers, 231.
Multiplication of Compound Numbers, ... 235.
Division of Compound Numbers 237

SECTION VIII.

FRACTIONS, ... 247.

SECTION IX.

INTEREST, ... 274
Partial Payments, ... 279.
Answers, ... 283.

To the Teacher.

Children generally begin the study of Arithmetic as treated in this book at about the age of *ten* or of *eleven* [*] years, and seldom finish the subject of Commercial Arithmetic before the age of *sixteen.*

This volume, which is meant to follow the Primary Arithmetic, is designed to occupy the pupil during *two and a half* [†] years, and prepare him to enter the Grammar School Arithmetic, which is the concluding number in the Common School Series.

The Author suggests the following course of study in Arithmetic:

1st yr.	Oral instruction from "Manual," with slate exercises.
2d yr.	Oral instruction from "Manual," with slate exercises from "First Lessons."
3d yr.	Oral lessons, with mental, slate, and blackboard exercises from "Primary," through Subtraction.
4th yr.	Mental, slate, and blackboard exercises, through Division and the Denominate tables in "Primary."
5th yr.	The Fundamental Rules, completed in "Intermediate."
6th yr.	U. S. Money; Reduction of Denominate Numbers, in "Intermediate."
7th yr.	Fractions and Applications; Decimals. "Grammar School."
8th yr.	Percentage; Interest; Banking. "Gram. School."
9th yr.	Exchange; Averaging Accounts; Alligation; Ratio and Proportion; Involution and Evolution; Mensuration, etc. "Gr. School."

N. B.—*Nothing is to be gained and much may be lost* by hurrying scholars on from subject to subject, and from book to book.

---

[*] Experience proves that the age of *eleven* is preferable, by far, for the good of the pupil, in the matter of mental discipline, as well as of economy in time.

[†] The last six months' course in Fractions and Interest, is supplementary, and may be omitted by those who wish to enter the "Grammar School."

ARITHMETIC.

SECTION I.

DEFINITIONS, NOTATION, AND NUMERATION.

LESSON I.

1. *A Unit* is one of any kind of things; as, 1 apple, 1 cow, 1 peach, 1 dozen, 1 fourth.

ILLUSTRATION.*—*Def.* A unit is one of any kind of things; since 1 cow, 1 peach, represent each a single thing, they are therefore, units.

2. *A Number* is an expression that tells *how many;* as, 1, 3, 5, 4 horses, 7 apples, 5 ninths.

3. *A Concrete* or ***Denominate Number*** is a unit or a collection of units, the kind of which is expressed; as, 1 bird, 7 sheep, 13 pears, 3 elevenths, 2 ounces.

4. *An Abstract Number* is a unit or a collection of units, the kind of which is not expressed; as, 1, 7, 8, 6.

5. *Arithmetic* treats of numbers; as, 4 apples and 3 apples are 7 apples, 8 less 5 are 3.

---

* NOTE FOR THE TEACHER.—The pupil should be required to illustrate at the black-board, each of the definitions in the work, in the same way.

QUESTIONS.*—*Lesson I.* [For a full development of the ideas of a unit and of a concrete and an abstract number, see "*Manual*," Sec. I., Exercise I.] What is the meaning of "expressed"? Of "represent"? What is meant by "collection"?

LESSON II.

NOTATION AND NUMERATION.

6. *Notation* is the method of expressing numbers either by words, letters, or figures. There are two methods in common use, the Roman and the Arabic.

7. *The Roman* method uses the letters I, V, X, L, C, D, M; and is chiefly used in numbering the chapters, sections, and other divisions of books.

(*a.*) TABLE OF THE ROMAN NOTATION.

I	denotes	one.	C	denotes	one hundred.
V	"	five.	D	"	five hundred.
X	"	ten.	M	"	one thousand.
L	"	fifty.	$\overline{M}$	"	one million.

(*b.*) All other numbers are expressed by the use of these letters repeated or combined, as:—

I.	one.	IX.	nine.	LXXX.	eighty.	DCC.	seven hundred.
II.	two.	X.	ten.	XC.	ninety.	DCCC.	eight hundred.
III.	three.	XX.	twenty.	C.	one hundred.	DCCCC.	nine hundred.
IV.	four.	XXX.	thirty.	CC.	two hundred.	M	one thousand.
V.	five.	XL.	forty.	CCC.	three hundred.	MD.	fifteen hundred.
VI.	six.	L.	fifty.	CCCC.	four hundred.	MM.	two thousand.
VII.	seven.	LX.	sixty.	D.	five hundred.	$\overline{X}$.	ten thousand.
VIII.	eight.	LXX.	seventy.	DC.	six hundred.	$\overline{M}$.	one million.

[*] The teacher should by no means omit to question the class thoroughly on each subject; and by illustrations and suggestions to develop in the mind of the pupil a clear *conception* of the *idea* contained in the definition, proposition, or problem, before requiring it to be committed to memory. The following questions and hints are merely suggestive, and are not *intended* to be either systematic or complete. No book can take the place of a living teacher, neither can any teacher follow any prescribed method of instruction, for by so doing he sacrifices *individuality* which is essential to success.

REVIEW QUESTIONS.*—What is a unit? (1.) What is a number? (2.) What is a concrete number? (3.) What is an abstract number? (4.) Of what does arithmetic treat? (5.)

[*] NOTE FOR THE TEACHER.—The numbers at the end of the questions refer to the paragraphs containing the answers.

N. B.—These review questions should by no means be omitted.

(*c.*) The value of a letter is repeated as many times as the letter itself is repeated; as, XXX, *thirty*, CC, *two hundred.*

(*d.*) If a letter of *less value* be written *before* a letter of *greater value*, the value of the less is *subtracted* from that of the greater; as, X., *ten*, IX., *nine;* L., *fifty*, XL., *forty.*

(*e.*) If a letter of *less value* be written *after* a letter of *greater value*, the value of the *less* is *added* to that of the greater; as, V., *five*, VI., *six;* M., *one thousand*, MD., *fifteen hundred.*

(*f.*) A dash (—) placed over a letter or a combination of letters repeats the value a thousand times; as X., *ten*, $\overline{X}$., *ten thousand;* XII., *twelve*, $\overline{XII}$., *twelve thousand.*

QUESTIONS.—*Less. II.*—What is the meaning of "notation"? Of "figures"? What is meant by "method"? Why is one method called Arabic? Why the other, Roman? What is the chapter of a book? What is a section of a book? Why are the divisions of a book numbered? What is the meaning of "multiplied"? Of "repeated"? Of "combination"?

LESSON III.

Read the following numbers from the book at recitation:—

1. XL.
2. XXIV.
3. XVII.
4. XIX.
5. XIV.
6. XXXIV.
7. XXIV.
8. XVII.
9. XXVIII.
10. XXXVI.
11. XXII.
12. XIII.
13. XLV.
14. XLIV.
15. XVIII.
16. XXIX.
17. XLVII.
18. LIV.
19. LVI.
20. LIX.
21. LXI.
22. XLIX.
23. LXXI.
24. LXIX.
25. C.
26. XC.
27. MD.
28. $\overline{M}$ D.
29. $\overline{MIX}$.
30. $\overline{MD}$CIX.
31. $\overline{MDXI}$.
32. $\overline{XII}$XDCIX.

QUESTIONS.—*Less. III.*—What is the value of the first number in the lesson? Is the value of the letter of less value added to, or taken from, the value of the greater? In the second number, which letter is subtracted? How does the notation of the 18th number differ from that of the 15th? Which is the greater?

REVIEW.—What is notation? (6.) How are numbers expressed by the Roman notation? (7.) How many letters are required? (7.) What is the value of each when standing alone? (7., *a.*) What effect has the repetition of a letter? (7., *c.*) What is the effect of placing a letter of less value before one of a greater? (7., *d.*) After one of a greater? (7., *e.*) What effect has a dash when placed over a letter or combination of letters? (7., *f.*)

LESSON IV.

Prepare the following lesson by writing the numbers in the Roman notation on slate or paper, to be read as a class exercise :—

1. 14.
2. 25.
3. 18.
4. 13.
5. 24.
6. 36.
7. 41.
8. 37.
9. 28.
10. 25.
11. 27.
12. 46.
13. 38.
14. 67.
15. 57.
16. 34.
17. 20.
18. 64.
19. 72.
20. 27.
21. 974.
22. 681.
23. 873.
24. 387.
25. 976.
26. 794.
27. 683.
28. 862.
29. 431.
30. 474.
31. 3466.
32. 9418.
33. 8633.
34. 7964.
35. 8946.
36. 9678.
37. 4138.
38. 91667.
39. 97664.
40. 41776.

QUESTIONS.—*Less. IV.*—Write 13 on the black-board in Roman characters. How many letters are required? Are the letters of less value added to, or subtracted from those of a greater value? Can "XIII" be made to represent 13,000? How? [If the above exercises be not sufficient to give the class skill in writing and reading numbers in the Roman notation, prepare on the black-board, columns of numbers similar to those given, to be copied and recited in the same manner. Care should be taken not to pass from the simple numbers to those more complex so rapidly as to embarrass the pupils.]

LESSON V.

8. *The Arabic Notation* is generally used in arithmetic. It consists of ten characters, or figures, nine of which are called digits, and have each a simple and a local value. They are :—

0, *cipher.* *naught.* *zero.*
1, *one.*
2, *two.*
3, *three.*
4, *four.*
5, *five.*
6, *six.*
7, *seven.*
8, *eight.*
9, *nine.*

9. *The Simple value* of a figure is the one expressed by it when standing alone, or in the units' place of a number.

10. *The Local value* of a figure is that which depends upon the place which the figure occupies in a number; thus, in 37, the figure 3 denotes *thirty;* and in 904, 9 denotes *nine hundred.*

11. ***The Naught or Cipher*** (0) has no value, but is used to show that the place of a denomination is vacant; thus, in the number 6074, the place of hundreds being vacant it is occupied by a cipher.

QUESTIONS.—*Less. V.*—What is the meaning of "local"? What is the meaning of "vacant"? Of "occupied"? How many names has the cipher? By which name is it commonly called?

LESSON VI.

NUMERATION.

12. ***Numeration*** is the art of naming in order the places occupied by the figures which represent a number.

13. ***Reading numbers*** is the art of expressing their written value orally.

(*a.*) A PERIOD.

Period.		
Hundreds.	Tens.	Units.
0	0	0

(*b.*) TABLE.

6th Period.*			5th Period.			4th Period.			3d Period.			2nd Period.			1st Period.		
Quadrillions.			Trillions.			Billions.			Millions.			Thousands.			Units.		
Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

---

* From quadrillions the table may be extended to quintillions, sextillions, septillions, octillions, nonillions, decillions, undecillions, duodecillions, &c.

REVIEW.—What is the Roman notation? (7.) Why so called? What is its use? Of how many letters does it consist? (7., *a.*) Name them? Which letter has the greatest value? Which the least? Which letter stands for fifty? Which for ten? Which for one hundred? Which for five hundred? What effect has the repetition of a letter? (7., *c.*)

14. Every place in each period, not occupied by a digit, with the exception of the left-hand places of the left-hand period, must be filled with a cipher.

15. Every period in each number, not occupied by digits, must be filled with ciphers.

(*a*.) In writing numbers, periods are often separated by commas.

QUESTIONS.—*Less. VI.*—Name the places of a period. Of what is a number composed? How do we generally express a number on the black-board? What is meant by "orally"? What is a digit? Why called a digit? How many places in a period? What is the first place called? What is the second called? What is the third called? Is a cipher ever called a digit? Name the digits?

LESSON VII.

1. Write 4 hundreds and 2 units, commencing at the left hand, and read the number.

Write 3 tens and 7 units in the same manner.

Write 5 hundreds, 3 tens, and 2 units.

Write 4 tens and 6 units.

Write 5 hundreds and 4 units.

Write 7 hundreds and 2 tens.

MODEL OPERATION.

Units' Period.		
H.	T.	U.
4	0	2
	3	7
5	3	2
	4	6
5	0	4
7	3	0

2. Write 5 hundreds, 7 tens, and 2 units, and read the number.
Write 3 tens and 1 unit.
Write 5 hundreds and 6 units.

3. Write 6 units;
9 tens and 4 units;
6 hundreds and 7 units;
5 hundreds and 6 tens;
9 hundreds and 5 tens.

4. Write 7 hundreds and six tens;
7 tens and 3 units;
5 hundreds and 8 tens;
6 hundreds and five units;
3 tens and seven units.

5. Write 8 tens and 3 units;
7 hundreds and 6 tens;
3 tens and four units;
8 tens and 3 units;
4 hundreds.

6. Write 3 tens and 8 units;
Six tens and 3 units;
Five hundreds and four tens;
9 hundreds and 3 tens;
7 tens and 8 units.

1. Write 2 in each period, commencing at the left hand, and read the number.

Write 22 in each period, commencing at the left hand, and read the number.

Write 222 in each period, commencing at the left hand, and read the number.

Note for the Teacher.—The pupil should be required to prepare the following exercises on slate or paper, thus arranged, to be read as a class exercise. The mechanical execution of the work should also be inspected.

MODEL OPERATION.

Tr.			Bill.			Mill.			Th.			U.		
H	T	U	H	T	U	H	T	U	H	T	U	H	T	U
		2	0	0	2	0	0	2	0	0	2	0	0	2
	2	2	0	2	2	0	2	2	0	2	2	0	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

To be read.—Two *trillions*, two *billions*, two *millions*, two *thousand* and two.

Twenty-two *trillions*, twenty-two *billions*, twenty-two *millions*, twenty-two *thousand* and twenty-two.

Two hundred and twenty-two *trillions*, two hundred and twenty-two *billions*, two hundred and twenty-two *millions*, two hundred and twenty-two *thousand*, two hundred and twenty-two.

2. Write 3 in each period in the same manner, and read the number.

Write 33 in each period, and read the number.
Write 333 in each period, and read the number.

3. Write 4 in each period, and read the number.
Write 44 in each period, and read the number.
Write 444 in each period, and read the number.

4. Write 5 in each period, and read the number.
Write 55 in each period, and read the number.
Write 555 in each period, and read the number.

5. Write 6 in each period, and read the number.
Write 66 in each period, and read the number.
Write 666 in each period, and read the number.

6. Write 7 in each period, and read the number.
Write 77 in each period, and read the number.
Write 777 in each period, and read the number.

7. Write 8 in each period, and read the number.
Write 88 in each period, and read the number.
Write 888 in each period and read the number.

8. In the same manner write 9, 99, 999, and read them.

9. Write ten in each period, and read the number.

10. Write in each period all the numbers from 11 to 20 inclusive, and read them.

11. Write all the numbers from 20 to 30, and read them.

12. Write all the numbers from 30 to 40, and read them.

13. Write all the numbers from 40 to 50, and read them.

14. Write all the numbers from 50 to 60, and read them.

15. Write all the numbers from 60 to 70, and read them.

16. Write all the numbers from 70 to 80, and read them.

17. Write all the numbers from 80 to 90, and read them.

18. Write all the numbers from 90 to 100, and read them.

QUESTIONS.—*Less VII.*—Which place is vacant in the first number of the model? How many hundreds in the number? How many tens? How many units? How is it read? How many units in the third number? How many tens? How many hundreds? How is it read? In reading, what do we call the 3 tens? In reading, do we repeat the name of the units? In reading periods, do we name the units' period? Do we name all other periods?

LESSON VIII.

Prepare the following lesson by copying the numbers upon a slate or paper, to be read as a class exercise.

REVIEW.—What is the principal use of the Arabic notation? (8.) What is the simple value of a figure? (9.) What is the local value of a figure? (10.) What is the use of the cipher? (11.) What is numeration? (12.) What is reading numbers? (13.) Name the places of a period? (13., *a.*) Name the periods of the numeration table from right to left? (13., *b.*) From left to right? What must be done with the vacant places of a period? (14.) Is there any exception? (14.) What is to be done with the vacant periods of a number? (15.) How are periods usually separated? (15., *a.*)

(1.)	(2.)	(3.)	(4.)	(5.)
347	4137	90607	860788	60302
689	6103	38064	301243	40004
043	402	40130	413072	301301
706	8603	70216	608307	103103
438	3001	41867	918603	190093
721	6300	91300	100032	401000
401	7000	40004	100100	400002
410	8640	40101	201201	800402
102	3704	80302	202202	800888
201	9603	39012	300300	808880
302	4413	16006	300303	908908
203	9001	401011	400034	300020

LESSON IX.

(1.)	(2.)	(3.)	(4.)
4130468	4138030	9900099	37060306
9130408	4000300	8088088	90909090
6091300	4000030	7007007	83838383
7000000	4030000	7700770	40060006
8010120	4300000	1011010	90909090
7080006	3023023	1010101	87000064
4030038	2200220	5020403	80070041
4130607	2000002	4030478	48567083
5017063	9002000	6130107	41893071
9001900	9900009	4070306	90101309

QUESTIONS.—*Less. IX.*—How many periods in the numbers of the first exercise? *Three.* In the first number, are the places of each of the periods full? Does the number occupy three full periods? How many places are vacant in the number? With what is the place filled? How many periods in the fourth number? [If more exercises are required to make the pupils expert in reading numbers, similar exercises should be placed on the black-board to be read as a class exercise.]

LESSON X.

	MODEL OPERATION.
1. Write one hundred and five;	105
Three hundred and two;	302
Five hundred and nineteen;	519
One thousand and four;	1,004
Eight thousand seven hundred and one;	8,701
Forty thousand four hundred and six.	40,406

2. Write,
Six hundred and seventy-three;
Nine hundred and eighty-four;
Five hundred and two;
Seven hundred;
Nine hundred and eight;
Fifty-four.

3. Write,
Five hundred;
Five;
Fifty;
Five hundred and fifty;
Fifty-five;
Five tens and five units.

4. Write,
Five hundreds, and five tens
Seven hundreds, three tens, and 8;
9 hundreds and five tens;
7 units and 5 hundreds;
9 hundreds and 7 tens.

5. Write,
Four hundreds and 6 tens;
Nine tens and 2 units;
Seven hundreds and 6 tens;
Four tens;
Six hundred and 48;
Five hundreds and 39.

6. Write,
Six hundred and 9;
Four tens and 3 units;
Eight hundred and 3 units;
Five hundred and 8;
Three hundred and 2;
One hundred and 1.

7. Write,
One thousand;
Seven thousand 387;
Six thousand and 46;
Nine thousand and 5;
Three thousand 497;
Five thousand and 2.

8. Write,
Two thousands, three tens;
Four thousands, 5 units;
Seven thousands, 2 tens;
Nine thousands, 3 hundreds;
One thousand, 378;
Five thousands and seven tens.

9. Write,
Twenty thousand;
Thirty-three thousand;
Sixty-seven thousand;
Forty-eight thousand;
Seventy-eight thousand;
Ninety-nine thousand.

10. Write,
36 thousand 347;
47 thousand 296;
87 thousand 483;
57 thousand 683;
97 thousand 647;
37 thousand 406.

11. Write,
40 thousand 2 hundred and 47;
39 thousand and forty-five;
37 thousand and sixty-eight;
93 thousand and twelve;
43 thousand and 2.

12. Write,
30 thousand 6 hundred and 2;
48 thousand 2 hundred 42;
67 thousand 576;
Fifty thousand and 3;
Sixty thousand and 30;
Ninety-nine thousand and 9.

13. Write,
746 thousand;
371 thousand;
496 thousand;
978 thousand;
400 thousand;
301 thousand.

14. Write,
307 thousand 896,
507 thousand 839;
760 thousand 837;
867 thousand 387;
401 thousand 207.

15. Write,
Two hundred 37 thousand;
Five hundred 67 thousand;
Forty thousand and three;
Three hundred and two thousand;
Two hundred thousand and two;
Seven hundred and two thousand.

16. Write,
Four hundred and three thousand;
Seven hundred thousand;
Six hundred and eleven thousand;
Four hundred and two thousand;
Nine hundred and nine thousand;
Forty thousand.

17. Write,
Forty thousand and forty;
Four hundred thousand 400;
Fifty-five thousand and 55;
Five hundred thousand and 5;
Thirty-three thousand 387;
Forty thousand and eighty-seven.

18. Write,
1 million 243 thousand 342;
8 millions 371 thousand 678;
4 millions 27 thousand and 26;
8 millions 46 thousand 396;
9 millions 2 thousand 39;
1 million 20 thousand 20.

19. Write,
14 millions 2 thousand 27;
21 millions 46 thousand 43;
80 millions 39 thousand 53;
41 millions 700 thousand;
83 millions 301 thousand;
40 millions 2 thousand.

16. A Rule is a concise direction for solving problems, and may be deduced from an analysis.

***17.* Rule for Notation.**—I. *Begin at the left hand, and write the figures belonging to the highest period.*

II. *Write the hundreds, tens, and units, of each successive period in their proper order, observing to write a cipher in each vacant place.*

NOTE FOR THE TEACHER.—For additional exercises in writing and reading numbers, the Examples in addition may be used.

LESSON XI.

REDUCTION.

TABLE.

10 units	make	1 ten.
10 tens	"	1 hundred.
10 hundreds	"	1 thousand.
10 thousands	"	1 ten-thousand.
10 ten-thousands	"	1 hundred-thousand.
10 ten-hundred-thousands	"	1 million.
10 millions, &c.		

MENTAL EXERCISES.

1. How many units in 1 ten? in 3 tens? in 7 tens? in 9 tens?

2. How many tens in 3 hundred? in 5 hundred? in 6 hundred? in 4 hundred? in 8 hundred?

3. How many hundreds in 1 thousand? in 3 thousand? in 7 thousand? in 11 thousand? in 15 thousand?

4. How many thousands in 1 ten thousand? in 7 ten-thousands? in 4 ten-thousands? in 13 ten-thousands?

5. How many hundred-thousands in 1 million? in 7 millions? in 14 millions? in 42 millions?

6. How many units in 1 hundred? in 7 hundreds? in 4 hundreds? in 11 hundreds? in 27 hundreds; in 3 hundreds and 4 tens? in 7 hundreds and 6 tens?

7. How many units in 6 thousand? in 3 thousands 3 hundreds? in 4 thousands, 2 hundreds, and 2 units? in 7 thousands, 2 hundreds, 5 tens? in 42 thousands? in 25 hundreds? in 27 tens? in 347 thousands? in 46 hundreds?

8. How many tens in 20 units? in 30 units? in 60 units? in 100 units? in 400 units?

9. How many tens in 13 units? *Ans.—One ten and* 3 *units.*

10. How many tens in 17 units? in 47 units? in 63 units? in 78 units? in 468 units?

11. How many hundreds in 30 tens? in 60 tens? in 83 tens? in 67 tens? in 463 tens? in 361 units?

Ans.—3 *hundreds*, 6 *tens*, and 1 *unit.*

12. How many millions in 37 hundred-thousands? in 468 ten-thousands? in 8643 thousands? in 96834 hundreds? in 496836 tens? in 9643726 units?

13. How many million in 13 hundreds-thousands? Thousands in 374 tens? Hundreds in 678 units? Thousands in 4986 tens?

SECTION II.

ADDITION

LESSON I.

18. *The Sum* of two or more numbers is a number which contains a number of units equal to the numbers taken together.

19. *Addition* is the process of finding the sum of two or more numbers.

20. *The Sign of Addition* is a small cross; thus (+), and is called *plus*, which signifies more. When placed between two numbers, it denotes that they are to be added together; as 4+6 equals 10; to be read, 4 plus 6 equals 10.

21. *The Sign of Equality* is two short, parallel, horizontal lines; thus (=), and when placed between two numbers or quantities, it denotes that they are equal; as 8+6=14; to be read, 8 plus 6 equals 14.

22. *The Dollar sign* is an S crossed by two, short, perpendicular, parallel lines; thus ($), and when prefixed to numbers, denotes that they represent dollars; as $86; to be read, 86 dollars.

23. *An Axiom* is a self-evident truth.

(*a*.) The following axioms are used in addition:

1. *Things can be added to like things only.*

2. *If the same or equal numbers be added to equal numbers, the sums will be equal.*

3. *A number is equal to the sum of all its parts.*

Questions.—*Less. I.*—What do you understand by a unit? If 5 apples were in one pile and 6 in another, how could you find the number in both piles? What would the answer be called? What name would apply to the process of finding the sum? What is the meaning of "sign"? Is a number a figure? What is the difference? Can an axiom be proved? Can it be illustrated?

Review.—What is a rule? (16.) What is the rule for notation? (17.) What is the sum of two or more numbers? (18.) What is addition? (19.) What is the sign of addition? (20.) What is the sign of equality? (21.) What is the dollar sign? (22.) What is an axiom? (23.) What axioms are used in addition? (23., *a*.)

LESSON II.

PROBLEMS.

24. *A Problem* in Arithmetic is a question with conditions, proposed for solution.

25. *A Simple Problem* is a problem containing but one elementary question.

26. *A Complex Problem* is a problem involving two or more elementary questions.

27. *An Analysis* is a simplification of a complex problem by resolving it into elementary questions.

(*a.*) NOTE.—The term Analysis is, also, somewhat loosely applied to the solution of an Elementary Question by means of an Arithmetical formula; thus:—

ELEMENTARY QUESTION.—If 1 pound of sugar costs 11 cents, what will 6 pounds cost?

ARITHMETICAL FORMULA.—If 1 pound of sugar costs 11 cents, 6 pounds will cost 6 times 11 cents, which is 66 cents.

28. *An Analytical Step* is the statement of one of the elementary questions composing a complex problem.

29. *Order* of solving problems.

(*a.*) State the problem.
(*b.*) Give necessary definitions.
(*c.*) Analyze.*
(*d.*) Give conclusion.
(*e.*) Deduce a rule.
(*f.*) Prove.

LESSON III.

ANALYSIS OF ADDITION.

PROBLEM.—3896 + 4896 + 3894 + 437 = how many?

(*a.*) **Statement.**—It is required to find the sum of the following numbers: 3896; 4896; 3894; and 437.

(*b.*) **Definition.**—The *sum* of two or more numbers is a number which contains a number of units equal to the numbers taken together.

---

* NOTE FOR THE TEACHER.—After the pupil becomes expert in applying the *arithmetical* formula in the solution of an *elementary question*, it is sufficient that he be required to give the analytical steps.

MODEL OPERATION.

3896
4896
3894
437
13123 sum.

(*c.*) **Analysis.—1.** For convenience, the numbers are written so that units shall stand in the column of units, tens in the column of tens, and hundreds in the column of hundreds, &c.

2. For convenience, begin by adding the column of units thus: 7, 11, 17, 23; 23 units are equal to 3 units and 2 tens: write the units under the column of units, and add the tens to the column of tens: thus,

3. 2, 5, 14, 23, 32; 32 tens are equal to 2 tens and three hundreds: write the tens under the column of tens, and add the hundreds to the column of hundreds: thus,

4. 3, 7, 15, 23, 31; 31 hundreds are equal to one hundred and three thousands: write the hundred under the column of hundreds, and add the thousands to the column of thousands: thus,

5. 3, 6, 10, 13; 13 thousands are equal to 3 thousands and one ten thousand: write the 3 thousands under the column of thousands, and the 1 ten thousand in the place of ten thousands.

(*d.*) **Conclusion.**—Therefore, the sum of 3896, 4896, 3894, and 437 is thirteen thousand one hundred and twenty-three.

From the preceding we derive the following

(*e.*) **Rule.**—I. *Write the numbers so that units shall stand in the column of units, tens in the column of tens, &c.*

II. *Beginning with units, add each column separately, and if the sum be less than ten, write it under the column added.*

III. *If the sum of any column be ten or more, write the unit figure only, and add the others with those of the next column.*

IV. *Write the sum of the last column.*

(*f.*) **Proof.**—*Addition may be proved by adding the columns downwards in the same manner as upwards, and if the result is the same, it is supposed to be correct.*

LESSON IV.

The classes should be thoroughly questioned on the following:—

ADDITION TABLE.

11+1= 12	12+1= 13	13+1= 14	14+1= 15	15+1= 16
21+2= 23	22+2= 24	23+2= 25	24+2= 26	25+2= 27
31+3= 34	32+3= 35	33+3= 36	34+3= 37	35+3= 38
41+4= 45	42+4= 46	43+4= 47	44+4= 48	45+4= 49
51+5= 56	52+5= 57	53+5= 58	54+5= 59	55+5= 60
61+6= 67	62+6= 68	63+6= 69	64+6= 70	65+6= 71
71+7= 78	72+7= 79	73+7= 80	74+7= 81	75+7= 82
81+8= 89	82+8= 90	83+8= 91	84+8= 92	85+8= 93
91+9=100	92+9=101	93+9=102	94+9=103	95+9=104
16+1= 17	17+1= 18	18+1= 19	19+1= 20	20+1= 21
26+2= 28	27+2= 29	28+2= 30	29+2= 31	30+2= 32
36+3= 39	37+3= 40	38+3= 41	39+3= 42	40+3= 43
46+4= 50	47+4= 51	48+4= 52	49+4= 53	50+4= 54
56+5= 61	57+5= 62	58+5= 63	59+5= 64	60+5= 65
66+6= 72	67+6= 73	68+6= 74	69+6= 75	70+6= 76
76+7= 83	77+7= 84	78+7= 85	79+7= 86	80+7= 87
86+8= 94	87+8= 95	88+8= 96	89+8= 97	90+8= 98
96+9=105	97+9=106	98+9=107	99+9=108	100+9=109

NOTE FOR THE TEACHER.—For additional tables and exercises in the combination of numbers, see the "*Intellectual Arithmetic.*"

LESSON V.

30. BLACKBOARD EXERCISES.

1. What is the sum of each of the following columns?

4	6	9	7	0	1	3	8	2	5
3	3	3	3	3	3	3	3	3	3

NOTE.—The Teacher should copy the foregoing and similar exercises on the blackboard, and pointing to each of the columns in succession require a pupil or the class in concert to name the sum instantly. Thus, pointing to the right-hand column, the pupil should say, "Eight;" not, "Three and five are eight." Pointing to the next column, the pupil should say, at once, "Five;" not, "Three and two are five."

These exercises are very useful in giving the pupil facility in adding rapidly and accurately, and they should be practiced until this is gained.

[illegible] 1891

2.	2	0	4	6	9	5	7	1	3	8
	4	4	4	4	4	4	4	4	4	4
3.	3	8	1	7	5	9	6	4	0	2
	5	5	5	5	5	5	5	5	5	5
4.	7	1	8	3	2	0	4	6	9	5
	6	6	6	6	6	6	6	6	6	6
5.	5	7	1	8	3	2	0	4	6	9
	7	7	7	7	7	7	7	7	7	7
	[illegible]	14		15	[illegible]	[illegible]	7	13		
6.	1	7	5	9	8	3	2	0	4	6
	8	8	8	8	8	8	8	8	8	8
7.	8	3	2	0	4	6	1	7	5	9
	9	9	9	9	9	9	9	9	9	9
8.	5	6	8	9	4	7	6	9	8	5
	9	9	9	9	9	9	9	9	9	9
9.	5	2	7	3	8	0	4	6	1	4
	3	3	7	2	9	8	5	4	9	6
10.	3	2	8	4	6	2	1	5	8	7
	6	5	3	9	8	7	6	4	3	2
11.	8	3	5	7	9	2	6	4	2	1
	3	5	7	9	2	3	5	4	8	7

LESSON VI.

The Teacher should not fail to require the class, department, or entire school to spend at least five minutes daily for ONE YEAR in concert drill on the following column exercises in addition. In order to prevent habits of hesitation and inaccuracy, each column exercise should be THOROUGHLY MASTERED before attempting the next.

BLACKBOARD EXERCISES IN COLUMNS.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.
2	2	2	3	3	3	1	4	4	4	4	1	5	5	5	5	5	4
2	2	1	3	3	3	2	4	4	4	4	2	5	5	5	5	5	3
2	2	2	3	3	3	3	4	4	4	4	3	5	5	5	5	5	4
2	2	1	3	3	3	2	4	4	4	4	4	5	5	5	5	5	5
2	2	1	3	3	3	3	4	4	4	4	1	5	5	5	5	5	4
2	2	2	3	3	3	1	4	4	4	4	3	5	5	5	5	5	3
2	2	2	3	3	3	2	4	4	4	4	2	5	5	5	5	5	5
2	2	2	3	3	3	3	4	4	4	4	4	5	5	5	5	5	4
2	2	1	3	3	3	3	4	4	4	4	1	5	5	5	5	5	1
2	2	2	3	3	3	2	4	4	4	4	5	5	5	5	5	5	4
2	2	2	3	3	3	1	4	4	4	4	2	5	5	5	5	5	5
2	1	1	3	1	2	3	4	1	3	3	4	5	1	2	3	4	3

(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)
&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.
6	6	6	6	6	6	1	7	7	7	7	7	7	7	1	8	8	8
6	6	6	6	6	6	5	7	7	7	7	7	7	7	7	8	8	8
6	6	6	6	6	6	4	7	7	7	7	7	7	7	2	8	8	8
6	6	6	6	6	6	3	7	7	7	7	7	7	7	7	8	8	8
6	6	6	6	6	6	4	7	7	7	7	7	7	7	5	8	8	8
6	6	6	6	6	6	6	7	7	7	7	7	7	7	7	8	8	8
6	6	6	6	6	6	6	7	7	7	7	7	7	7	7	8	8	8
6	6	6	6	6	6	6	7	7	7	7	7	7	7	7	8	8	8
6	6	6	6	6	6	6	7	7	7	7	7	7	7	7	8	8	8
6	6	6	6	6	6	3	7	7	7	7	7	7	7	2	8	8	8
6	1	2	3	4	5	2	7	1	2	3	4	5	6	7	8	1	2

(37)	(38)	(39)	(40)	(41)	(42)	(43)	(44)	(45)	(46)	(47)	(48)	(49)	(50)	(51)	(52)
&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.
8	8	8	8	8	4	9	9	9	9	9	9	9	9	9	3
8	8	8	8	8	8	9	9	9	9	9	9	9	9	9	5
8	8	8	8	8	3	9	9	9	9	9	9	9	9	9	8
8	8	8	8	8	7	9	9	9	9	9	9	9	9	9	7
8	8	8	8	8	3	9	9	9	9	9	9	9	9	9	6
3	4	5	6	7	4	9	1	2	3	4	5	6	7	8	9

Note.—The above blackboard exercises in adding columns should be practised as follows.—Copy the exercise on the blackboard, and let a pupil or class in concert commence at the bottom and and, repeating only the result of each addition, thus: (Exercise 1st.) 2, 4, 6, 8, 10, 12, 14, &c., to fifty or more, as the teacher may judge best. When the first exercise is THOROUGHLY mastered, commence with the second in the same manner, thus MASTERING each exercise successively, until all are finished. It must not be understood that the exercises in this lesson are to be completed before the pupil proceeds; but they should, however, form a part of each recitation during his entire arithmetical course, if not finished sooner.

LESSON VII.

ABSTRACT PROBLEMS.

Note for the Teacher.—The following problems should be prepared from books WITHOUT ANSWERS, on slates or paper, by the pupils at their seats. They may be recited as follows:—

Those who are prepared on the first problem rise. One pupil is required to read *his* answer, and those whose answers agree with it, then sit down. If any remain standing, they also are required to read, after which the teacher announces the correct result from the key. Those who are wrong draw a line across the incorrect answer, and report at the close of the recitation. The other problems may be recited in the same way. The recitation of ten problems should only take from *five* to *seven* minutes. At each recitation, one or more problems should be placed on the blackboard, to be analyzed by the class after the model on page 15.

(1.)	(2.)	(3.)	(4.)	(5.)
1502134	45345	451245	404531	453454
5342435	30400	342412	142312	301523
3501243	43235	452154	532135	241342
5434514	54345	345435	445554	451353

(6.)	(7.)	(8.)	(9.)	(10.)	(11.)
410534	105345	45043	1243	3854	3311
254235	351423	13342	5081	5030	5422
523413	424532	54215	1005	1035	3580
304052	515343	32343	3254	2350	2105
350543	402534	14355	4305	1144	1044

QUESTIONS.—*Less. VII.*—In the first problem, what is the sum of the units' column? How many tens is it equal to? Why add the tens to the sum of the column of tens? Upon what axiom is it based?

LESSON VIII.

(12.)	(13.)	(14.)	(15.)	(16.)
5540	2041	153	210043214	321025421
2375	4323	5214	162153010	315543454
4334	124	1435	2153111	2302500
2117	2135	032	21532	42253201
4251	412	5334	10155	4354221
3532	343	1413	512345	544435

(17.)	(18.)	(19.)	(20.)	(21.)	(22.)
4324	1253	4004	512	555	4451154
4351	524	123	43	44	322244
3415	3415	5035	5	251	423453
2543	5324	1721	54	305	500432
4076	4188	9716	445	404	334455

LESSON IX.

(23.)	(24.)	(25.)	(26.)	(27.)
3454	1500	431015	241412	425500
4123	4450	251503	154213	324144
2545	2551	325344	334244	552154
3453	1334	423454	554235	432455
155	253	325014	434221	532445
2113	342	542151	544321	234354

REVIEW.—What is a problem? (24.) What is a simple problem? (25.) What is a complex problem? (26.) What is an analysis? (27.) How many elementary questions can there be in a simple problem? What is an analytical step? (28.)

(28.)	(29.)	(30.)	(31.)
35501124	453213	21454321	554431124
4302142	54410	1443224	43521453
1304215	103420	1003200	234211034
425430	55433	544322	434500110
214234	2344	543212	25334024
524543	43234	432123	534200
21141	145543	321234	1542

QUESTIONS.—*Less. IX.*—In adding the units' column of the 23d Ex., why do we add the column downward in the same manner as upward, when accuracy is required? Should the fingers ever be counted to assist in adding?

LESSON X.

PRACTICAL PROBLEMS.

PROBLEM.—A farmer had 324 cows in one field, 433 in another, and 242 in another; how many had he in all?

MODEL OPERATION.

324 cows in 1st field,
433 " 2d "
242 " 3d "
———
999 " all the fields.

ARITHMETICAL FORMULA.—If a farmer had 324 cows in one field, 433 in a second, 242 in a third, he had in all the fields the sum of these numbers, which is 999 cows; therefore, the farmer had 999 cows.

NOTE FOR THE TEACHER.—The following practical problems should be solved on slates or paper by the pupils, at their seats, from books WITHOUT ANSWERS, in the same way as the model. The problems should be read from the slates at recitation, without the aid of the books, omitting all the circumstances and conditions not affecting the result. Different pupils should

---

REVIEW.—Name the order of solving problems. (29.) What is the first step of the rule for addition? (29., *e.*, I.) Repeat the second step. (29., *e.*, II.) What is the fourth? How may a problem in addition be proved? (29., *f.*)

sometimes be called upon for parts of the analysis of the same problem. At each recitation, two or more problems should be placed upon the blackboard, the analysis of which should be subject to special criticism by the members of the class. Promptness and accuracy should be rigidly required. Not over *ten minutes* should be spent in the analysis of ten problems.

32. A boy had 324 apples in one basket, and 465 in another; how many had he in both?

33. A man bought 3416 sheep at one time, and 4343 at another time; how many did he buy in all?

34. There are 413 marbles in one bag, and 186 marbles in another; how many in both?

35. There are 903 apples in one barrel, and 94 in another; how many in both?

36. There are 53862 pigeons in one flock, and 24131 in another; how many are there in both?

37. If there are 413 peaches in one box, and 6141 in another; how many are there in both?

38. If there are 41832 words in one book, and 8116 in another; how many words in both?

39. There are 5312 men in one corps, and 1541 in another; how many men in both?

40. If there are 6378 houses in one city, and 1411 in another city; how many houses in both cities?

QUESTIONS.—*Less. X.*—What is uniting two or more numbers into one sum called? What is meant by "formula"? What by "conclusion"? What by "conditions"? How many questions in the 32d problem? Why are the problems in this lesson called "practical"?

LESSON XI.

41. In a battle there were 5387 shots fired by an army, and 1501 by the enemy; how many shots were fired by both?

42. In a certain orchard there are 6235 apple trees and 2653 pear trees; how many trees in the orchard?

43. A butcher sells at one time 26728 pounds of beef, and

REVIEW.—What is a problem? (24.) Illustrate each of the axioms used in addition. (23., *a.*) What is a unit? (1.) What is a number? (2.) What is the difference between a concrete and an abstract number? (3.)(4.) Of what does arithmetic treat? (5.)

at another time 61261 pounds; how many pounds does he sell in all?

44. If a grocer buys at one time 3778 pounds of sugar, at another, 4211 pounds; how many pounds does he buy in all?

45. If a man's farm is worth $354532, and his other property is worth $40456; what is the value of his property?

46. If a merchant sells at one time 56436 yards of calico, at another, 42422 yards of silk; how many yards does he sell in all?

47. There are 125746 feet of lumber in one pile, and 316423 in another; how many feet in both?

48. Two boats started from New York for Albany the same day: the first carried 245216 pounds of coal; the second, 153772 pounds: how many pounds did they both carry?

49. The purser of a California steamer received $48364 from the passengers for fare, and $31432 for carrying goods; how many dollars did he receive in all.

50. At a certain period, the white population of the United States amounted to about 26234517, and the colored, to about 3743371; what was the whole population of the United States?

LESSON XII.

51. A merchant bought 138646 pecks of Brazil-nuts in South America, 941242 pecks of pea-nuts in Florida; how many pecks of nuts did he buy in all?

52. The captain of a merchant ship from New York, bought 389764 pounds of cotton at Mobile, and stopping at Charleston on his return home, he bought 311225 pounds more; how many pounds did he buy in all?

53. A retired merchant invests his money in bank and railroad stocks; the amount of his bank stock is $4132635, his railroad stock amounts to $2854243; what is the amount of his property?

REVIEW.—What is the difference between the Roman and the Arabic notation? (7.)(8.) What is the use of each? Name the letters used in the Roman notation, and give the value of each. (7., *a.*) What is the effect of repeating a letter? (7., *c.*)

54. The captain of a whaling vessel went to the coast of Greenland for oil; the four first whales afforded 6172 gallons of oil, and the three next 3827 gallons; how much oil did they yield?

55. A broker receives yearly from commissions $17275, and from speculations $81423; what is the amount of his yearly income?

56. There are three bins of corn; the first contains 342 bushels, the second 323, and the third 314; how many bushels do they all contain?

57. In one cornfield there are 3103 hills, in a second 2444 hills, in a third 3321; how many hills in all the cornfields?

58. There are 545 pins on one paper, 231 on another, and 103 on another; how many pins are there on all?

59. There are 324 apples in one bag, 273 in another, and 101 in another; how many apples in all?

60. A farmer has 410 acres of land planted with corn, 327 acres planted with potatoes, and 261 planted with beans; how many acres has he planted in all?

Questions.—*Less. XII.*—In the 34th problem, what is meant by a "merchant-ship"? What is the meaning of "invests"? What is the meaning of "stocks"? [The teacher should explain the meanings of these terms by familiar illustrations. A definition is often as incomprehensible as the word defined; hence an illustration is far preferable.]

LESSON XIII.

61. If I have 103 sheep, and my father gives me 444, and I buy 321 more, how many sheep have I in all?

62. If I have 413 peach trees in one orchard, 34 in another, and 212 in another, how many trees have I in all?

63. There are three boxes containing marbles: the first contains 1582; the second, 4404; the third, 4002: how many marbles are there in all the boxes?

Review.—What is the effect of placing a letter before one of greater value? (7., *d.*) What is the effect of placing a letter after? (7., *e*) What effect has a dash, when placed over a letter or a combination of letters in the Roman notation? (7., *f.*)

64. If I have $141 in one bank, $636 in another, and $211 in another; how many dollars have I in all three banks?

65. If I sell 3 horses for $430, 3 cows for $334, and 55 sheep for $223; for how many dollars do I sell them all?

66. If there are 2483 sheets of paper in one pile, 1211 in another, and 2105 in a third; how many sheets are there in all?

67. I bought 3 barrels of nuts; the first contained 5391 nuts, the second 1413 the third 2030; how many nuts in all my barrels?

68. In the year 1859, I bought 98764 sheep; in the year 1860, 3181 sheep; in the year 1861, 1401 sheep: how many did I buy in all?

69. I bought 98764 sheep and sold 5371; I then bought 8648 more, and sold 1316; how many sheep did I sell in all?

70. I had two fields of melons; in the first I had 9764, from which 5324 were stolen; in the second I had 7896, from which 2543 were stolen: how many melons were stolen in all?

QUESTIONS.—*Less. XIII.*—Which numbers in the 68th example do not affect the result? In the solution is it necessary to mention the year? What question is asked in the 69th Ex.? Does the number bought affect the result? How might the problem have been written? What conditions should be omitted in the statement of the formula? Write the 70th problem on the black-board, retaining only the numbers and conditions which affect the result.

LESSON XIV.

71. A farmer sold 3 loads of hay. There were 1341 pounds in the first load, 2225 in the second, and in the third 2213 pounds; how many pounds in the three loads?

72. A man by his will gave his wife $12235, his daughter $2340, his son $2422; what was the amount of his bequests?

73. James Smith has three lots of wild land; the first contains 1352 acres, the second 3236 acres, the third 4310 acres; how many acres do the three lots contain?

REVIEW.—What is the Arabic notation? (8.) How many figures or characters are used? What is the simple value of a figure? (9.) What is the local value? (10.) What is the use of the cipher? (11.) What is numeration? (12.) What is reading numbers? (13.) Repeat the numeration table by periods from right to left. (13., *b.*)

74. A lumber merchant sold at one time 32225 feet of pine lumber, at another 41431 feet, at a third 24232; how many feet did he sell in all?

75. If a carriage wheel turns round 10432 times in going from New York to Poughkeepsie, and 15355 times in going from Poughkeepsie to Hudson, and 4112 times in going from Hudson to Albany; how many times will it turn round in going from New York to Albany?

76. If I owe A 13257 cents, and B 51311 cents, and C 21300 cents; how many do I owe them all?

77. There are three piles of lumber; the first contains 12321 feet, the second 111232 feet, the third 10017 feet; how many feet in the three piles?

78. If there are 12351 pints of wheat in one bin, 103125 in another, and 71213 in another; how many pints are there in all the bins?

79. How many persons are there in four townships, if there are 3125 in the first, 11172 in the second, 1501 in the third, and 10100 in the fourth?

80. How much freight is there on a train of three cars, if the first carries 115412 pounds, the second 171483 pounds, and the third 112100 pounds?

QUESTIONS.—*Less. XIV.*—Illustrate the meaning of "bequests." What is a "township"? Illustrate the meaning of "freight."

LESSON XV.

EXERCISES IN NOTATION AND ADDITION.

81. Add,

		MODEL OPERATION.
36 thousand 243,	=	36243
46 thousand 396,	=	46396
87 thousand 946,	=	87946
86 thousand 857,	=	86857
96 thousand and 7,	=	96007
417 thousand and 27,	=	417027
		770476

REVIEW.—What should be done with the vacant places of a period? (14.) What with vacant periods? (15.) How are the periods of a number usually separated? (15., *a.*) What is a rule? (16.) What is the rule for notation? (17.)

82. Add,

Eighty thousand and 7 ;
Forty thousand and forty;
Seventy thousand and twenty;
Fifty thousand and 5 ;
Ninety thousand and ninety ,
Four thousand and four.

83. Add,

Three hundred thousand and 6 ;
Four hundred thousand and 60 ;
Nine hundred thousand 600 ;
Eleven thousand and eleven ;
Fourteen thousand and nine ;
Seven hundred thousand 683.

84. Add,

Five hundred thousand and 32 ;
Six hundred thousand 438 ;
Two hundred thousand and 29 ;
Nine hundred thousand and 30 ;
Eight hundred thousand 284 ;
Twenty thousand, 384.

85. Add,

5 hundred and 38 thousand 224 ,
8 hundred and 29 thousand 720 ;
Eighty thousand and 9 ;
4 hundred thousand 560 ;
Nine hundred and 40 thousand and 12 ;
82 thousand and 36.

86. Add,

7 hundred and 50 thousand and 20 ;
8 hundred and 70 thousand 496 ;
Ten hundred thousand and 20 ;
99 thousand and fifty ;
Thirty thousand and 98 ;
7 hundred thousand 561.

87. Add,

350 thousand 880 ;
Sixty thousand and 56 ;
Four hundred and 99 thousand ;
22 thousand and 85 ;
456 thousand 790 ;
6 hundred thousand and 70.

QUESTION.—*Less. XV.*—[These problems may be prepared and recited in the same manner as the preceding problems in addition. The principles of numeration should be fully illustrated on the blackboard, and the class questioned thoroughly on the same.]

LESSON XVI.

88. Add,

3 millions, 387 thousand, and 24 ;
4 millions, 46 thousand, and 4 ;
9 millions, 8 thousand, and six ;
Twenty millions, 7 thousand, 720;
10 millions, 9 hundred thousand ;
5 millions, 248 thousand, and 6.

89. Add,

14 millions, 14 thousand, and 14 ;
Three millions, 760 thousand, 480 ;
8 millions, 20 thousand, and 2 ;
50 millions, 640 thousand ;
22 millions, 321 thousand, and 56 ;
8 millions, 777 thousand, 988.

90. Add,

Twenty-five millions, 927 thousand ;
14 millions, 26 thousand, 327 ;
32 millions, 591 thousand, and 33 ;
Forty millions, 800 thousand ;
8 millions, 700 thousand, and 7 ;
11 millions, 256 thousand, 277.

91. Add,

65 millions, 999 thousand, and 80 ;
8 millions, 2 hundred thousand ;
38 millions, 45 thousand, 240 ;
17 millions, 590 thousand ;
25 millions, 25 thousand, and 25 ;
20 millions, 444 thousand, 960.

92. Add,

4 hundred millions, 900 thousand ;
1 hundred millions, and 450 ;
7 hundred millions, 7 thousand and 9 ;
9 hundred millions, 489 thousand ;
3 hundred millions, 7 thousand and 60 ;
800 millions, 700 thousand and 19.

93. Add,

11 hundred millions and 920 ;
18 hundred millions ;
300 millions, 4 thousand, and 50 ;
5 hundred millions, 200 thousand ;
800 millions, 800 thousand ;
1 hundred millions 567.

94. Add,

400 millions, 9 thousand, and 30 ;
9 hundred millions and 99 ;
600 millions, 24 thousand ;
12 hundred millions 884 ;
300 millions, 350 thousand ;
Seven hundred millions 256.

95. Add,

12 hundred millions, 96 thousand ;
18 hundred millions and 27 ;
200 millions, 426 thousand 200 ;
500 millions, 956 thousand ;
Nine hundred millions ;
300 millions and fifty.

QUESTIONS.—*Less. XVI.*—How many periods do hundreds occupy? Hundred-thousands? Hundred-millions? How many places do units occupy? Tens? Hundreds? How many places make a period?

LESSON XVII.

96. Add,

Five hundred and 20 ;
Ninety-nine ;
3 thousand and 96 ;
Nine hundred ;
26 thousand 780 ;
1 million 480 thousand.

97. Add,

Thirty-six ;
9 thousand nine hundred ;
2 hundred and 28 thousand ;
4 millions 26 thousand 580 ;
500 millions, 4 thousand, and 10 ;
Four hundred and 65.

98. Add,

11 hundred millions ;
Ninety-eight ;
24 thousand 420 ;
7 millions 20 thousand ;
Fifty-eight ;
200 millions, 4 thousand, and 50.

99. Add,

Ten thousand and ten ;
11 millions 726 thousand ;
Five hundred and 90 ;
Seventy-eight ;
3 hundred millions 989 ;
66 thousand 251.

REVIEW.—What is the sum of two or more numbers? (18.) What is addition? (19.) What is the sign of addition? (20.) What is the sign of equality? (21.) What is the dollar sign? (22.) What is an axiom? (23.) Illustrate on the blackboard the axioms used in addition. (23., *a*.)

100. Add,
Four hundred and fifty;
720 thousand 787;
500 millions 66 thousand;
Sixty-nine;
400 millions, 228 thousand, and 90;
26 thousand and 10.

101. Add,
6 millions 84 thousand 291;
Nine hundred millions;
Twenty-nine;
Six hundred and fifty;
3 thousand and 80;
400 millions and 86.

102. Add,
Seven hundred and 90;
800 millions 596 thousand;
5 hundred thousand 970;
Eighty-eight;
3 millions, seven hundred;
26 thousand 777.

103. Add,
9 hundred thousand;
Forty-seven;
19 millions 19 thousand;
7 thousand and 47;
6 hundred millions and 59;
Nine hundred and sixty.

Questions.—*Less. XVII.*—How many periods and places do thousands occupy? Hundred-thousands? Ten-thousands? Millions? Ten-millions? Tens? Hundred-millions? Hundreds? Units?

LESSON XVIII.

104. Add,
Three billions, 769;
2 billions, 96 millions, and 60;
Ten billions, 520 thousand;
Seven billions, 428 millions;
1 billion, 25 millions, 756 thousand;
3 billions, 250 thousand, and 9.

105. Add,
Six billions, 32 millions;
7 billions, 650 thousand;
9 billions, 27 millions, and 58;
3 billions, 599 thousand, 298;
Eight billions, and twenty-nine;
1 billion, 300 millions, 450 thousand.

106. Add,
Four billions, 237 thousand;
3 billions, 650 millions, and 23;
6 billions, 13 millions, 28 thousand;
Two billions, 350 thousand, 960;
Seven billions, 890 millions;
Nine billions, 27 millions, and 48.

107. Add,
Three billions, fifty thousand, and 7;
1 billion, twenty-three millions;
9 billions, 999 millions, 999;
Four billions, 951 thousand;
8 billions, and seventy-two;
Six billions, 676 millions, 627.

108. Add,
Twenty-five billions, 679;
60 billions, 99 millions, and 76;
42 billions, 2 thousand, 451;
Thirty-three billions, and 75;
52 billions, 726 thousand, 250;
Eighty-nine billions, and 97.

109. Add,
68 billions, 420 millions;
Seventy-nine billions;
87 billions, 327 thousand;
10 billions, 24 millions;
Nineteen billions, and 68;
99 billions, 4 thousand, and 36.

110. Add,
20 billions, five hundred;
Ninety-six billions, 987;
33 billions, 777 millions;
Twenty-one billions;
17 billions, 560 thousand, 599;
26 billions, 37 millions, and 60.

111. Add,
29 billions, 327 millions, and 6;
Eleven billions, 426 thousand;
80 billions, 2 millions, and 59;
23 billions, 796 millions;
10 billions, and seventy-six;
14 billions, 17 thousand, and 17.

QUESTIONS.—*Less. XVIII.*—How many periods and places do billions occupy? Thousands? Units? Ten-thousands? Ten-millions? Ten-billions? Hundreds? Hundred-thousands? Hundred-millions? Hundred-billions?

LESSON XIX.

112. Add,
560 billions, 76 millions, and 98;
Three hundred and 27 billions;
891 billions, 7 hundred millions;
230 billions, 750 thousand, and 90;
777 billions, 757 millions, 767;
9 hundred and 56 billions, and 59.

113. Add,
Two hundred and 99 billions;
789 billions, 276 thousand;
3 hundred billions, 850 millions;
967 billions, 4 millions, 570 thousand;
275 billions, 121 millions, 469;
7 hundred billions, and 326.

114. Add,
596 billions, 329 thousand;
677 billions, 12 millions, 779;
Two hundred and nine billions;
1 hundred billions, 39 millions;
277 billions, 500 thousand, and 70;
333 billions, 421 millions, 469.

115. Add,
Nine hundred and 76 billions;
520 billions, 1 million, 6 thousand;
311 billions, 500 thousand, and 20;
269 billions, 37 millions, and 69;
397 billions, 42 thousand;
500 billions, 27 thousand, 376.

116. Add,
Four hundred and ninety;
2 billions, 700 millions;
Fifty-four;
36 billions, 4 millions, and 27;
700 millions, 4 hundred thousand;
Seven hundred and 98.

117. Add,
99 millions, 327 thousand, and 7;
42 billions, 26 millions;
9 hundred billions, and 50;
Three hundred;
Forty-two;
63 millions, 56 thousand, 900.

118. Add,
Fifty-six;
6 millions, 479;
26 billions, 700 thousand;
4 thousand, and twenty;
5 hundred billions, 9 millions, 768;
3 thousand and seventeen.

119. Add,
4 billions, 921 millions;
323 thousand, 560;
Ninety-nine;
769 millions, 426 thousand;
12 billions, and 42;
Four hundred and 90.

REVIEW.—What is a problem? (24.) What is a simple problem? (25.) What is a complex problem? (26.) What is an analysis (27.) Repeat an elementary question. (27., *a.*) Repeat an arithmetical formula. (27., *a.*) What is an analytical step? (28.)

LESSON XX.

PRACTICAL PROBLEMS.

120. A man dying left his property to his three children: to the eldest he left $671344; to the second he left $713472; to the youngest, $417134: how much was he worth?

121. There are three barrels of eggs: the first contains 675; the second, 1276; the third, 1575: how many eggs do the barrels contain?

122. Three ships sailed from New York to Liverpool laden with cotton: the first contained 671343 pounds; the second, 417634 pounds; and the third, 1734167 pounds: how many pounds did all the ships contain?

123. There were three batteries firing at a fort: the first fired 6783 shots; the second, 4637 shots; the third, 57163: how many were fired from the three batteries?

124. During an attack on a fort by an enemy's fleet, one vessel fired 577 shots at it, another 463, and another 960; how many shots were fired at the fort?

125. A farmer sold three loads of potatoes: the first contained 73764 potatoes; the second, 67164; and the third 173645: how many potatoes did all the loads contain?

126. Three ships sailed from Boston with potatoes: the first contained 5364 bushels; the second, 7321; and the third, 3463: how many bushels did all of them contain?

127. Gave 4366 cents for a watch, 76432 cents for a carriage, 125700 cents for a house; how many cents did they all cost?

128. In an orchard there are 137 cherry trees, 634 apple trees, 72 pear trees, 65 plum trees; how many trees in the orchard?

129. The owner sold the cherries for 3416 cents, the apples for 76737 cents, the pears for 17432 cents, and the plums for 13134 cents; how many cents did he receive for the fruit?

REVIEW.—What is the order of solving problems? (29.) Give the analysis of a problem in addition. (29., *e.*) Repeat the rule. (29., *e.*) Give the method of proof. (29., *f.*)

LESSON XXI.

130. The hind quarters of an ox weighed 375 pounds each; the two fore quarters weighed 621 pounds; the hide and tallow, 112 pounds: what was the entire weight?

131. A merchant bought three pieces of cloth: the first measured 2864 inches in length; the second, 1772 inches; the third, 2136 inches: how many inches in all the pieces?

132. He sold the first piece for 37644 cents, the second for 17634 cents, the third for 21361 cents; how many cents did he receive for all the pieces?

133. A merchant bought 1345 hogsheads of molasses for $34176, 231 barrels of sugar for $3463; he sold his sugar and molasses so as to gain $3176; for how much did he sell them?

134. A ship owner bought two vessels: for the first he gave $53643, for the second $38691. He then sold them so as to gain $23462; for how much did he sell them?

135. A man bought a lot for $13673, and built a house on it which cost $4346; he then sold it so as to gain $2463; how much did he receive for it?

136. A drover bought 1864 oxen for $53167, 921 cows for $21641, and sold them so as to gain $2643; for how much did he sell them?

137. Henry and James were playing games; Henry won in the first 43, in the second 57, and in the third 156: how many did he have when he finished?

138. John Smith's farm is worth $73467, he has money in a bank to the amount of $34174, and he has $7134 in gold; how much is he worth?

139. In the year 1840 there were 501763 persons in Maine, 274574 in New Hampshire, 737621 in Massachusetts; what was the entire number of people in these States?

QUESTIONS.—How many numbers in the 130th problem? Which are the important numbers in the 133d problem? Which numbers might be omitted without affecting the result? What is a bank?

LESSON XXII.

140. A man by will divides his property between his wife and three children: to his wife he gives forty-six thousand six hundred and seventy-three dollars; and to each of his children forty-three thousand three hundred and five dollars: what is the amount of his property?

141. There are three piles of boards: the first contains eighty-four thousand three hundred and fifty-seven feet, worth $637; the second contains forty-three thousand six hundred and thirty-four feet, worth $676; the third contains seventy-six thousand three hundred and seventy-four feet, worth $387: how many feet in the three piles?

142. A lumber merchant bought the first pile of 84357 feet for seven hundred and fifty-seven dollars, the second pile of 43634 feet for seven hundred and thirty dollars, and the third pile of 76374 feet for four hundred and eighty-four dollars; how much did he pay for all?

143. If the first pile of boards was worth $637, and the merchant paid one hundred and twenty dollars more than it was worth; and the second pile was worth six hundred and seventy-six dollars, and the merchant paid $114 more than it was worth; and the third pile was worth $384, and the merchant paid $100 more than it was worth; how much more did the merchant pay for all the boards than they were worth?

144. A cheese merchant bought of one man 86673 pounds of cheese for $8646, forty-six thousand seven hundred and eighty-three pounds of another for four thousand and eighty-three dollars, and ninety-six thousand seven hundred and forty-four pounds of another for six thousand and four dollars; how much did his cheese cost him.

LESSON XXIII.

145. There are three banks: the first has one hundred and eighty thousand and three dollars in gold, three millions seven thousand and seven dollars in bills; the second bank has

two hundred and four thousand and sixty dollars in gold, and nine hundred and thirty-seven thousand and five dollars in bills; the third bank has twenty thousand and twenty hundred dollars in gold, and one million three thousand and three hundred dollars in bills. How many dollars in gold in the banks? How many dollars in bills? How many dollars in all?

146. A man borrows at one time $378643, and then lends thirty-four thousand six hundred and seventy-eight dollars; at another time he borrows four hundred and sixty-seven thousand one hundred and thirty-four dollars, and lends $43467; at a third time he borrows one million eight hundred and seven thousand five hundred and nine dollars. How much does he borrow in all?

147. There are three flocks of pigeons: the first contains one billion two hundred and fifty thousand three hundred and five; the second, nine hundred and fifty thousand and five; the third, two billions twenty millions twenty thousand twenty hundred. How many pigeons in all the flocks?

148. One of Hoe's patent printing presses can print 26 thousand three hundred and eighty-six sheets in one hour; how many can it print in three hours?

149. If a man takes one hundred and fifty thousand four hundred and thirty-six steps in going from West Point to Albany, how many steps will he take in going three times as far?

150. If one vessel can carry two thousand four hundred and sixty-two tons of coal, how many tons can three vessels of the same size carry?

151. A man owned a farm worth sixteen thousand seven hundred and eight dollars; he sold it for two thousand seven hundred and sixteen dollars more than it was worth, and his father gave him sixty thousand and four dollars; what was the amount of his money?

QUESTIONS.—*Less. XXIII.*—How many questions in the 145th problem? Which are the numbers in the first? Which are the numbers in the second? Which are the numbers in the third? How many questions in the 146th problem? Which numbers could be omitted in the problem? Write it on the blackboard in the shortest way possible.

LESSON XXIV.

152. Add together 3876 barrels of flour, 3876 bushels of potatoes, 5306 barrels of flour, 5867 bushels of potatoes, 6784 bushels of potatoes, and 3463 barrels of flour. What is the amount?

153. Find the number of bushels in 6738 bushels of corn, 39768 bushels of wheat, 25672 bushels of oats, 52371 bushels of corn, 31264 bushels of oats, 7541 bushels of wheat, 38421 bushels of corn, 2854 bushels of wheat, and 15231 bushels of corn.

154. A man has one farm of 384 acres, which affords pasture for 183 head of cattle and 671 sheep; he has another of 287 acres, which affords pasture for 128 head of cattle and 432 sheep; and a third farm of 300 acres, which affords pasture for 75 head of cattle and 1032 sheep: how many acres do the farms contain?

155. By the census of 1850, the number of persons in Maine was 581793; in New Hampshire, 314574; in Connecticut, 371978; in Vermont, 314948. Required the whole number of persons in those States.

156. The number of persons in New York in 1850 was 3120921; in New Jersey, 490306; in Pennsylvania, 2312033; and in Delaware, 92085. What was the number of persons in the Middle States?

157. Required the number of persons in the following Southern States in 1850: there being in Virginia 1422797, in North Carolina 869349, in South Carolina 699398, in Georgia 906392, in Alabama 722756, in Mississippi 607651, and in Louisiana 518411.

158. The population of the Western States in 1850 was as follows: Tennessee, 1003210; Kentucky, 982828; Ohio, 989467; Indiana, 988866; Illinois, 851183; Missouri, 682702; Arkansas, 210000; and Michigan, 398267. What was their total population?

159. How many persons in the following Territories and in the District of Columbia in 1850: there being in Oregon 53873,

in New Mexico 61878, in Indian Territory 110878, and in the District of Columbia 52878 ?

160. A carpenter built a house for $2464, a barn for $496, and outhouses for $309 ; how much did he receive for building all ?

161. A merchant bought at auction 520 yards of broadcloth, 386 yards of muslin, 92 yards of flannel, and 156 yards of silk ; how many yards did he buy in all ?

162. A father divided his money among his four sons, giving each $2087 ; how much was he worth ?

163. Three persons put money in a bank ; the first put in $4780, the second $3042, the third $407 ; how much did all of them deposit ?

164. In a certain army there are 26840 infantry, 6370 cavalry, 3250 artillerymen, and 1070 engineers ; how many men in the army ?

165. By selling a farm for $34683, I lost $4876 ; what was the cost of the farm ?

QUESTIONS.—*Less. XXIV.*—In the 152d problem, can barrels and bushels be added together ? Why not ? How many questions in the problem ? How many questions in the 153d problem ? Can bushels of corn be added to bushels of oats to find the number of bushels ? What numbers can be omitted in the 154th problem ? Write the problem on the blackboard, omitting all unnecessary conditions. What is meant by "census" ? What is meant by "population" ? What is an "auction" ? What is meant by "infantry" ? "Artillerymen" ? "Engineers" ? In the 165th problem, did the farm cost less, or more, than the money received for it ?

LESSON XXV.

(166.)	(167.)	(168.)	(169.)	(170.)
1643041	4164237	5917541	8641	28291
6713890	2798735	2053263	2534	85432
4016783	1005672	4107329	9871	37561
9134167	3584579	4271826	1235	12429
9386432	2354602	5732651	4683	73541
2541285	9547207	8453798	3296	21763
5017385	2005391	6254321	5432	44228
6426214	5720910	9823546	2150	63427

(171.)	(172.)	(173.)	(174.)	(175.)
693854	2732654	27635201	257631	4251
421572	2114307	17123517	138425	1673
714090	1005224	20301621	486932	1075
232543	6192735	15763250	145723	4823
124239	3214365	75426735	928437	3425
615471	7243463	43652891	624915	6217
254345	1673241	58327965	843214	9824
829378	3259623	13235716	327421	8437

(176.)	(177.)	(178.)
2678352917483	257426451	4275356534
8435702196417	321143754	2143425417
5832462792251	129563217	6215143847
1734125812433	754437856	9823214675
5842765324154	625143253	2435867854

LESSON XXVI.

(179.)	(180.)	(181.)	(182.)
95432742	75842344	22892446	248925
58972361	63921217	37549281	164437
28547232	54112532	24756343	218944
68412750	11258437	81247542	184437
14326725	62437542	91324510	214865
92135721	12007324	37542674	721374
25653247	58293467	25423744	241563
14357116	12445376	25179636	751212

REVIEW.—What is a unit? (1.) What is a number? (2.) What is the difference between a concrete and an abstract number? (3.)(4.) Of what does arithmetic treat? (5.) What is notation? (6.) What is the difference between the Roman and the Arabic notation? (7.)(8.) What is the difference between the simple and the local value of figures? (9.)(10.) What is the use of the cipher? (11.) What is the difference between numerating and reading numbers? (12.)(13.)

(183.)	(184.)	(185.)	(186.)
86427354	14620354	33472684	24267854
28793421	84371647	19414375	92345426
37586724	10054282	92386524	86754234
84100754	21448672	25371421	25468421
32484267	98437207	91846009	94237544
98547542	16243275	12342274	91272105
18654261	12447810	58673245	63732450
17283445	25327456	13436423	12345726
41678341	38924345	56323872	31542175

(187.)	(188.)	(189.)	(190.)
357264321	24527463	17754234	487218364
082937454	17379180	64372001	857446213
244375632	23450644	24371044	325721445
134247854	18309725	61754299	257364284
319402161	43017516	17344321	327468321
214371840	12434897	27348624	244672325
123754216	63841154	12821435	463726384
844513740	32730014	62784327	982736437
724453996	68437700	98232748	725486734
173745711	43037101	17234451	992171663

QUESTIONS.—How many periods in the 183d problem? How many places does 25 occupy? Does 156? 430? Six thousand and 8? 26 thousand and 38? 437 thousand and 36? 1 million, 1 thousand, and one? 22 millions, 22 thousand, and 22? 999 millions, 999 thousand, and 999? What is the greatest number that can be expressed by 3 and 4? By 1, 2, and 3? By 2, 3, and 7? What is the greatest number that can be expressed by 3, 4, and 6? What is the least?

REVIEW.—What is done with vacant places in any period of a number? (14.) What is done with the vacant periods of a number? (15.) What is a rule? (16.) What is the rule for notation? (17.) What is the sum of two or more numbers? (18.) What is addition? (19.) What is the sign of addition? (20.) How is it used? What is the sign of equality? (21.) What is the use of the dollar sign? (22.) What is the answer called in addition? (18.) Write 28 on the blackboard in the Roman notation. Change it to twenty thousand and eight. Change it to 25003. To 10018. To 26002.

Martha

LESSON XXVII.

(191.)	(192.)	(193.)	(194.)
8468729	8426342	327254364	987586421
3785642	9833475	122756210	243728486
9118841	2931812	932457225	175384342
6327451	7543245	124372864	814267243
2134275	6237214	921443586	211724375
6222437	3274216	724984354	862375417
1845415	7521445	126237243	214342867
7524672	3274263	842586721	433216754
4421187	8542726	937548875	998374333

(195.)	(196.)	(197.)	(198.)
2573846211	2486723	243726	4263789
9148456732	9172415	345867	2437284
9837248465	2732416	542241	6342001
1783438464	8563243	283756	8001104
2793842630	7234457	854274	2230048
1002734585	1248863	637844	2961245
6344584348	7114263	243786	2186445
2438627210	1951120	272437	2843754
1238424750	2437243	485431	8684243
1638427892	6237299	632781	8297214

(199.)	(200.)	(201.)	(202.)
3572462784	2634275	58672724	2483745
8437567215	2542817	32184621	8612433
4821437426	2432784	24723867	2143754
8142612754	1612437	18434842	6192342
2614327842	1243245	27632412	9237584
1243726424	6213754	24375421	1963724
2486722483	1246417	41186727	2434375
2742344261	2437854	75834261	6274284
9872317242	6247212	24375424	3742648
6729842426	3865432	61753241	6842745

LESSON XXXIII.

(203.)	(204.)	(205.)	(206.)
12572432	586427843	627342	637542
62785417	126754234	145327	983245
25637842	923854789	542634	432754
12642783	264832455	219853	327426
24867375	587527621	642372	298252
58267443	342844655	983214	454200
27844250	724326832	642875	637230
21637242	987244867	210736	242757
37267214	238542673	432898	932481
42632088	437523431	442157	675231

(207.)	(208.)	(209.)	(210.)
3854276324	2437245	24991143	2483752
8915437263	6784213	42332434	9574623
9524375861	7158454	72893755	2484730
3725426784	2100310	12923846	9543622
2534692375	7502344	27529381	2437458
8142753746	6837251	44713241	3726143
9537426117	7528381	37585484	7244524
2734516271	4286799	64852734	1368537
9527374284	4384911	95862447	2437586

QUESTIONS.—How many periods in the 203d problem? Is each of the periods full? How many places in a period? At which hand do you begin to count in separating a number into periods? At which hand do you begin to write a number? At which hand do you begin to read a number? At which hand do you begin to numerate a number? What is the first step in addition? At which column do you begin to add? How are the numbers written? What do you write under the first column? What is meant by "carrying *one* for every ten"? How may addition be proved? In how many ways may addition be proved?

REVIEW.—What are the axioms in addition? (23, *a.*) Illustrate each at the black-board What is the difference between a simple and a complex problem? (25.) (26.) What is an analysis? (27.) What is an analytical step? (28.) Repeat the formula for addition. (30.)

LESSON XXXIV.

(211.)	(212.)	(213.)	(214.)
258673842	8437246372	25673	426327
953724541	9843841187	48307	934452
684372411	3216257221	67245	173744
753427545	1543274385	84324	246327
937426384	7238146732	72364	983741
253748237	9438114275	85345	125634
942738645	2435774146	24371	756114
238427423	2321552734	68424	324867
689923483	1981430073	37452	586729
427144348	2764321102	34167	375256
127291911	2942384662	24375	983754

(215.)	(216.)	(217.)	(218.)	(219.)	(220.)
3256	3572	7521	2437	2576	25672
4354	6891	6835	2561	3248	85742
2753	7584	4575	1671	4267	94386
8460	3271	2938	2805	8543	75243
7243	8456	4261	8431	2756	62984
8653	2758	1724	7264	3248	24572
7584	3321	8437	8543	1724	37586
9237	7564	6124	1724	3425	18437
2456	3752	8234	7524	7258	28434
9117	9874	1724	3654	1964	64279
2435	6234	8463	2431	3241	25732
6475	8472	3117	1725	2871	43864
2138	9972	2134	1632	4386	37290
7560	9176	6217	1372	2564	87263

QUESTIONS.—*Less. XXXIV.* How many periods in the numbers of the 211th problem? How many in the numbers of the 212th problem? [Dictation or other class exercises should often be introduced to enliven the recitation. The pupils should frequently be timed in their work in order to secure the greatest rapidity possible.] Twenty-five units are equal to how many units and tens? 28 tens are equal to what? 48 hundreds? 69 thousands? 78 ten-thousands? 120 thousands? 168 hundreds?

LESSON XXXV.

221. In the year 1854, James Brown bought 1302 sheep; in 1855, 1291 sheep; in 1857, 867 sheep; how many sheep did he purchase in all?

222. I bought at one time 385 sheep, and sold 281; at another time I bought 645, and sold 413; at another I bought 1007, and sold 913; how many sheep did I sell in all?

223. I had three fields of melons: from the first 285 were stolen, and there remained 1072; from the second, 1252 were stolen, and 624 remained; and from the third, 108 were stolen, and 265 remained; how many were stolen in all?

224. A man bought three barrels of nuts; the first contained 9876 nuts, the second contained 6285, the third contained 6423; how many nuts were in the barrels when he bought them?

225. There were three flocks of blackbirds; the first contained 38964, the second 48257, the third 896423. John fired into the first flock and frightened away 12851; from the second he frightened 13264; and from the third 123604. How many were there in all the flocks, before he fired into them?

226. On Monday, there were deposited in a certain bank $8964, and $3864 were drawn out of it; on Tuesday, $7526 were deposited and $3215 were drawn out; on Wednesday, $12900 were deposited, and $5270 were drawn out: how many dollars were deposited in the three days?

227. If William can count 5898 in one hour, how many can he count in nine hours?

228. There are 7 books, and each book contains 78987 words; how many words in all the books?

229. How many letters in all the books, supposing each book to contain 368926 letters?

QUESTIONS.—Which numbers are not important in the 221st problem? Which numbers and conditions can be omitted in the 222d and 223d problems without affecting the solution? What is meant by "depositing money in a bank"?

LESSON XXXVI.

230. A man paid for flour, $1655; for pork, $1478; for

beef, $1508; for sugar, $189; for cheese, $170; what did they all cost him?

231. I paid $1349 to one man, $961 to another; I have $429 left; how much had I at first?

232. I paid away $396 in gold, $5386 in silver, and had $397 left; how much had I at first?

233. I bought 730 bushels of potatoes, of which I sold 396 bushels; I then bought 4786 bushels more, of which I sold 387 bushels; how many bushels did I buy in all?

234. My father gave me $3968, and I earned $374; he then gave me an amount equal to the sum I earned; afterward I earned $4378; how much did my father give me? How much did I earn? How much did I have in all?

235. My father had $2380; he gave me $278, and to my sister $467; he then bought me a farm for $1008, and my sister a horse, for which he paid $347; how much did he give us?

236. With my money I bought a cow for $86, five sheep for $33, and had $159 left; how much did my father give me?

237. By selling a farm for $6479, the owner lost $2734; what did it cost at first?

238. A man sold 327 bushels of potatoes for $296, 486 bushels for $329, and 327 bushels for $211; this amount lacked $437 of what they cost him; what did they cost him?

239. A lady went shopping, and bought some calico for 96 cents, some lace for 70 cents, some pins for 20 cents, and had 380 cents left; how much had she when she started?

LESSON XXXVII.

240. The population of a certain village in the year 1861 was 1866, the next year it gained 421, the third year 923, the fourth year 340, and the fifth year 386; what was the number of inhabitants at the end of the fifth year?

241. A man started from New York and travelled 371 miles the first day, 463 the second day, and 468 the third day; how far was he then from New York?

242. Two boys started from the school-house, one went north 76 rods, and the other went south 125 rods; how far were they then apart?

243. John and Henry started from the same place; John ran north 25 rods the first minute, and 27 rods the next minute; Henry ran south 23 rods the first minute, and 24 rods the next minute. How far did John run north? How far did Henry run south? How far were they apart at the end of the second minute?

244. John stood in the school-house door and shot an arrow from his cross-bow 20 rods up the road; he wished a little boy to get it for him; how many rods was he obliged to travel to bring it to John?

245. He then shot two arrows; one up the road 17 rods, and the other down the road 22 rods; how far did the little boy have to go to bring them to John?

246. Henry lends $9683 to Thomas, $4637 to Samuel, $3463 to Theodore; and has $4678 remaining; how many dollars had he at first?

247. A man was 37 years old, when his son was born; if he live, how old will he be, when his son is 59 years old?

248. Marcus had $3876, and Oren had $3786 more than Marcus; how many dollars had Oren?

249. Kate has 17 roses, Mary has 11 more than Kate, Samuel has 18 dahlias more than Mary's roses, and William has 14 dahlias more than Samuel; how many dahlias has William?

250. John's book has 876 problems, James' book has 41 more than John's, Louise's book has 27 more than James', and Clara's book has 23 more than Louise's; how many problems has Clara's book?

251. Andrew's purse contains 397 cents, Caroline's contains 43 cents more than Andrew's, and Andrew's contains as many

---

REVIEW.—What is a unit? (1.) What is a number? (2.) What is a concrete number? (3.) What is an abstract number? (4.) Of what does arithmetic treat? (5.) What is notation? (6.) What is the use of the Roman notation? (7.) What is the use of the Arabic notation? (8.) What is the difference between the simple and the local value of a figure? (9.)(10.)

cents as Minnie's, lacking 27 cents; how many cents does Minnie's purse contain?

LESSON XXXVIII.*

(252.)	(253.)	(254.)	(255.)	(256.)
2154	2583	2583	2572	38572
1384	6487	7521	3645	61438
2572	2426	3269	8154	17243
3256	3851	7583	3542	86543
8445	1742	3325	9859	17432
2137	8143	8146	1723	19865
6042	2516	7543	6154	42394
1724	7243	2438	3421	17238
6123	2415	9124	1748	16251
8429	7584	6178	9813	92374
7548	2563	3211	2486	45861
6438	3742	1543	1725	12923
9217	9114	1785	8374	75428
1120	6285	6144	3257	19175
1632	7543	3054	8443	43867
2584	8624	7521	2764	24361
3752	1789	4288	2119	17237
1725	2438	7561	1784	35872
1263	6427	3385	3265	91174
1924	2743	7543	3257	62584
3017	8625	1725	2578	27364
2742	2113	6434	4321	25267
8243	6475	2175	1642	19354
2586	8743	1864	1937	21624
1724	6114	8575	8543	17273
3648	2524	2386	2785	25723
1924	1837	2125	6438	86434

---

* NOTE FOR THE TEACHER.—The art of adding columns of figures with readiness and accuracy requires practice, but it must be practice of the right kind. There are many accountants who have *practised* for years with but indifferent success, because their practice was *wrong;* they did not learn to take *leaps*, but touched at every step. In this connection we wish again to call the attention of the teacher to the black-board exercise in columns. (30.) Arrange columns of figures on the black-board, and require the pupils to repeat in concert the sum of each consecutive addition. This exercise will greatly reduce the liability to error. Another exercise which, while it gives variety, will secure rapidity, is to dictate for the class to copy, simultaneously on their slates, columns of figures containing only certain digits. At a certain signal, let all commence adding, and as soon as the work is done, let the pupils rise, the teacher announcing the time spent by each in performing the work.

LESSON XXXIV.

(257.)	(258.)	(259.)	(260.)	(261.)
2546	2572	1725	2729	1297
7237	3846	4621	8436	8436
2143	1725	2403	1248	2516
8672	1834	7114	7584	1744
9831	1217	8623	6116	8437
1557	5243	2144	8521	2161
2834	2136	6213	1424	7245
1540	1727	2771	7564	4237
4272	9236	5852	3244	1844
1934	4587	8144	1724	1612
6291	2914	7585	3861	2144
2842	1724	1937	1295	3743
7586	1643	2881	1754	2748
2737	1851	1744	2756	1644
1895	2142	6385	1834	8212
6425	8148	1219	4381	7543
1724	1967	2463	2746	2149
3861	2434	2445	2111	1678
1724	1615	1728	6425	1243
2730	2742	4213	1624	4276
2148	3846	2712	2761	1723
7264	1759	1954	3254	8645
3427	1240	6374	8621	8543
6131	1786	2144	1242	1275
2725	2572	5813	3721	8434
4219	1938	2774	4258	1937
2114	5427	7516	1617	2973
1857	1643	2112	8432	1654
2143	2534	7219	1240	2362
2462	8115	8441	1728	5385
9837	2574	8684	5842	7294

NOTE FOR THE TEACHER.—The extra time of recitation should be spent in drill on the black-board exercises.—See (30.)

LESSON XXXV.

NOTE.—The following problems in addition may be solved directly from the book without being copied. It is desirable that the pupils should have practice in adding numbers arranged in horizontal lines. The recitation may be conducted in the usual manner.

What is the value of each of the following expressions?

262. 257+6324+21+84376+201+7586+30+5+25.
263. 3542–75+100+6164+724+384672+17+8540+7.
264. 27543+85674+120+1742+3856+172+10+1854.
265. 4+2001+174+867243+43+186+1728+252+40.
266. 240+7516+1345+6724+384+177+2443+1724+6.
267. 78+1008+2438+1860051+75843+12446+105+8.
268. 1774–8+90+84372+27+84386+4482+9854.
269. 2278+423+86000+9843+25+672+48572+47.
270. 2572+324600+17438+17+9249+61854+19+1000.
271. 467+85243+7261+36+758421+7386+5723+26+7584+12.
272. 17214+16+20001+31457+728+186+4385+24+75+61+5.
273. 44811+1674+897485+47+29+365+42+1+201+758+864.
274. 2744+758436+1744+256184+27211+32654+177246.
275. 4417+256+78001+49+273+8+43867+483729+24+889.

NOTE FOR THE TEACHER.—Much time should be spent in dictating exercises in addition, as the pupils will learn to combine numbers much more readily by this means than by any other.

LESSON XXXVI.

276. 257+38654+38567+648+273856+7584326+8+9+5
277. 85726+727–3586+27434+386437+29+864+35+9+857.
278. 275+8643+2+10+758+59+242+666+75+866+754+8667.
279. 7546+92+8+85764+38+924+75864+8738+4266+29+4.
280. 2767+24413+164+2741+8544+67211+1+87+100+72+8.
281. 728+60+4294+9+6+72+1001+27+38+587+2+364+72.
282. 286+754+21147+38464+272+4484+2272+3+8456.
283. 386+42+299+8672+3+4429+67294+9873+8542.
284. 75854+246+72+849729+270+4372+80+600+729+10.
285. 1857+24875+16+72+90011+2738564+81+2+10057.
286. 2257+321+18566+75423+2438+111+67254.
287. 84+3275+674511+2738564+357+2344+7+642+1.
288. 3274+67324+17+23856+89+921+385762+29+811+1008.
289. 2564+8572+95+2054+36244+3867+923+86742+987+1.

LESSON XXXVII.

290. 3256+231+4285+101+268+785112+3758+261+8434.
291. 2564+34211+2687+95438+36421+194372+4458+25.
292. 382+64+9872+75856+2438526+8423+25+461+8.
293. 3857+256+8576+33+846+211+85438+67+2558+7.
294. 2174+6729+24457+8461+844+372+867+24110+84.
295. 3527+6113+84211+67523+21542+4822+6424.
296. 108+2122+925+643+10+845+74+616+2375+19.
297. 227+6114+382+75483+22405+62345+7296+824.
298. 1864+2994+6384+253+98+4+6854+21+8564.
299. 1263+784+3654+9+8758+6114+84345.
300. 875+21+66411+8350+611+28648+9238.
301. 98254+365421+344+6293+8564+2975.
302. 3254+7565+857+64+3874+48432+7538.
303. 9367+2243+8575+293+6421+3854+37.
304. 225×86492+4381+1728+9637+25843+74.
305. 22675+38729+6430+2146+17+4+853+41.
306. 43291+5843+22+611+25846+38542+11.

LESSON XXXVIII.

COMPOSITION OF PROBLEMS.*

Containing but one elementary question.

NOTE FOR THE TEACHER.—The composition of abstract and practical problems may be carried as far as the teacher may deem necessary to give skill and expertness in computation and analysis. The following suggestions and modes will assist in making these exercises exceedingly interesting, if properly varied and adapted to the capacity of the pupils by the ingenious teacher.

EXERCISE 1.—Let each pupil arrange on *one-fourth* of a page of foolscap, five abstract problems in addition, each problem to contain *ten* numbers, and each number not to exceed *five* figures.† After correctly solving the same, let him present them to the teacher with the answers on separate pieces of paper, and numbered to correspond. Let the teacher distribute these problems to the class *without* answers, by lot, to be solved by the pupils receiving them, and recited in the usual manner at the next recitation.

---

* NOTE.—The number and character of the problems should be adapted to the capacity and degree of advancement of the pupils.

† NOTE FOR THE TEACHER.—The teacher should not be discouraged because his pupils at first may not do this work to his satisfaction. To persevere will be to succeed.

Exercise 2.—Arrange in the same manner five problems after the following model, to be solved, distributed, and recited in the same manner as in the preceding exercise.

MODEL.

Add
46 millions 24 thousands and 46;
54 millions 4 thousands and 7;
Forty millions 8 thousands, 467;
300 millions 400 thousands;
Fifty-four thousand 3 hundred 84.

Exercise 3.—Compose five practical problems like those found in the preceding lessons in addition, to be distributed and recited in the same way as before.

SECTION III.

SUBTRACTION.

LESSON I.

31. *Subtraction* is the process of finding the difference between two numbers, or of taking a less number from a greater.

32. *The Minuend* is the number from which another number is to be subtracted.

33. *The Subtrahend* is the number to be subtracted.

34. *The Remainder* is the number left after taking a less number from a greater.

35. *The Difference* of two numbers is a number, such that if added to the less the sum will equal the greater.

36. *The Sign of Subtraction* is a short, horizontal line, thus: (—), and is called *minus*, meaning *less*. When it is placed between two numbers it denotes that the number at the right is to be taken from the number at the left; thus, 6—4=2, to be read, 6 minus 4 equals 2.

37. The Axioms used in Subtraction are,

1. *Things can be subtracted from like things only.*

2. *If the same or equal numbers be subtracted from equal numbers, the remainders will be equal.*

3. *If the same number be both added to and subtracted from another number, its value will not be changed.*

4. *If two numbers be equally increased or diminished, their difference will not be changed.*

LESSON II.

38. ANALYSIS OF SUBTRACTION.

(*a.*) **Problem.**—It is required to find the difference between 3897 and 2938.

MODEL OPERATION.

	Proof.
3897, minuend.	2938, subtrahend.
2938, subtrahend.	959, difference.
959, difference.	3897, a sum equal to the minuend.

(*b.*) **Analysis.—1.** For convenience, write the *subtrahend* under the *minuend*, so that units may stand under units, tens under tens, &c.

2. For convenience, begin at the units' column to subtract, thus:—

3. As 8 units can not be taken from 7 units, take 1 ten which equals 10 units, from the 9 tens; 10 units and 7 units equal 17 units; 8 units from 17 units leave 9 units to be written in the place of units.

4. 3 tens from 8 tens leave 5 tens to be written in the place of tens.

5. As 9 hundreds can not be taken from 8 hundreds, take 1 thousand which equals 10 hundreds, from the 3 thousands; 10 hundreds added to 8 hundreds equals 18 hundreds; 9 hundreds from 18 hundreds leave 9 hundreds to be written in the place of hundreds.

6. 2 thousands from 2 thousands leave no thousands.

(*c.*) **Conclusion.**—Therefore, the difference between 3897 and 2938 is 959.

(*d.*) **Rule.**—I. *Write the numbers so that the units of the subtrahend shall stand under the units of the minuend, the tens under the tens, etc.*

II. *Begin at the right hand and subtract each figure of the subtrahend from the figure above it, if it can be done, and write the result underneath.*

III. *If any figure of the subtrahend is greater than the corresponding figure of the minuend, add ten to the figure of the minuend before subtracting. In this case, before subtracting, make the figure over the next left-hand figure in the subtrahend,* ONE *less.*

(*e.*) **Proof.**—*Add the difference to the subtrahend, and if the sum equals the minuend, the work is correct.*

LESSON III.

(*a.*) The pupils should be thoroughly questioned on the following

SUBTRACTION TABLE.*

10-1= 9.	11-1=10.	12-1=11.	13-1=12.
20-2=18.	21-2=19.	22-2=20.	23-2=21.
30-3=27.	31-3=28.	32-3=29.	33-3=30.
40-4=36.	41-4=37.	42-4=38.	43-4=39.
50-5=45.	51-5=46.	52-5=47.	53-5=48.
60-6=54.	61-6=55.	62-6=56.	63-6=57.
70-7=63.	71-7=64.	72-7=65.	73-7=66.
80-8=72.	81-8=73.	82-8=74.	83-8=75.
90-9=81.	91-9=82.	92-9=83.	93-9=84.

---

* NOTE FOR THE TEACHER.—Portions of the table should be copied on the black-board *without* the answers; thus, 20—3, and the pupils drilled in concert in the same way as in the black-board exercises.

14-1=13.	15-1=14.	16-1=15.	17-1=16.
24-2=22.	25-2=23.	26-2=24.	27-2=25.
34-3=31.	35-3=32.	36-3=33.	37-3=34.
44-4=40.	45-4=41.	46-4=42.	47-4=43.
54-5=49.	55-5=50.	56-5=51.	57-5=52.
64-6=58.	65-6=59.	66-6=60.	67-6=61.
74-7=67.	75-7=68.	76-7=69.	77-7=70.
84-8=76.	85-8=77.	86-8=78.	87-8=79.
94-9=85.	95-9=86.	96-9=87.	97-9=88.
18-1=17.	19-1=18.	20-1=19.	21-1=20.
28-2=26.	29-2=27.	30-2=28.	31-2=29.
38-3=35.	39-3=36.	40-3=37.	41-3=38.
48-4=44.	49-4=45.	50-4=46.	51-4=47.
58-5=53.	59-5=54.	60-5=55.	61-5=56.
68-6=62.	69-6=63.	70-6=64.	71-6=65.
78-7=71.	79-7=72.	80-7=73.	81-7=74.
88-8=80.	89-8=81.	90-8=82.	91-8=83.
98-9=89.	99-9=90.	100-9=91.	101-9=92.

LESSON IV.

39. BLACK-BOARD EXERCISES.

What is the difference of the numbers in each of the following columns?

1.	7	2	4	7	6	9	5	8	3	6
	2	2	2	2	2	2	2	2	2	2
2.	8	4	7	6	9	5	8	3	9	7
	3	3	3	3	3	3	3	3	3	3
3.	9	4	7	6	9	5	8	5	7	5
	4	4	4	4	4	4	4	4	4	4

NOTE.—Let the teacher write the above examples upon the black-board, and pointing to each, require a pupil or the class in concert, to name the difference in each, instantly.

This should be repeated, until the pupil has thoroughly mastered these exercises.

4.	9	8	7	6	9	5	8	7	8	9
	5	5	5	5	5	5	5	5	5	5
5.	7	9	8	9	8	9	9	8	6	5
	6	6	6	7	7	8	9	3	2	2
6.	10	12	15	13	14	19	17	18	16	11
	3	3	3	3	3	3	3	3	3	3
7.	10	12	15	13	14	19	17	18	16	11
	4	4	4	4	4	4	4	4	4	4
8.	10	12	15	13	14	19	17	18	16	11
	5	5	5	5	5	5	5	5	5	5
9.	10	12	15	13	14	19	17	18	16	11
	6	6	6	6	6	6	6	6	6	6
10.	10	12	14	13	14	19	17	18	16	11
	7	7	7	7	7	7	7	7	7	7
11.	10	12	15	13	14	18	17	19	16	11
	8	8	8	8	8	8	8	8	8	8
12.	10	12	15	13	14	19	17	18	16	11
	9	9	9	9	9	9	9	9	9	9

NOTE FOR THE TEACHER.—For other and more comprehensive black-board exercises, see "Intellectual Arithmetic."

(*a.*) GENERAL EXERCISES IN SUBTRACTION.*

EXERCISE 1.—Count from ten to one, thus, 10, 9, 8, 7, &c.

2.—Count from 100 to ten, by tens, thus, 100, 90, 80, &c.

3.—Count from 100 to 5, by fives, thus, 100, 95, 90, 85, &c.

4.—Count from 100 to 2, by twos, thus, 100, 98, 96, 94, 92, 90, 88, &c.

5.—In the same manner count by threes, fours, fives, sixes, &c.

LESSON V.

ABSTRACT PROBLEMS.

What is the value of each of the following expressions?

1. 277777—139645.
2. 3777777—248976.
3. 4777777—3279685.
4. 577777—189788.
5. 888888—589678.
6. 3888887—1788999.
7. 888888—788889.
8. 999998—199999.
9. 999997—179908.
10. 1000000—999999.
11. 1000000—111111.
12. 1000000—222223.
13. 1006003—300700.
14. 1010101—521212.
15. 3070700—854267.
16. 300000—233334.
17. 2845167—1237959.
18. 35003500—29114711.

LESSON VI.

PRACTICAL PROBLEMS.

PROBLEM.—A boy had 97 marbles, and gave 24 to his playmate; how many had he remaining?

MODEL OPERATIONS.

(*a.*)

97 No. of marbles.
24 No. he gave away.
——
73 No. remaining.

(*b.*)

97—24=73. *Ans.*

---

* NOTE FOR THE TEACHER.—These exercises should generally be practised in concert.

REVIEW.—What is a unit? (1.) What is a number? (2.) What is a concrete number? (3.) What is an abstract number? (4.) Of what does arithmetic treat? (5.) What is notation? (6.) What is the difference between the Roman and the Arabic notation? (7.)(8.) Name the letters used in the Roman notation? (7., *a.*)

Formula.—If a boy had 97 marbles and gave 24 to his playmate, he must have had remaining the difference between these numbers, which is 73 marbles; therefore, he must have had 73 marbles remaining.

Mental problems for illustration.*—John has five apples less than James, and James has 15; how many has John?

19. Robert has 85 cents and William 53; how many cents has Robert more than William?

20. James caught 397 pigeons, and John caught 143; how many did James catch more than John?

21. Henry having 165 cents, gave 82 of them to his sister; how many had he left?

22. A man bought a yoke of oxen for $175, a wagon for $65; how much more did he give for the oxen than for the wagon?

23. A farmer has 536 sheep in one field and 305 in another; how many more had he in the first field than in the second?

24. A gentleman owns a farm worth $3397 and a store worth $1085; how much more is the farm worth than the store?

25. A gentleman gave for a house and lot $7947, for a cotton factory $9503; how much more did the factory cost than the house and lot?

26. Bought a factory for $9410 and a farm for $8964; what is the difference of their values?

27. A man bought some land for $9870, a tannery for $5630; how much more did the land cost than the tannery?

28. A merchant had 1987 yards of cloth, and sold 874 yards of it; how many yards had he remaining?

29. A speculator sold a quantity of flour for $7897, and by so doing gained $332; what did it cost him?

30. A man sold an estate for $9873, and gained $3896; how much did the estate cost him?

---

* Note for the Teacher.—Mental problems are introduced at the head of the lesson whenever they are thought necessary to illustrate any new combination of the elementary questions.

31. A gentleman having $57897, gave his eldest son $26805; how much had he remaining?

32. Henry has 97 marbles less than John, and John has 200; how many has Henry?

Questions.—*Less. VI.*—What is a factory? What is meant by the "difference of their values"? What is a tannery? What is a "speculator"? What is meant by "estate"?

LESSON VII.

Ment. Prob.—The sum of two numbers is 8; one of the numbers is 3; what is the other? The difference of two numbers is 5; the greater is 11; what is the less?

33. A farmer had 7012 sheep, and as many lambs lacking 749; how many lambs had he?

34. Joseph had 5312 cents, and Newton as many lacking 1896; how many cents had Newton?

35. The sum of two numbers is 348964, and one of the numbers is 34867; what is the other?

36. The sum of two numbers is 8783764, the greater of the numbers is 5874256; what is the less?

37. The sum of two numbers is 12578640, and the less number is 5423865; what is the greater?

38. Our national independence was declared in the year 1776; how many years from that period to the close of the war in 1815?

39. The war of the Revolution broke out in the year 1775; how many years to the breaking out of the great rebellion in 1861?

40. America was discovered in the year 1492; how many years from that period to the landing of the Pilgrims in 1620?

41. The mariner's compass was invented in Naples 190 years

Review.—What is the difference between the simple and the local value of a figure? (9.) (10.) What is the use of a cipher? (11.) What is numeration? (12.) What is reading numbers? (13.) With what are the vacant places and periods of a number filled? (14.) (15.)

before America was discovered, which was in 1492; in what year was the mariner's compass invented?

42. Bonaparte was declared emperor in the year 1804, at which time he was 35 years of age; in what year was he born?

43. Glass windows began to be used in private houses in England in the year 1180, which was 516 years after the discovery of glass by a monk; in what year was it discovered by the moderns?

44. Sir Isaac Newton was born in 1642, and died in 1727; how old was he when he died?

45. Musical characters were invented by Guido Arctino, in the year 1025, which was 34 years after the Arabic numerals were introduced into Europe by the Saracens; in what year were they introduced?

46. Gunpowder was invented by Schwartz, in the year 1330; how long was this before the invention of printing, which took place in the year 1441?

QUESTIONS.—*Less. VII.*—How is the sum of two numbers to be found? If the sum and one number are given, how is the other number found? How do we find the time between two dates? When the difference of two numbers and one of the numbers are given, how do we find the other?

LESSON VIII.

MENT. PROB.—What number is that which if taken from 6 will leave 4? If the remainder is 15 and the subtrahend 3, what is the minuend?

47. Supposing a man to have been 80 years old in 1854, in what year was he born?

48. There are two numbers whose difference is 8764; the greater number is 15287; what is the less?

49. What number is that which taken from 3794 will leave 865?

50. What number is that to which if you add 789, it will equal 6350?

51. The minuend is 98764, the subtrahend is 48765; what is the difference?

52. The sum of two numbers is 1408; the less 216; what is the greater?

53. The minuend is 8786438, the subtrahend is three millions four hundred and sixty; what is the remainder?

54. The minuend is nine hundred millions two hundred and thirty; the difference is seventy-eight millions four hundred and thirty-eight thousand six hundred and seventy-four; what is the subtrahend?

55. Find the difference between forty-eight millions and one, and six hundred and twenty thousand and four.

56. From three thousand two hundred and seventy-six, take two thousand one hundred and forty-three.

57. From eight hundred millions seven thousand and seven, take ninety millions ninety thousand and ninety.

Questions.—*Less. VIII.*—Which is the minuend in the 54th problem? Which is the difference? How is the subtrahend found? When the difference and minuend are given, how do you find the subtrahend? When the subtrahend and difference are given, how is the minuend found? See illustrations (39., *a.*).

LESSON IX.

Find the difference of each of the following expressions.—

58. 8903607—1726784.
59. 5876241—2879346.
60. 2100051—972644.
61. 1711110—882222.
62. 1200000—700001.
63. 5421100—3572114.
64. 200184—73595.
65. 1178542—164257.
66. 3854275—246723.
67. 2008572—1345875.
68. 10000001—8922234.
69. 12576425—5872644.
70. 10897246—3214845.
71. 7851674—5972911.
72. 1267425—354726.
73. 785426—242948.
74. 337292—99999.
75. 532444—88888.
76. 112222—12345.
77. 275644—157324.

Questions.—In the 58th problem, which number is the minuend? Which is the subtrahend? How many periods in the minuend? How many in the subtrahend? What sign is between the minuend and subtrahend? When the subtrahend is taken from the minuend, what is the number left called? Which is the greater number, the minuend or subtrahend?

Review.—What is a rule? (16.) What is the rule for notation? (17.) What is the sum of two or more numbers? (18.) What is the difference between two numbers? (35.) What is the difference between addition and subtraction? (19.) (31.) What is the sign of addition? (20.) What is the sign of subtraction? (36.)

LESSON X.

78. 1260325—745872.
79. 3729642—932191.
80. 4274385—3219674.
81. 1325943—449278.
82. 758543—84275.
83. 1484371—932465.
84. 486725—187295.
85. 5327935—435672.
86. 1724375—964327.
87. 2560091—2534000.
88. 8489267—3243875.
89. 1432467—582437.
90. 287964—178949.
91. 385427—199999.
92. 3111111—1222222.
93. 255555—166666.
94. 344444—255555.
95. 2900011—2811111.
96. 3246000—1231111.
97. 58764325—486732.

LESSON XI.

Ment. Prob.—What number is that from which if 5 is taken the remainder will be 8? What number is that to which if 7 is added the sum will be 11?

Five boys have 10 cents, 4 of them have 7 cents; how many has the fifth boy?

98. From eight billions and eight thousand, take one hundred and forty-seven millions thirty-eight thousand and five.

99. From ninety billions ninety millions eighty thousand and eighty, take eighty-four billions four hundred thousand six hundred and one.

100. From one hundred and four millions, take five millions seventy-eight thousand and five.

101. What number is that which if taken from eighty-nine millions seventy-eight thousand and three hundred, will leave seventeen millions and twenty-five?

102. What number is that to which if you add eighty-nine thousand and four hundred, the sum will be nine millions?

103. Four hundred millions and eighty-nine thousand less five thousand and five are how many?

104. If John was born in the year eighteen hundred

Review.—What is the sign of equality? (21.) What is the dollar sign? (22.) What are the axioms in addition? (23.) Illustrate each.

and fifteen, and James in the year eighteen hundred and twenty-eight; how much older is John than James?

105. A man was born in 1785; what was his age in 1833?

106. If I lend a man $8634, and he dies owing me $4678; how much has he paid me?

107. In five bags of money there are $10000, and in four of them there are $7003; how many dollars does the fifth contain?

108. In 1850 there were in the State of New York 3097394 inhabitants, and in the State of Pennsylvania 2311786 inhabitants; how many more inhabitants were there in New York than in Pennsylvania?

QUESTIONS.—*Less. XI.*—What name in subtraction should be given to $10000 in the 107th problem? What name should be given to $7003? What should the answer be called? What number in the 108th problem does not affect the result?

LESSON XII.

109. In 1850 there were in New York city 515547 inhabitants, in Philadelphia 340045; how many more inhabitants were there in New York than in Philadelphia?

110. A man died worth $12000; he left $7864 to his son and the remainder to his daughter; how much did he leave to his daughter?

111. A merchant bought 500 barrels of flour for $3586; he sold 250 barrels for $2004: how many remained on hand?

112. A lumber merchant bought 98764 feet of pine boards for $1470; he then sold 4386 feet for $1247; how many feet had he remaining?

113. A man bought $89876 worth of lumber; he sold $25072 worth to A, and $43287 worth to B; how much more in value did B buy than A?

114. A drover bought of A 4687 sheep and 910 oxen; he bought of B 1697 sheep and 467 oxen; how many more sheep did he purchase of A than of B?

115. A merchant bought a vessel for $41320 less $5840; he sold it for $38275, and received in payment 150 acres of land and the balance in cash; how much did he receive for the vessel?

116. 987643 less 437864 are how many?

117. A man lost $9742 less $3896; how much did he lose?

118. A man bought a ship for $25430 less $2542, and sold it for $37491; how much did the ship cost?

119. A merchant bought a quantity of goods worth $47324, and paid for them in money to the amount of $21372, and gave his note for the balance; what was the amount of his note?

QUESTIONS.—*Less. XII.*—What is there peculiar about the 115th problem? What is the meaning of "cost"? What is meant by "balance"? [The teacher should *illustrate* these terms rather than *define* them.]

LESSON XIII.

120. Two persons were 478 miles apart; they travel toward each other; how far are they apart when both have traveled 367 miles?

121. John and James stand 220 rods apart; John shoots an arrow 76 rods toward James; how far does the arrow strike from James?

122. James has $646 dollars, and Henry has as many lacking $379; how many dollars has Henry?

123. A man worth $46896 leaves $46732 to his wife and the rest to his son; what is the son's portion?

124. At an election 137864 votes were cast, of which the successful candidate received 86834; how many votes did the other candidate receive?

125. How many votes did one candidate receive more than the other?

126. Mary has 372 cents, and Susan has as many lacking 278; how many cents has Susan?

127. A has $3964 less than B, and B has $40374; how many dollars has A?

128. A farmer has 478 sheep in one field, and 67 more in a second than in the first; how many has he in the second?

129. A man sold some flour for $8964, and by so doing gained $386; how much did it cost him?

QUESTIONS.—*Less. XIII.*—What is meant by a "candidate"? In the 122d problem, which is the minuend? Which the subtrahend? Which the remainder? In the 128th problem, what is "478 sheep" called? What is "67" called? By what rule is this problem solved?

LESSON XIV.

130. From 386 thousand 378, take 41 thousand and 29.
131. From 328 millions, take 346 thousand and 27.
132. From 490 thousand 658, take 24 thousand 386.
133. From 7 millions 600 thousand, take 4 thousand and 2.
134. From 12 thousand and 29, take 8 thousand 620.
135. From 65 millions, take 650 thousand 980.
136. From nine hundred thousand, take five hundred and 50.
137. From 12 millions 12 hundred, take 400 thousand 4 hundred.
138. From 280 millions and 11, take 24 millions 650 thousand.
139. From twelve hundred and ninety, take seventy-five.
140. From 6 hundred millions, take 500 millions 5 hundred.
141. From 7 thousand and 20, take 9 hundred and 99.
142. From 1 million 100 thousand, take 98 thousand.
143. From 296 million 560, take 48 millions 7 thousand.
144. From 26 thousand and 77, take 4 thousand 980.
145. From 900 millions, take 3 millions 99 thousand.
146. From six hundred and 48, take one hundred and 70.
147. From 460 millions and 10, take 920 thousand 750.
148. From 1 million and 20, take 960 thousand.
149. From 756 million 3 thousand take 657 million and 8.
150. From six hundred and 90, take seventy-five.
151. From 42 thousand, take 9 thousand 768.

40. *A Parenthesis* ( ) denotes that all the numbers enclosed are to be taken as a whole, or that the same thing is to be done to them all. Thus 14—(3+4) signifies, that the sum of 3 and 4 is to be taken from 14; (13—4)—(5+2) signifies that the sum of 5 and 2 is to be taken from the difference of 13 and 4.

N. B.—NOTE FOR THE TEACHER.—The teacher should not fail by numerous illustrations to make the pupil familiar with the power and use of the parenthesis.

REVIEW.—What is a problem? (24.) What is a simple problem? (25.) What is a complex problem? (26.) What is an analysis? (27.) How are problems proved in addition? (29., *f.*) Illustrate the axioms used in addition. (23., *a.*)

LESSON XV.

COMPLEX PROBLEMS,

Which contain not more than two analytical steps.

PROBLEMS.—(*a.*) 1413—684—246=what ?
(*b.*) 1413—(684+246)=what ?

MODEL OPERATION.

(*a.*)

1413	minuend.
684	1st subtrahend.
729	1st remainder.
246	2nd subtrahend.
483	2nd rem., *Ans.*

(*b.*)

(1.)		(2.)	
684	1st subtrahend.	1413	minuend.
246	2nd "	930	sum of subtrs.
930	sum of subtrs.	483	rem., *Ans.*

What is the value of each of the following expressions?

152. 6843—462—413.
153. 4674—213—1413.
154. 9230—578—1600.
155. 7620—381—500.
156. 3410—231—660.
157. 9456—658—1260.
158. 6762—591—1001.
159. 4981—450—890.
160. 7769—588—960.
161. 4200—213—1913.
162. 2610—(386+400).
163. 1101—(972+129).
164. 3444—(760+421).
165. 3864—(461+900).
166. 2777—(346+560).
167. 6834—(463+246).
168. 7000—(500+980).
169. 3620—(941+462).
170. 7684—(661+320).
171. 4332—(260+1100).

LESSON XVI.

MISCELLANEOUS PRACTICAL PROBLEMS,

Which contain not more than two analytical steps.

Problem.—A boy had 378 cents; he gave 127 cents for a book and 40 cents for a slate; how many cents had he remaining?

MODEL OPERATION.

(*a.*)

1. {
378 no. cents boy had.
127 " " given for book.
———
251 " " left.
40 " " given for slate. } 2.
———
211 " " left. *Ans.*

(*b.*)

cts.
378—127—40=211 cts. *Ans.*

Anal. Steps.—1. *Find how many cents remained after paying for the book.*

2. *Find how many cents remained after paying for the slate.*

Note for the Teacher.—The teacher should require the solution of each step until the pupils become prompt and accurate.

172. A drover bought a drove of cattle for $6843; he paid at one time $462, at another time $413; how much did he still owe?

173. John had 4378 examples in his arithmetic; the first year he solved 2004, the next year 1976; how many had he yet to solve?

174. There was a field containing 3467 hills of potatoes; Henry dug 1367 hills the first week and 2003 hills the next week; how many hills still remained?

175. There were 3748 barrels of apples at the depot; 287 barrels were sent away on one train, and 467 on another train; how many barrels remained?

176. A man had 3467 pounds of iron; he sold 1764 pounds to one man and 976 pounds to another; how many pounds had he left?

177. A boy had 100 cents; he bought an arithmetic costing 63 cents, and a slate which cost 30 cents, and gave the remainder for a sponge; what did the sponge cost him?

178. A merchant bought 946 yards of calico; he sold 137

yards to one person, and 673 to another; how many yards had he remaining?

179. I lent a man $1374; he paid me at one time $387, at another $476; how much does he still owe me?

180. There is a farm consisting of 437 acres; 78 acres of which are planted with corn, and 156 are sown with wheat; the remainder is meadow; how many acres of meadow land are there?

181. Kate has a book containing 378 pages; on Monday she read 146 pages; on Tuesday, 98 pages; how many pages has she yet to read?

Questions.—*Less. XVI.*—Give the formula for the first step in the model problem? For the second step? [It is not desirable when the pupils can readily give the formulas to require them for every problem. They should, however, be required to point out each step of the analysis.]

LESSON XVII.

Ment. Prob.—A boy gave some nuts to his little brother and two sisters; to his brother he gave 12, to his elder sister 3 less than to his brother, and to his younger, 4 less than to the elder; how many nuts did the younger receive?

182. A man dying, left his property to his wife and two children; to his wife he left $89644, to his son $7384 less than to his wife, and to his daughter $1641 less than to his son; how much did the daughter receive?

183. A man had 832 sheep; on Monday he sold 199, on Tuesday he sold 259 more; how many had he then remaining?

184. A man invested his capital amounting to $89878 in New York and Erie R. R. stock; at one time he lost $48678, at another time he lost $38716; how much was he worth after his losses?

185 A man had $9864 in cash; he bought a building-lot for which he paid $3417, a house for which he paid $4187; how much cash had he left?

186. A boy went to a store with a ten-dollar bill; he bought an algebra, a history and a chemistry for $3; on his way

home, he met a boy with a little carriage; he immediately gave all the money he had left for it, excepting $2; how much did he give for the carriage?

187. A planter sold 5879 bales of cotton for $293950; he purchased a fine carriage and horses, costing $1098; he then gave the remainder of his money, excepting $121036, for a house in the city; what was the cost of the house?

188. Mr. E. Foster owned 6765 acres of wild land; he gave to his eldest brother 2196 acres, and to his youngest brother 1981 acres; how many acres remained?

189. A gentleman having $564 deposited $319 in a bank, and spent $86 of the remainder; how much then remained?

LESSON XVIII.

Problem.—A boy had 374 cents; he gave 37 cents for a dove, 90 cents for a rabbit, and 114 cents for a pair of pigeons; how many cents had he left?

MODEL OPERATIONS.

(*a.*)

37 cts., cost of dove.
90 " " rabbit.
114 " " pigeons.
——
241 cost of all.

374 cts. he had at first.
241 " spent.
——
133 cts. left, *Ans.*

(*b.*)

cts.
374—(37+90+114)=133 cts. *Ans.*

Anal. Steps.—**1.** *Find the total amount expended.*
2. *Find the amount remaining.*

190. Suppose a gentleman has an income of $3090; he pays for taxes $150, and for his other expenses $1071; how much of his income does he save?

191. A farmer having 554 sheep, sold 168, and then bought 896 more; how many had he then?

192. A drover having 97 cattle sold 25 of them, and then bought 89 more; how many had he then?

193. In a certain army there were 3512 men; in a battle 513 of them were killed, 627 were wounded, and 97 of them deserted; how many remained?

194. A farmer having 625 bushels of grain sold to A 97 bushels, to B 127, to C 197, and gave 110 bushels to the poor; how many bushels had he remaining?

195. A man traveled by railroad 372 miles, and started to return on foot; on the first day he traveled 35 miles, on the second day 43, on the third 44, on the fourth day he took the cars and came home; how many miles did he travel on the last day?

196. A man left an estate of $370129 to his wife and two children as follows: to his son $139468, to his daughter $98579, and to his wife the remainder; how much did his wife receive?

197. A man invested $37000 in a ship; the first year he made $997; the second year he made $547; and the third year by an accident at sea he lost $5782; did he make or lose by the operation, and how much?

198. A merchant having 78967 barrels of flour sold 796 barrels to A, 1473 barrels to B, 7810 barrels to C, 10847 barrels to D, and the remainder to E; how many barrels did E buy?

QUESTIONS.—*Less. XVIII.*—What is meant by "income"? What are the steps of the 198th problem? Which is solved by the formula of addition? Which by the formula of subtraction?

LESSON XIX.

199. Four persons propose purchasing a factory valued at $75960; A is to pay $5896, B $10847, C $27982, and D the remainder; how much is D to pay?

200. A man owing $379 paid at one time $37, at another time $84, at another $23, at another $84, and at another time $121; how much did he then owe?

201. A man divides his fortune of $2543 among his four sons, James, John, Harry, and Charles; he gives John $504, James $600, and Harry $725; how much remains for Charles?

202. My yearly income is $10000; if I pay $275 for rent, $220 for fuel, $75 to the doctor, $3675 for my other expenses, how much shall I have left at the end of the year?

203. A merchant owes $450120, and has property as follows: bank stock valued at $350000, western lands valued at $225000, furniture worth $400, and a stock of goods worth $96000; how much is he worth?

204. If a man's income is $3467 a year, and he spends $269 for clothing, $467 for house rent, $879 for provision, and $146 for traveling expenses; how much does he have left at the end of the year?

205. If I agree to pay a man $36 for plowing 25 acres of land, $200 for fencing it, and $150 for cultivating it; how much shall I owe him after paying him $331?

206. A merchant bought 83 hogsheads of sugar for $4312; he paid $431 for freight, and then sold them for $168 less than they cost him, including freight; how much did he receive for them?

207. If I buy 489 oranges for 912 cents, and sell at one time 125 for 186 cents, and at another time 125 oranges for 199 cents, how many oranges shall I have left?

208. A farmer having 450 sheep sold 124 at one time, and 96 at another; how many had he remaining?

LESSON XX.

Problems.—(*a.*) $4678 - 467 + 374 =$ what?
(*b.*) $4678 - (467 - 374) =$ what?

MODEL OPERATION.

(*a.*)

$$\begin{array}{r} 4678 \\ 467 \\ \hline 4211 \\ 374 \\ \hline 4585 \end{array}$$
Ans.

$$\begin{array}{r} 467 \\ 374 \\ \hline 93 \end{array}$$

(*b.*)

$$\begin{array}{r} 4678 \\ 93 \\ \hline 4585 \end{array}$$
Ans.

What is the value of each of the following expressions?

209. 5698—361+1428.
210. 4720—1159+3026.
211. 9620—333+1986.
212. 9786—8889+769.
213. 6602—438+796.
214. 9800—201+9860.
215. 7886—650+989.
216. 5561—255+3400.
217. 8989—743+9870.
218. 2010—988+400.
219. 4683—(4673—246).
220. 3976—(3860—420).
221. 2113—(1765—961).
222. 3898—(3762—556).
223. 4678—(2796—998).
224. 7868—(6600—456).
225. 9876—(8898—366).
226. 4516—(3716—201).
227. 5565—(4208—396).
228. 6854—(3201—469).

LESSON XXI.

229. If a man's income is $175 per month, and he pays $25 for rent, $44 for provisions, and $18 for other expenses, how much will he have left?

230. A man gave his note for $3245; he paid at one time $780, and at another time $484; how much remained unpaid?

231. A man paid $140 for a horse and $165 for a carriage; he afterwards sold them for $300; did he gain or lose, and how much?

232. A flour merchant having 700 barrels of flour on hand sold 278 barrels to one man and 142 to another; how many barrels had he left?

233. Three men bought a farm for $9840; the first paid $2672, the second paid $3089, and the third the remainder; how much did the third pay?

234. A man bought a house for $1500, and having expended $315 for repairs, sold it for $2000; how much was his gain?

235. A man deposited $5000 in a bank; he drew out at one

Review.—What is subtraction? (31.) What is the minuend? (32.) What is the subtrahend? (33.) What is the remainder? (34.) What is the difference? (35.) What is the sign of subtraction? (36.) What is its use? Illustrate the axioms used in subtraction. (37., *a.*) Analyze a problem in subtraction at the blackboard. (38., *b.*) Give a rule. (38., *d.*) Give method of proof. (38., *e.*) What is the use of the parenthesis? (40.)

time $2356, at another $1242, and at another $737; how much had he remaining in bank?

236. Borrowed of my neighbor at one time $680, at another $910, and at another $218; I paid him at one time $1309; how much do I still owe him?

237. A merchant buys 245 yards of cloth of one person, 125 of another, and 1183 of another, and then sells 803 yards; how many yards has he remaining?

LESSON XXII.

238. A farmer bought one farm for $12250; he bought a second for $15342 lacking $6340; how much more did he pay for the first than for the second?

239. A drover bought a drove of cattle of one farmer for $8641, and a drove of a second farmer for $9876 lacking $1896; how much more did he pay for the first than for the second?

240. Two boys, John and Harry, went to gather chestnuts; John gathered 864 gills, and Harry 986 gills lacking 34 gills; how many more chestnuts did one gather than the other?

241. What is the difference between 8878386 and 89643 less 41863?

242. A man asked his neighbor where he was going with his five hundred turkeys; he replied that he had not five hundred, but that if some one would give him 19, he would then have five hundred; how many had he?

243. A boy had 33 cents given him to buy a slate and sponge; he paid 9 cents for his sponge and his slate cost 30; how much did he owe for the slate?

244. A man had $18764; he bought a store for which he paid $9978, and goods for which he paid $27466; how much did he owe for the goods?

245. A man owns three farms: the first contains 864 acres and is worth $39464; the second contains 396 acres, and is worth $18043; the third is worth as many dollars as the difference in value of the first and second farms: what is the value of the third farm?

LESSON XXIII.

246. In 1821 there were imported into the United States 21273659 pounds of coffee, and in 1839 106696992 pounds were imported; what was the increase?

247. According to the census of 1840, 11853507 bushels of wheat were raised in the State of New York, and 13029756 bushels were raised in Pennsylvania; how many more bushels were raised in the latter State than in the former?

248. How many more inhabitants were there in the city of New York than in Boston, in 1840, there being by census 312710 inhabitants in the former, and 93383 in the latter city?

249. In 1830 the number of inhabitants in Bradford was 1856, and in 1840 it was 2222; what was the increase?

250. If it costs a man $2 per day to support his family, and he receives $5 per day for his work, how much can he save in one week?

251. The city of New York owed $42000000 and Boston owed $1698232; how much more did New York owe than Boston?

252. A merchant bought 500 barrels of flour for $3500; he sold 250 barrels for $2000; how many barrels has he on hand?

253. At a certain period there were 4338472 children in the United States, between the ages of 5 and 15 years; of this number, 2477676 were in schools; how many did not attend school?

254. The circulation of the blood was discovered in the year 1616; how many years to 1855?

255. If I lend a man $1565, and he should die owing me $450, how much would he have paid me?

LESSON XXIV.

256. If there are 60 minutes in an hour, how many minutes are there between 7 minutes after 9 o'clock and 2 minutes before 10 o'clock?

257. If there are 60 minutes in an hour, how many minutes are there between 8 minutes after 12 M. and 1 o'clock P. M.?

258. If there are 30 days in June, how many days will elapse between the 7th of June and the 23d of the same month?

259. A man bought a span of horses and a wagon for $936; he then sold the wagon for $140, and the horses for $708; how much did he lose by the operation?

260. Having in my possession $9860, I wish to know how much I must add to this amount to be able to purchase a farm valued at $14642 and yet save $576 for other purposes?

261. A boy bought a sled for 28 cents, and gave 14 cents for having it repaired; he sold it for 40 cents; did he gain or lose by the operation, and how much?

262. One man travels 67 miles in a day, another man follows at the rate of 42 miles in a day; if they both start from the same place at the same time, how far apart will they be at the close of the day?

263. A starts from Boston on Monday morning, and travels at the rate of 40 miles per day; B starts from the same place on Tuesday morning, and follows at the rate of 70 miles a day; how far will they be apart on Tuesday night?

264. A person borrowed of his neighbor at one time $355, at another time $673, at another time $403; he paid $977; how much did he then owe?

265. The difference of two numbers is 9086, the less is 9806; what is the greater?

LESSON XXV.

266. The difference of two numbers is 5001, the greater is 9009; what is the less?

267. 987641 added to a certain number makes 9876432; what is the number?

268. 4895632 subtracted from a certain number leaves a remainder of 2351420; what is that number?

269. The difference of two numbers is 31208, the greater is 54607; what is the sum of the numbers?

270. There is a farm consisting of 732 acres, 25 of which are planted with corn and potatoes, 197 are sown with wheat,

156 with oats, 97 with rye, 199 are pastured, and the remainder is meadow; how many acres of meadow-land are there?

271. A lady having $312 paid for a bonnet $20, for a shawl $75, for a silk dress $97, and for some delaine $83; how much money had she remaining?

272. A farmer's yearly income was $1710; he paid for repairing his house $342; for farming utensils $110; for hired help $411; how much had he remaining?

273. A farmer having 749 sheep sold 136 sheep to A, 128 to B, 297 to C, and 119 to D; how many had he remaining?

LESSON XXVI.

Perform the operations indicated in each of the following problems.

274. 3416—3246+2413=what?
275. 4167+4132—4167—741=what?
276. 2426—1619+2543=what?
277. 3654+8426—6536—3003=what?
278. (2685—1036)—560+1940=what?
279. (4321+261)+1436—4563=what?
280. 5672+(256+498)+596=what?
281. 3211+(5679—4208)—3101=what?
282. 4620—1426+(3561—428)=what?
283. 5400—3690+(400—350)=what?
284. 9374—(413+267+934)=what?
285. (4360—4020+3500)—900=what?
286. 4629+(4261—320—1002)=what?
287. 5678+(1376—598+945)=what?
288. (7432+2038—1560)—2360=what?

NOTE.—Should the above problems be found too difficult, they may be omitted until a review of the fundamental rules.

To the Teacher.

The subjects of notation, addition, and subtraction are intended to furnish a course of study sufficient for six months. Each pupil's daily lesson should average at least *ten* problems.

At *four*[*] lessons a week this will enable the class to perform the work given in the book, in fifteen weeks, leaving several weeks to be devoted to a special review of the subjects which the class have studied. Should additional problems be required, they may be supplied by composition. Special and frequent drill in the black-board exercises will greatly facilitate the pupil's progress.

LESSON XXVII.

COMPOSITION OF PROBLEMS.

Require each pupil to prepare five problems, after the following models:—

1. 34+648—264=what?
2. 24+86+34—346=what?
3. 8964—(413—64)=what?
4. 9167—(413+24+37)=what?
5. (6713+413+24)—413+64=what?
6. (434+3+4+6)—463—444=what?

Require each member of the class to compose five practical problems containing but two elementary questions. These problems should be recited in the same way as those given in addition.

TEST QUESTIONS FOR EXAMINATION.

1. What is a unit? Name a unit. Is a book a unit? A horse? A house?

2. What is a number? Write a number on the black-board. Show a number of books. Can there be numbers of different kinds? Give an example.

3. What is a concrete number? Write a denominate number on the black-board. Name five denominate numbers of different kinds. Name five concrete numbers of the same kind.

4. What is an abstract number? Name an abstract number. What is the difference between a concrete and an abstract number? Name three abstract numbers. Can one abstract number be greater than another? Write an abstract number on the black-board. Write a concrete number on the black-board. What is the difference between a number and a unit? Is a unit always a number? Is a number always a unit? Give examples on the black-board.

5. Of what does arithmetic treat?

6. What is notation? Illustrate the meaning of notation. Give an example of each way. Which is the most convenient? Why? Which is chiefly used in arithmetic? Why?

---

[*] NOTE.—It is desirable that one day in the week be spent in a weekly review of the principles over which the class have passed. The *review questions* will suggest the method.

7. What is the Roman notation? Why called Roman? What characters are used in the Roman notation? How many? What is the value of each? Which has the greatest value? Which the least? What is the effect of repeating a letter? What is the effect of placing a letter at the left of one of greater value? What of placing a letter at the right of one of greater value? When a dash is placed over a letter or combination of letters, how is the value affected? Write forty-eight on the black-board in Roman characters. Change it to forty-eight thousand. To forty thousand and eight. Ten thousand fifty-eight. Forty-five thousand and three. Forty-seven thousand and one.

8. What is the Arabic notation? Why so called? What characters are used in the Arabic notation? How many? By what name are nine of them sometimes called? How many single things does each represent? Which character has no value? How many names has it? Which is the most common? What do figures represent? Is a figure or combination of figures a number? Why? What is the difference between a picture and a thing? When you look at figures, what do you think about?

9. What is the simple value of a figure?

10. What is the local value? Has every digit a simple value? When is the simple value the same as the local value? When are they different? In the number 395 how many values has the figure five? Does the local value change the simple value? How many values has the figure three? Does its local value change its simple value? Does it increase it, or decrease it?

11. What is the use of the cipher? What does it show in writing a number? Has it any value?

12. What is numeration? What is a period? How many figures in a period? What is the name of the first period? Of the second period? Of the third period? How many places in a period? Name the periods from right to left. From left to right.

13. What is reading numbers? Write on the black-board a number containing four periods. Numerate it. Read it. At which hand do you commence to write a number? Repeat the numeration table by periods from right to left. From left to right.

14. What should be done with a vacant place in the period of a number? Write a number on the black-board, each period of which has a vacant place.

15. What should be done with vacant periods of a number? Write a number on the black-board having a vacant period. What should be done with the vacant left-hand places of the left-hand period? Tell how a number is written. Tell how a number is read. How are periods usually separated in writing numbers?

16. What is a rule? How may it be obtained? Which is the more important, a rule or an analysis? Can a rule be made from an analysis? Why are rules used in arithmetic? [Let the teacher explain to the pupil that rules are deduced from analyses, and that an analysis is far more important than a rule.

17. Recite the rule for notation.

18. What is the sum of two or more numbers? When two numbers are added together, what is the result called? Of what then is the sum of two numbers composed? What is the sum of 6 and 8? What is the 14 called? Of what numbers is the 14 composed? Can 14 be separated into more than two parts? Which is the greater, 14 or the sum of all the parts of 14? What is putting together two or more numbers into one sum called?

19. What is addition? What is the first process in arithmetic? *Notation.* What is the next? *Numeration.* What is the next? *Reading Numbers.* What is the next? *Addition.*

20. What is the sign of addition? What is a sign? Illustrate the use of the sign of addition.

21. What is the sign of equality? Illustrate its use.

[illegible]

22. What is the dollar sign? Illustrate its use. What is its supposed origin?

23. What is an axiom? What is meant by "a self-evident truth"? Can an axiom be proved? Can it be illustrated? Do axioms need to be proved? Illustrate at the black-board the axioms used in addition.

24. What is a problem? What is meant by the solution of a problem?

25. What are simple problems? How many questions in a simple problem? How many answers can be given? What is the meaning of "complex"?

26. What is a complex problem? Can a complex problem have more than one elementary question? Can it have more than one answer? [The teacher, if he thinks best, at some time in the course, may distinguish between a compound and a complex problem. The latter uses the answer of one question as a means for solving a second, and admits of but one final answer; while the former admits of as many answers as there are independent questions in the problem. Many problems, however, are both complex and compound.]

27. What is an analysis? What is an elementary question? Give an example. Give an arithmetical formula. How many formulas are there in addition? How many are there in subtraction? Repeat the formula for addition. The formula for subtraction.

28. What is an analytical step? How many steps can there be in a simple problem? How many in a complex problem? Show at the board the analytical steps of a complex problem.

29. What is the order of solving problems? Give a complete analysis of a problem in addition, following the given order.

30. What is the use of the black-board exercises?

31. What is subtraction? What is the process of taking 7 from 14 called? Why?

32. What is the minuend? Is the minuend the greater or the less number?

33. What is the subtrahend? Which is the greater, the subtrahend or the minuend? How should the subtrahend be written? Why?

34. What is the remainder? Which is the greater, the remainder or the minuend? Can the remainder be less than the subtrahend? Can it be greater? What two numbers added together will give the minuend?

35. What is the difference of two numbers? How does the difference differ from the remainder? Are they always alike? If the difference and minuend are given, how can the subtrahend be found? When the remainder and the subtrehend are given, how can the minuend be found?

36. Illustrate the use of the sign in subtraction.

37. What are the axioms used in subtraction?

38. Illustrate the first on the black-board. The second. The third. The fourth.

39. Give a complete analysis of an example in subtraction?

40. Illustrate the use of the parenthesis in arithmetic.

SECTION IV.

MULTIPLICATION.

LESSON I.

41. *Multiplication* is the process of taking a number a given number of times; and is therefore a short method of addition.

42. *The Multiplicand* is the number to be taken or multiplied.

43. *The Multiplier* is the number by which we multiply, and shows how many times the multiplicand is to be taken.

44. *The Product* is the result of multiplication.

45. *The Factors* of the *product* are the multiplicand and multiplier.

46. *The sign of Multiplication* is a small cross; thus (×), and when placed between numbers, it denotes that they are to be multiplied together; as, $4\times8=32$; to be read, 4 multiplied by 8 equals 32.

47. *Axioms* used in multiplication:

1. *If equal numbers be multiplied by the same or equal numbers, the products will be equal.*

2. *The product is the same whichever factor is taken for the multiplier.*

LESSON II.

48. ANALYSIS OF MULTIPLICATION.

(*a.*) ***Problem.***—It is required to multiply 9873 by 24.

MODEL OPERATION.

9873	multiplicand		factors.
24	multiplier		
39492	product of	4	times multiplicand.
19746	"	20	"
236952	"	24	"

(*b.*) **Analysis.—1.** For convenience write the multiplier under the multiplicand, so that units may stand under units, tens under tens, &c.

2. For convenience begin to multiply with the right hand or units' figure.

3. 4 times 3 units are 12 units; 12 units are equal to one ten and 2 units; write the units in units' place, and add the 1 ten to the product of tens.

4. 4 times 7 tens are 28 tens, which with the 1 ten added are 29 tens; 29 tens are equal to 2 hundreds and 9 tens; write the 9 tens in tens' place, and add the 2 hundreds to the product of the hundreds.

5. 4 times 8 hundreds are 32 hundreds, which with the 2 hundreds added are 34 hundreds; 34 hundreds are equal to 3 thousands and 4 hundreds; write the 4 hundreds in the place of hundreds, and add the 3 thousands to the product of the thousands.

6. 4 times 9 thousands are 36 thousands, which with the 3 thousands added are 39 thousands; 39 thousands are equal to 3 tens of thousands and 9 thousands; write the 9 thousands in the place of thousands, and the 3 tens of thousands in the place of tens of thousands.

7.* Next multiply by the 2 tens of the multiplier. 2 tens are equal to 20 units, and 20 times 3 units are 60 units; 60 units are equal to 6 tens and 0 units; the 0 units need not be written, but write the 6 tens in the place of tens.

8. 20 times 7 tens are 140 tens; 140 tens are equal to 1 thousand 4 hundreds and 0 tens; write the 4 hundreds in the place of hundreds, and add the 1 thousand to the thousands of the product.

9. 20 times 8 hundreds are 160 hundreds; 160 hundreds are equal to 1 ten of thousands 6 thousands and 0 hundreds; 6 thousands with the 1 thousand added are 7 thousands; write them in the place of thousands, and add the 1 ten of thousands to the tens of thousands of the product.

10. 20 times 9 thousands are 180 thousands; 180 thousands are equal to 1 hundred of thousands 8 tens of thousands and 0 thousands; 8 tens of thousands with 1 ten of thousands added, &c.

11. Next proceed to add the partial products 39,492, and 197,460, which give the total product 236,952.

(*c.*) **Conclusion.**—Therefore 9,873 multiplied by 24 equals 236,952.

(*d*) **Rule.**—*I. Write the multiplier under the multiplicand, so that units may stand under units, tens under tens, &c.*

II. Beginning with the unit figure, multiply each figure of the multiplicand by each figure of the multiplier successively, observing

---

* NOTE.—Any number of figures in the multiplier may be analyzed after this method by changing all the different orders to units.

5.	2	4	7	3	6	9	1	0	5	8
	6	6	6	6	6	6	6	6	6	6
6.	2	4	7	3	6	9	1	0	5	8
	7	7	7	7	7	7	7	7	7	7
7.	2	4	7	3	6	9	1	0	5	8
	8	8	8	8	8	8	8	8	8	8
8.	2	4	7	3	6	9	1	0	5	8
	9	9	9	9	9	9	9	9	9	9

LESSON V.

ABSTRACT PROBLEMS.

Problem.—It is required to find the product of 134071834 by 24.

MODEL OPERATIONS.

(*a*.)

```
134071834 multiplicand.
       24 multiplier.
---------
536287336
268143668
---------
3217724016 total product.
```

(*b*.) PROOF.

```
134071834 multiplicand,
       23 multiplier less one.
---------
402215502
268143668
---------
3083652182 product.
 134071834 multiplicand.
----------
3217724016 sum, equal to product of operation (a.)
```

1. 4343824 × 2 = what ?
2. 4137186 × 2 = what ?
3. 48713876 × 3 = what ?
4. 41314136 × 4 = what ?
5. 41871387 × 5 = what ?
6. 87834013 × 6 = what ?
7. 187183841 × 7 = what ?
8. 86783418 × 8 = what ?
9. 3871383 × 9 = what ?
10. 38718341 × 11 = what ?

LESSON VI.

Let the pupil prove each of the following examples:—

11. 371834 × 22 = what?
12. 4187134 × 33 = “
13. 9187183 × 44 = “
14. 3871864 × 55 = “
15. 9187183 × 66 = “
16. 4187183 × 77 = “
17. 8918783 × 88 = “
18. 4108713 × 99 = what?
19. 418341 × 24 = “
20. 3871864 × 39 = “
21. 4187183 × 42 = “
22. 6783871 × 73 = “
23. 91867183 × 78 = “
24. 871836 × 418 = “

LESSON VII.

25. 4187134 × 312 = what?
26. 418343 × 413 = “
27. 671834 × 871 = “
28. 871387 × 834 = “
29. 91864 × 913 = “
30. 418374 × 937 = “
31. 58718398 × 870 = what?
32. 4183783 × 4180 = “
33. 41918713 × 6713 = “
34. 8713871 × 6341 = “
35. 9871344 × 3781 = “
36. 87039416 × 8709 = “

LESSON VIII.

Problem.—What will 43 gallons of vinegar cost at 6 cents a gallon?

MODEL OPERATION.[*]

(*a.*)
43 no. of gallons.
cts. 6 cents.
258 “

(*b.*)
6 × 43 = 258 cts. *Ans.*

[*] Throughout the work, the name of each denominate number is affixed to it for the purpose of aiding the memory, the *analysis* in multiplication and division determining *which* number is to be used as *abstract.*

REVIEW.—What is a unit? (1.) What is a number? (2.) What is a concrete number? (3.) What is an abstract number? (4.) Of what does arithmetic treat? (5).

Formula.*—If 1 gallon costs 6 cents, 43 gallons will cost 43 times 6 cents, which are 258 cents.

Conclusion.—Therefore 43 gallons will cost 258 cents.

NOTE.—If necessary, the pupil may be required to give a full analysis of each *question* in the following problems.

37. If one orange cost 3 cents, what will 8483 oranges cost?

38. If 1 bag of apples cost $2, what will 1783 bags cost? 4189 bags? 38976 bags?

39. If 1 quart of peanuts cost 9 cents, how many cents will 1873 quarts cost? 4183 qts.? 39187 qts.? 918374 qts.? 418367 qts.? 137183 qts.?

40. If 1 square foot of pine boards cost 8 cents, how many cents will 18378 sq. feet cost? 3781637 sq. ft.?

41. Bought 1876 writing books at 7 cents each; what did they cost? 418578 books? 3918713 books? 3718374 books? 387183718 books? 324137 books?

42. Bought 1876 pounds of veal at 8 cents a pound; what did it cost? 3718367 pounds? 3918713 pounds? 41374 pounds? 3871867 pounds? 918713 pounds?

43. What cost 198 quarts of milk at 6 cents a quart? 8713718 quarts? 371834 quarts? 4137 quarts? 371834 quarts? 4187138 quarts?

44. What cost 538 yards of braid at 4 cents per yard; 4183713 yards? 3718641 yards? 39187134 yards? 678341 yards? 34187138 yards?

45. If a horse trot 7 miles an hour, how far will he trot in 873 hours? In 418713 hours? In 671834 hours?

LESSON IX.

46. There are 4 gills in one pint: how many gills in 913 pints? In 67137 pints? In 3841871 pints?

47. There are 8 furlongs in 1 mile; how many furlongs in 413 miles? In 4187134 miles? In 3714673 miles? In 316713 miles?

---

* NOTE FOR THE TEACHER.—The following *formula* may be used if the teacher prefers it :—

FORMULA.—43 *gallons will cost* 43 *times as much as* 1 *gallon;* 1 *gallon costs* 6 *cents, therefore* 43 *gallons will cost* 43 *times* 6 *cents, which are* 258 *cents.*

48. In apothecaries' weight 8 drams make one ounce; how many drams in 1896 ounces? In 37184 ounces? In 418713 ounces? In 37186712 ounces?

49. In 1 dram apothecaries' weight there are 3 scruples; how many scruples in 886 drams? In 34187 drams? In 3718674 drams? In 318783 drams?

50. There are 4 farthings in 1 penny, how many farthings in 896 pence? In 387167 pence? In 371841 pence?

51. There are 7 days in one week, how many days in 52 weeks? How many days in 38413 weeks? In 613718 weeks? In 3871863 weeks? In 3918130 weeks?

52. Eight quarts make a peck; how many quarts in 684 pecks? In 37841 pecks? In 314 pecks? In 3814187 pecks? In 38418713 pecks?

53. Nine square feet make one square yard; how many sq. feet in 38718 sq. yards? In 389187 sq. yards?

54. In 1 month there are 4 weeks; how many weeks in 184 months? In 341872 months? In 413718 months?

55. What will 640 pairs of boots cost at $6 per pair? At $4? At $8? At $3? At $2? At $5?

LESSON X.

56. What will 4386 bushels of cranberries cost at $3 a bushel? At $5? At $7? At $9?

57. At three dollars apiece, what will 98713 hats cost? At $5? At $7? At $9?

58. If a man walks 2 miles an hour, how many miles will he walk in 371837 hours? In 3918712 hours? In 418786 hours?

59. If a man walks 5 miles an hour, how many miles will he walk in 418713 hours? At 3 miles? At 7 miles?

REVIEW.—What is notation? (6.) What is the Roman notation? (7.) Repeat the table? (7., *a.*) What is the effect of repeating a letter? (7., *c.*) How is the value of a letter affected, when a letter of less value is placed before it? (7., *d.*) How, when a letter of less value is placed after it? (7., *e.*) How does a dash over a letter affect its value? (7., *f.*)

60. How much will 8783 yards of broadcloth cost at $3 a yard? At $4? At $7? At $6?

61. How much will 198764 cords of wood cost at $5 per cord? At $9? At $7? At $3?

62. When $6 are paid for 1 cord of wood, how much must be paid for 37184 cords? For 3784 cords? For 418786 cords? For 4187864 cords? For 3718641 cords?

63. What cost 1387 baskets of peaches, if 1 basket costs $2? 918713 baskets? 3918713 baskets? 4187183 baskets? 413418 baskets?

LESSON XI.

64. If there are 324 pins in one paper, how many pins in 24 papers? In 36 papers? In 14 papers?

65. If 19 men eat a barrel of flour in one day, how long will it last 1 man? If they eat it in 5 days? In 7 days? In 13 days?

66. If 49 men dig a ditch in 9 days, how long will it take one man to do the same work? If they dig it in 17 days? In 3713 days?

67. If 38 men can build a wall in 14 days, how long will it take one man working at the same rate? If they build it in 3716 days? In 387134 days? In 418713 days?

68. If 67 men cut 14 cords of wood in 47 days, how many days will it take one man to do the same work?

69. If 136 men can hoe 198 acres of corn in 18 days, how long will it take one man to do the same work?

70. If a garrison of 398 soldiers can consume 19 barrels of flour and 11 barrels of beef worth $328, in 14 days, how long would it take 1 man to consume the same quantity?

71. If 1 horse can consume the grass of 13 acres in 6 months, how many acres will 13 horses require in the same time? Will 46 horses? Will 8913 horses? Will 37134 horses?

REVIEW.—What is the use of Arabic notation? (8.) What is the difference between the simple and the local value of a figure? (9.) (10.) What is the use of the cipher? (11.) What is numeration? (12.) What is reading numbers? (13.) Repeat the numeration table by periods from right to left. (13., *b.*) From left to right. What is to be done with vacant places and periods? (14.) (15.)

72. If 29 horses in 13 days eat 186 bushels of oats worth $92, how long will it take one horse to eat the same quantity?

73. If one man can drink 7 barrels of beer in 9 months, how many barrels can 46 men drink at the same rate in the same time? Can 974 men? Can 38741 men?

Questions.—*Less. XI.*—Which can eat the most, one man or six men? Which will use up a barrel of flour the soonest, one man or six men? Which can dig the longest ditch in the same time, one man or six men? How much longer will a ditch furnish work for one man than it will for six men? How much longer will an acre of grass last one horse than six horses?

LESSON XII.

74. 3408764 × 38716 = what?
75. 418764384 × 3813001 = “
76. 4187180031 × 898003 = “
77. 419001374 × 83039 = “
78. 58764183 × 900191 = “
79. 98000391 × 80009 = “
80. 37198379 × 801901 = “
81. 98713871 × 918701 = “
82. 37891864 × 891870 = “
83. 3900399 × 900009 = “
84. 58703890 × 80000 = what?
85. 37800000 × 89000 = “
86. 41003999 × 88800 = “
87. 91089988 × 88830 = “
88. 98780384 × 891878 = “
89. 37186418 × 398718 = “
90. 671838718 × 37834 = “
91. 918783874 × 30418 = “
92. 418718341 × 30418 = “
93. 37184134 × 67183 = “

LESSON XIII.

94. 91830040 × 401800 = what?
95. 9988800 × 89999 = “
96. 37186703 × 89834 = “
97. 3917834 × 89167 = “
98. 3718964 × 83716 = “
99. 41871384 × 38718 = “
100. 4187834 × 6783 = “
101. 4187183 × 41386 = “
102. 91871834 × 67183 = “
103. 918713878 × 3671 = what?
104. 9800030009 × 41830 = “
105. 10000099 × 400004 = “
106. 900009000 × 3000900 = “
107. 4998009139 × 3901301 = “
108. 837103418 × 371864 = “
109. 987378384 × 87836 = “
110. 5918783913 × 871671 = “
111. 3871341864 × 891372 = “

Review.—What is a rule? (16.) What is the rule for notation? (17.) What is the difference between *sum* and *remainder?* (18.) (34.) What is the difference between addition and subtraction? (19.) (31.) What is the difference between the signs of addition and subtraction? (20.) (36.) What is the difference between the sign of equality and the dollar sign? (21.) (22.)

LESSON XIV.

112. If 46 barrels of flour will last 1 man 36 months, how many barrels will be required by 183 men for the same time? By 41678 men? By 371864 men? By 387186 men?

113. If 430 pounds of hay will last 86 horses 1 day, how many pounds will they eat in 136 days? In 418713 days? In 391870 days? In 40183 days?

114. Since there are 40 rods in 1 furlong, how many rods in 864 furlongs? In 389167 furlongs? In 371 furlongs?

115. There are 100 cents in $1; how many cents in $3? In $5? In $9187? In $371834? In $4187134? In $6713?

116. There are ten mills in 1 cent; how many mills in 38 cents? In 47837 cents? In 367183 cents?

117. There are 1000 mills in 1 dollar; how many mills in $4? In $1347? In $918764? In $37184?

118. If a railroad car runs 38 miles in 1 hour, how many miles will it run in 58 hours? In 9671 hours? In 71864 hours?

119. There are 18764 letters in a book; how many letters will there be in 4137 copies of the same book? In 41878 copies? In 341783 copies? In 391718 copies?

120. A merchant bought 4139 yards of silk at $3 per yard; what was the whole cost? At 9? At $13?

121. If I give $384 for 1 acre of land, how much will a farm of 3713 acres cost? Of 418713 acres?

122. How much will 3896 sheep cost at 346 cents a head? At 483 cents? At 934 cents? At 874 cents?

LESSON XV.

123. 640 acres make 1 square mile; how many acres in 4137 square miles? In 3917164 square miles? In 34178 square miles? In 3718643 square miles?

REVIEW.—Illustrate the axioms used in addition. (23.) Illustrate the axioms used in subtraction. (37.) What is the difference between a simple problem and a complex problem? (25.) (26.)

124. Since there are 365 days in 1 year, how many days in 9 years? In 4173 years? In 371634 years?

125. There are 4 quarters in 1 unit; how many quarters in 87 units? In 37164 units? In 3718643 units?

126. There are 3 thirds in 1 apple; how many thirds in 86 apples? In 37184 apples? In 37186 apples?

127. Since there are 5 fifths in 1 unit, how many fifths in 148 units? In 371868 units? In 37893 units?

128. If 9 ninths make one thing, how many ninths in 3784 things? In 37186 things? In 38714 things?

129. If I cut 1 pie into 16 pieces, into how many pieces of the same size can I cut 139 pies? 37186 pies? 371841 pies?

130. How many halves in 98 units? In 734 units? In 37864 units? In 4678384 units? In 4187186 units?

131. An army of 5746 men having plundered a city took so much money that each man received $37; how much was taken?

132. If it costs $47384 to build one mile of railroad, how much will it cost to build 378 miles? 37184 miles?

133. A gentleman bought 307 horses for shipping, at the rate of $105 each; how much did he pay for the whole?

QUESTIONS.—*Less. XV.*—What is meant by a half of anything? By a quarter? Which is the greater, a whole pie or a half of a pie? A whole apple or a quarter of an apple? How many quarters does it take to make a half? How many quarters does it take to make a whole?

LESSON XVI.

134. What is the value of 9764 shares of railroad stock at $98 per share? At $67? At $89?

135. A man bought 48 building lots at $1236 each; how much did they all cost him?

136. How many yards of broadcloth are there in 4876 pieces, if each piece contains 37 yards?

137. If it requires 135 tons of iron for one mile of railroad, how many tons will be required for 398 miles?

138. How many oranges in 356 boxes, if each box contains 264 oranges?

139. If it requires 68934 shingles for the roof of a house, how many shingles will be required for 146 such houses?

140. Multiply 12 millions and thirteen by twelve thousand and four.

141. Multiply three millions and three thousand by nine thousand and nine.

142. Multiply 9 million two hundred and forty thousand by nine thousand and seven.

143. What will be the cost of building 128 miles of railroad at $8913 per mile?

144. A cotton crop was packed in 183 bales, each bale containing 589 pounds; what was the entire weight of the crop?

145. Two towns 243 miles apart are to be connected by railroad at the cost of $39760 per mile; what will be the entire cost of the road?

146. If an acre of land produce 380 bushels of potatoes, how many bushels will 917 acres produce?

147. If a garrison of soldiers consumes 7897 pounds of bread a day, how much will they consume in 365 days?

148. There are 987396 pigeons in 1 flock; how many pigeons would there be in 876 flocks of the same size?

LESSON XVII.

MISCELLANEOUS ABSTRACT PROBLEMS,*

Which contain not more than two analytical steps.

Perform the operations indicated in each of the following expressions:—

---

* NOTE FOR THE TEACHER.—Should the teacher find these problems too difficult for the pupils, they may be omitted until a review of the fundamental rules.

REVIEW.—Repeat the rule for notation. (17.) For addition. (29., *e.*) For subtraction. (38., *d.*) How is addition proved? (29., *f.*) How is subtraction proved? (38., *e.*) What is the answer called in addition? (18.) In subtraction? (34.) (35.) What are the terms used in subtraction? (32.) (33.) (34.) (35.) What does the parenthesis in arithmetic show? (40.) Illustrate its use.

149. $2467 \times 3 - 4678$.
150. $4134 \times 2 + 6789$.
151. $8641 - 374 + 467$.
152. $9134 - 467 \times 63$.
153. $9137 - 8693 \times 41$.
154. $913 + 673 \times 25$.
155. $916 + 413 \times 46$.
156. $403 \times 403 \times 4069$.
157. $(24 \times 34) - 467$.
158. $8413 - (672 \times 2)$.
159. $9637 \times (467 \times 4)$.
160. $274 \times (37 \times 4)$.
161. $967 \times (764 - 634)$.
162. $(876 \times 4) - 643$.
163. $(864 \times 3) + 413$.
164. $(913 - 674) \times 46$.

LESSON XVIII.

MISCELLANEOUS PRACTICAL PROBLEMS,

Which contain not more than two elementary questions.

NOTE FOR THE TEACHER.—For mental exercises illustrating the following problem, see "Intellectual Arithmetic."

Problem.—A man sold 38 horses for $248 each; he received in cash $1348; how much was still due him?

MODEL OPERATION.

(*a.*)
$248
38 num. of horses.
____
1984
744
____
$9424 total amount.
1348 received in cash.
____
$8076 amount still due him.

(*b.*)
$\$248 \times 38 - \$1348 = \$8076$, *Ans.*

Anal. Steps.—1. *Find the amount for which the horses were sold.*

FORMULA.*—If for 1 horse he received $248, for 38 horses he received 38 times $248, which are $9424.

2. *Find the amount due him after he was paid* $1348.

FORMULA.—If he sold his horses for $9424, and was paid $1348, there must have been still due him the difference of the amounts, which is $8076; therefore there was $8076 due him.

---

* NOTE.—The following formula has its advantages:—

FORMULA.—For 38 horses he should receive 38 times as much as for 1 horse; for 1 horse he received $248, therefore for 38 horses he should have received 38 times $248, which are $9424.

165. A drover bought 138 cows at $19 per head, and sold them for $1980; how much did he lose by the operation?

166. A farmer has an orchard which contains 43 rows of apple-trees with 38 trees in each row; he cut down 125 of the trees; how many remained?

167. There is a book which contains 999 pages, and each page averages 346 words, and each word averages 7 letters; how many letters in the book?

168. There is a pile of lumber containing 47 boards; what is it worth at 3 cents per foot, if each board contains 19 feet?

169. A carpenter bought 30 boards; each board contained 14 feet, and he gave 7 cents per foot; what did the boards cost him?

170. A merchant bought 46 firkins of butter containing 119 pounds each, at 38 cents per lb. How much did his butter cost him?

171. A merchant bought 39 pieces of silk containing 42 yds. each, at 50 cents per yard. How much did the silk cost?

172. If in 1 pound there are 20 shillings, how many shillings are there in 40 pounds?

173. If there are 20 shillings in 1 pound, and twelve pence in one shilling; how many pence in 24 pounds?

174. If there are 12 pence in one shilling, and 4 farthings in one penny; how many farthings in 18 shillings?

175. A man sold 38 lbs. of raisins at 28 cents per pound; he then gave 183 cents to a poor woman; how much had he left?

176. A boy hired himself to a farmer for 37 cents per day; after working 43 days, the farmer in addition to his wages gave him for good conduct 169 cents; how much had he then?

177. Another boy agreed to work for the same farmer for

REVIEW.—What is multiplication? (41.) What is the multiplicand? (42.) What is the multiplier? (43.) What is the product? (44.) Which are the factors? (45.) What is the sign of multiplication? Illustrate its use. Illustrate at the blackboard each of the axioms used in multiplication. (47.) Give the complete analysis of a problem in multiplication. (48., *b*.)

the same wages; but after staying 22 days, the farmer sent him away, and kept 467 cents of his wages on account of his idleness; how much did the boy receive?

178. Henry earned $46 at one time and $27 at another time; his father then gave him three times as much as he had earned. How much did his father give him?

179. Mr. Jones bought 147 cows at one time, 371 at another time, and 416 at another time; and sold them all at $20 per head. How much did he receive for them?

LESSON XIX.

180. Mr. Wm. Nash bought at one time 341 pounds of tin, at another 864 pounds, and at a third time 678 pounds; and paid for it all $700. He sold it at 44 cents per pound; how much did he receive for it?

181. Mr. J. Mailer can set 1500 ems of type in one hour; how many ems can he set in a day of 8 hours? How many ems can he set in 46 days?

182. A man owned 4376 acres of land; he sold 468 acres at $19 per acre. What is the remainder worth at $20 per acre?

183. How many grains in 17 pennyweights; if there are 24 grains in 1 pennyweight?

184. How many grains in 21 ounces; if there are 20 pennyweights in one ounce and 24 grains in one pennyweight?

185. How many pennyweights in 46 pounds; if there are 12 ounces in one pound, and 20 pennyweights in one ounce?

186. How many drams in 18 pounds; if 16 drams make 1 ounce, and 16 ounces make 1 pound?

187. How many ounces in 3 quarters; if 25 pounds make one quarter, and 16 ounces make one pound?

REVIEW.—Repeat the rule for multiplication. (48., *d.*) Give the method of proof. (48., *e.*) What is the difference between addition and subtraction? (19.) (31.). What is the difference in the signs? (20.) (36.) What is the answer called in multiplication? (44.) What in subtraction? (31.) What in addition? (18.)

188. In 1 bushel there are 4 pecks; how many pecks in 396 bushels?

189. What will be the cost of 467 bushels of apples at 43 cents a peck; if there are 4 pecks in 1 bushel?

LESSON XX.

190. Multiply 467 millions 346 thousand by 9.

191. Multiply 4 hundred and 60 thousand 4 hundred and 98 by 12.

192. Multiply 27 billions 463 thousand six hundred and 8 by 7.

193. Multiply 250 millions 656 thousands 4 hundred and 70 by 5.

194. Multiply 17 billions 620 millions 596 by 25.

195. Multiply 54 millions 980 thousand and 98 by 11.

196. Multiply 7 billions 276 millions 560 by 13.

197. Multiply 110 millions 55 thousand and 76 by 8.

198. Multiply 298 billions 946 millions 552 thousand by 9.

199. Multiply 14 billions 556 thousand 556 by 14.

200. Multiply 131 millions 651 thousand 986 by 3.

201. Multiply 269 billions 4 thousand and 50 by 6.

202. Multiply 99 billions 99 millions 999 thousand by 19.

203. Multiply 72 millions 572 thousand and 68 by 22.

204. Multiply 626 millions five hundred and seventy by 27.

205. Multiply 36 billions 141 millions 652 thousand by 15.

206. Multiply 156 millions 277 thousand 568 by 24.

207. Multiply 272 billions 8 hundred thousand by 31.

208. Multiply 546 billions 645 millions 428 by 42.

209. Multiply 79 millions 568 thousand 578 by 29.

210. Multiply 37 millions by 3.
211. Multiply 38 tens by 4.
212. Multiply 34 hundreds by 6.
213. Multiply 56 billions by 5.
214. Multiply 46 millions by 6.
215. Multiply 28 thousand by 9.
216. Multiply 59 tens by 7.
217. Multiply 28 billions by 8.

REVIEW.—What is addition? (19.) Subtraction? (31.) What is multiplication? (41.) What are the terms used in subtraction? (32.) (33.) (34.) (35.) What are the terms used in multiplication? (42.) (43.) (44.) (45.)

218. Multiply 16 thousand by 4.
219. Multiply 97 tens by 3.
220. Multiply 57 billions by 8.
221. Multiply 88 tens by 8.
222. Multiply 76 thousands by 7.
223. Multiply 64 millions by 9.
224. Multiply 44 tens by 6.
225. Multiply 96 thousands by 3.
226. Multiply 54 hundreds by 4.
227. Multiply 77 millions by 6.
228. Multiply 58 thousands by 5.
229. Multiply 93 tens by 2.

LESSON XXI.

A boy having 437 cents, bought 3 algebras at 75 cents apiece; how many cents had he left?

MODEL OPERATIONS.

(a.)

(1.)
75 cents.
3 no. books.
225 cts. cost of books.

(2.)
437 cents.
225 cents spent.
212 cents remaining.

(b.)

cts.
$437-(75\times 3)=212$ cts. *Ans.*

Anal. steps.—1. *Find the cost of books.*
2. *Find amount left.*

230. A butcher bought 4768 pounds of beef, and sold 3864; how much would the remainder be worth at 14 cents per pound?

231. A grocer bought at one time 3864 pounds of sugar; at another time, 4673 pounds. How much did it cost him at 12 cents per pound?

232. If a merchant sells at one time 467 yards of calico; and at another, 378 yards; how much should he receive for it at 27 cents a yard?

233. A merchant bought 467 pounds of coffee at one time; at another, 347 pounds. How much did it cost at 37 cents per pound?

234. There are two hogsheads of sugar: the first contains 467 pounds; the other contains 374 pounds. How much is it worth at 14 cents per pound?

235. A farmer had three loads of hay: the first contained

1437 pounds; the second 1413 pounds; and the third contained 1413 pounds. What is it worth at 1 cent per pound?

236. If I borrow at one time $347, and at another time $6783; how much do I owe after paying $5000?

237. N. Mendal Shafer bought three farms; the first contained 123 acres; the second contained 273 acres, and the third 467 acres; how much did they cost him at $30 per acre?

238. A lady has three daughters aged respectively 5, 8, and 10 years; what will be their united ages in nine years?

239. A man sold 9 umbrellas that were worth only 75 cents each for 120 cents each; how much did he get for them more than they were worth?

240. A farmer sold 25 bushels of hickory nuts to one man, 13 bushels to another, and 34 to another; how much is due him at 499 cents per bushel?

241. If a milliner can make 12 bonnets in one week, for which she receives $96; how many can she make in five weeks?

242. Julia had 120 cents; Mary had 326 cents more than Julia; but afterward Julia found 320 cents; how many cents had Mary?

243. A gentleman sold his sleigh for $180, his carriage for $190, and his horse for $150; how much did his sales lack of $600?

LESSON XXII.

244. If a man earns $25 per week, and pays $7 a week for his expenses; how much can he save in five weeks?

245. A lady bought 20 yards of muslin for 370 cents, 12 yards of calico for 460 cents, and three pieces of tape for 24 cents; after paying for the articles, how much had she left of 1000 cents?

246. A man bought a house for $3500, and sold it for $5000 lacking $350; how much did he make by his bargain?

247. If I have 220 pencils, and give 10 to Mary, 16 to John, and 35 to Louise; how many have I left, and what are they worth at 5 cents apiece?

248. If a man starts for Washington, 250 miles distant, and travels 59 miles each day for four days; how many miles farther must he travel before reaching it?

249. If Dr. Jones has 12 patients in one town, 13 in another, and 21 in a third; and each pays him $15 for his services; how much does he receive?

250. A merchant bought 800 barrels of flour of one man, and 750 barrels of another; how much did it cost him at $8 per barrel?

LESSON XXIII.

COMPOSITION OF PROBLEMS.

Problems containing not more than two elementary questions may be composed in the same way as in addition and subtraction.

MODEL.—1. $34+673\times6=$ what?
2. $4678-(346\times2)=$ what?
3. $1864+3+246-(432\times3)=$ what?
4. $9134\times(674-489)=$ what?
5. $764-(34+64)\times2=$ what?

NOTE.—For models in practical problems, see the preceding lessons.

SECTION V.

DIVISION.

LESSON I.

50. Division is the method of finding how many times one number is contained in another, or of separating one number into as many equal parts as there are units in another.

51. The Dividend is the number to be *divided.*

52. The Divisor is the number by which the *dividend* is divided.

53. The Quotient is the number of times the *divisor* is contained in the *dividend.*

54. In Short Division the quotient is set down at once. It is the preferable method, when it can be as readily performed.

55. ***In Long Division*** each step of the process is written. It is the method generally used, when the *divisor* is greater than *twelve.*

56. ***A Remainder*** in Division is the part of the *dividend* which remains undivided.

57. ***The Sign of Division*** (÷) shows that the number before it is to be divided by the number after it. Thus $20 \div 4$ means that 20 is to be divided by 4.

58. ***The Fractional Sign of Division*** consists in writing the dividend above a line and the divisor below it. Thus, $\frac{12}{3}$ means that 12 is to be divided by 3.

59. ***The Axioms*** used in Division are :—

1. *If equal numbers be divided by the same or equal numbers, the quotients will be equal.*

2. *If a number be both multiplied and divided by the same number, its value will not be changed.*

QUESTIONS.—What is the process of dividing a number into two equal parts called? What is the number to be divided called? What do we call the number that shows how many parts a thing or number is divided into?

LESSON II.

60. ANALYSIS OF SHORT DIVISION.

(*a*.) **Problem.**—Divide 70301 by 4.

MODEL OPERATION.

Divisor. 4)70301 Dividend.
$17575\frac{1}{4}$ Quotient.

Proof.
$17575\frac{1}{4}$ Quotient.
4 Divisor.
70301 = Dividend.

(*b*). **Analysis.***—**1.** For convenience write the divisor at the left of the dividend, and commence by dividing the left hand digit.

---

* In this analysis we prefer the *fractional* form, as it is simpler and more readily understood, although liable to some objection. The teacher will find another form in the analysis of Long Division, (62) which he can use here if he prefers it.

2. *One-fourth* of 7 tens of thousands is 1 ten of thousands with a remainder of 3 tens of thousands equal to 30 thousand; write the 1 in the place of tens of thousands, and add the 30 thousands to the thousands.

3. *One-fourth* of 30 thousands is 7 thousands with a remainder of two thousands which is equal to 20 hundreds; write the 7 in the place of thousands, and add the 20 hundreds to the hundreds; 20 hundreds and 3 hundreds are 23 hundreds.

4. *One-fourth* of 23 hundreds is 5 hundreds with a remainder of 3 hundreds equal to 30 tens; write the 5 in hundreds' place, and add the 30 tens to the tens.

5. *One-fourth* of 30 tens is 7 tens with a remainder of 2 tens equal to 20 units; write the 7 in the place of tens, and add the 20 units to the units; 20 units and 1 unit are 21 units.

6. *One-fourth* of 21 units is 5 units with a remainder of 1 unit, which divided by 4 gives 1 fourth of a unit, written $\frac{1}{4}$; write the 5 in the place of units, and annex the $\frac{1}{4}$ of a unit.

(*c.*) **Conclusion.**—Therefore *one-fourth* of 70301 is $17575\frac{1}{4}$.

(*d.*) **Rule for Short Division.**—I. *Write the divisor at the left of the dividend.*

II. *Beginning at the left, divide each figure of the dividend by the divisor, and set each quotient figure under its dividend.*

III. *If there is a remainder after any division, prefix it to the next figure of the dividend, and divide as before.*

IV. *If any partial dividend is less than the divisor, write a cipher for the quotient, prefix the partial dividend to the next figure of the dividend, and divide as before.*

V. *If there is a remainder after dividing the last figure of the dividend, it can be changed to a* FRACTIONAL QUOTIENT *by writing the divisor under it and annexing the result to the quotient.*

(*e.*) **Proof.**—*Division may be proved by multiplying the* QUOTIENT *by the integral part of the* DIVISOR *and adding to the product the remainder, if there is any. The result will be equal to the dividend, if the work is correct.*

QUESTIONS.—What is meant by *one-fourth?* In what other way can one-fourth be written? Can a problem in division be solved by subtraction? Explain at the blackboard how this can be done?

LESSON III.

61. BLACKBOARD EXERCISES.

NOTE.—Recite the following exercises in the same way as the exercises in Subtraction and Multiplication. Let the teacher point and the pupil simply give the quotients.

1. 2)2 6 8 4 12 10 18 16 14

2. 3)9 12 3 6 15 21 18 27 24

3. 4)8 4 16 20 12 24 32 28 36

4. 5)10 5 15 25 35 20 40 30 45

5. 6)24 12 6 18 42 36 48 24 30

6. 7)14 35 21 42 28 56 7 49 63

7. 8)8 24 16 40 32 72 56 48 64

8. 9)27 18 9 45 36 63 54 81 72

9. 10)30 50 20 10 40 60 80 70 90

10. 12)12 96 108 36 24 48 72 60 84

11. 2)26 30 40 28 24 36 32 38 34

12. 3)57 39 36 54 51 42 57 45 48

13. 4)52 60 56 76 68 64 72

14. 5)65 95 70 80 75 90 85

15. 6)78 90 84 102 96 114 108

16. 7)133 91 98 126 105 119 112

17. 8)104 152 112 144 120 136 128

18. 9)126 117 135 162 144 162 153

19. 13)13 78 65 26 52 39 91 104 117

20. 14)14 42 126 28 112 56 70 84 98

21. 15)15 135 30 120 45 105 60 90 75

22. 16)16 48 32 144 80 64 128 96 112

23. 17)34 17 68 51 119 102 85 153 136

24. 18)162 18 144 36 126 54 90 72 108

25. 19)171 152 38 19 76 57 114 95 133

Give the quotients and the remainders of the following examples, thus, 42 and 1 remainder, &c.

26. 4)9 13 12 27 34 29 6 38 22

27. 5)12 8 19 29 36 21 41 33 37

28. 6)27 15 18 21 45 39 52 25 34

29. 7)17 39 28 43 34 62 59 50 65

LESSON IV.

MENTAL DIVISION.

ILLUSTRATION.

One. 2 halves. 3 thirds. 4 fourths. 5 fifths.

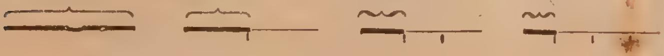
How many *halves* in one apple? How many *thirds* in one pear? How many *fourths* in one orange? How many *fifths* in one melon? How many *sixths* in one dollar? How many *sevenths* in one dime? How many *eighths* in one pound?

How many halves make one apple? How many thirds make one pear? How many fourths make one orange? How many fifths make one melon? How many tenths make one mile? How many twenty-fifths make one peach?

1 half. 1 third. 2 fourths. 3 fifths.

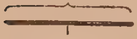
What is meant by one *half?* Ans.—*One of the two equal parts into which a unit or number of units is divided.* What is meant by one *third?* What by 2 fourths? What by 3 fifths? What by 4 sevenths? What by 7 twelfths? What by 4 fifths?

One. 1 half. 1 third. 1 fourth.

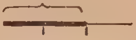


Which is the greater, one or one *half?* One *half* or one *third?* One *third* or one *fourth?* One *half* or one *fourth?* *One* or one *third?* *One* or one *fourth?* One *fourth* or one *fifth?* One *seventh* or one *tenth?* One *sixteenth* or one *seventeenth?*

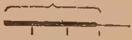
2 halves.



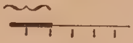
2 thirds.



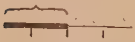
3 fourths.



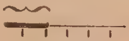
2 sixths.



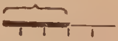
2 fourths.



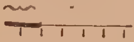
2 sixths.



3 fifths.

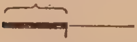


2 sevenths.

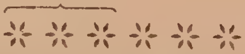


Which is the greater, 2 halves or 2 fourths? 2 thirds or 2 sixths? 3 fourths or 3 fifths? 2 sixths or 2 sevenths? 6 sevenths or 6 eighths? 5 sevenths or 5 elevenths? 5 ninths or 5 thirteenths?

1 half of one.



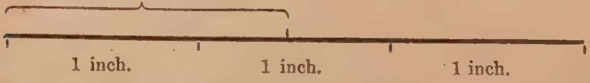
1 half of six.



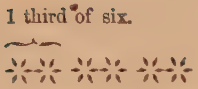
What do we mean by one half of one? Ans.—*One of the two equal parts into which one is divided.* What do we mean by one half of two? Ans.—*One of the two equal parts into which two is divided.* What do we mean by one half of six? What by one half of eight? What by one half of twelve? What by one half of 16? Of 18? Of 20? Of 10?

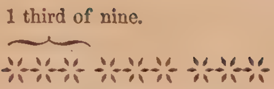
How many units in one half of one? Ans.—*One half* of one unit. How many units in one half of two? Ans.—*One* unit. How many units in one half of six? Ans.—*Three.* In one half of eight? In one half of twelve? In one half of 16? Of 18? Of 20? Of 10?

1 half of three.

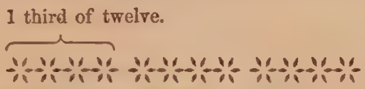


How many units in one half of three? Ans.—*One unit and a half.* How many units in one half of five? Ans.—*Two units and a half.* How many units in one half of seven? In one half of nine? In one half of 13? Of 15? Of 17? Of 19?

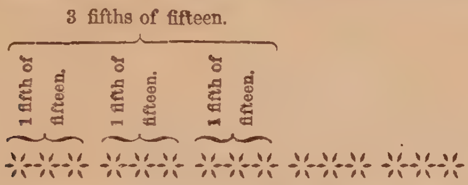




What do we mean by one third of six? Ans.—*One of the three equal parts into which six is divided.* How many units in one third of six? Ans.—*Two.* What is meant by one third of nine? How many units in one third of nine?



What is 1 third of 12? Of 18? Of 27? Of 24? Of 36? What is 1 fifth of 20? One seventh of 14? One ninth of 18?



What is meant by one fifth of 15? Ans.—*One of the five equal parts into which fifteen is divided.* What is meant by 3 fifths of 15? Ans.—*Three of the five equal parts into which fifteen is divided.* How many units in 1 fifth of 15? Ans.—*Three.* How many units in 3 fifths of 15? Ans.—*Three times 3 units, or 9 units.* What is meant by 1 sixth of 18? What by 3 sixths of 18? How many in 1 sixth of 18? How many in 3 sixths of 18? How many in 1 fourth of 12? Of 16? Of 24?

LESSON V.

MENTAL EXERCISES.

1. How many times can 2 be subtracted from 10? 3 from 9? 4 from 12? 8 from 16? 4 from 36? 7 from 21?

2. How many times can 1 be subtracted from 3? from 8? from 9? from 36? 4 from 24? 10 from 30? 25 from 100?

3. How many times is 2 contained in 10? in 12? in 18? in 20? 5 in 30? 8 in 72? 11 in 44? 12 in 96?

4. How many times is 8 contained in 16? 5 in 15? 5 in 30? 7 in 63? 6 in 42? 9 in 36? 2 in 18?

5. How many times is 9 contained in 18? 7 in 21? 11 in 55? 4 in 24? 6 in 24? 7 in 35? 8 in 48? 7 in 56?

6. What is *one half* ($\frac{1}{2}$) of 2? Of 6? Of 8? Of 12? Of 16? Of 22? Of 4? *One third* ($\frac{1}{3}$) of 9? *One seventh* ($\frac{3}{7}$) of 14? *One ninth* ($\frac{1}{9}$) of 27? *One eleventh* ($\frac{1}{11}$) of 33? *One eighth* ($\frac{1}{8}$) of 16? *One twelfth* ($\frac{1}{12}$) of 24? *One fifth* ($\frac{1}{5}$) of 15? *One ninth* ($\frac{1}{9}$) of 63?

7. What is $\frac{1}{9}$ Of 18? Of 27? Of 36? Of 45? Of 54?

8. What is $\frac{1}{7}$ of 14? Of 21? Of 28? Of 42? Of 56?

9. What is $\frac{1}{11}$ of 22? Of 55? Of 88? Of 77? Of 66?

10. What is $\frac{1}{6}$ of 18? Of 54? Of 42? Of 48? Of 72?

LESSON VI.

1. 1 is what part of 2? Of 3? Of 4? Of 5? Of 6?

2. 2 is what part of 3? Of 4? Of 5? Of 6? Of 7?

3. 3 is what part of 4? Of 5? Of 16? Of 21? Of 25?

4. 4 is what part of 11? Of 16? Of 25? Of 89? Of 71?

5. 5 is what part of 87? Of 49? Of 371? Of 346?

6. 25 is what part of 69? Of 378? Of 546? Of 8913?

7. 97 is what part of 478? Of 39183? Of 48718?

8. 147 is what part of 3896? Of 87184? Of 6783?

9. How many times is 2 contained in 1? Ans. *One-half of a time.*

10. How many times is 3 contained in 1? Ans. *One-third* ($\frac{1}{3}$) *of a time.*

11. How many times is 3 contained in 2? Ans. *Two-thirds* ($\frac{2}{3}$) *of a time.*

LESSON VII.

1. How many times is 2 contained in 1? Ans.—$\frac{1}{2}$ *of a time.* In 2? Ans—1 *time.* In 3? Ans.—1 *time and a half* (1$\frac{1}{2}$). In 4? Ans.—2 *times.* In 5? Ans.—2$\frac{1}{2}$ *times.* In 6? Ans.—3 *times.* In 7? Ans—3$\frac{1}{2}$ *times.*

2. How many times is 3 contained in 1? Ans.—$\frac{1}{3}$ *of a*

time. In 2? ANS.—$\frac{2}{3}$ *of a time.* In 3? ANS.—1 *time.* In 4? ANS.—$1\frac{1}{3}$ *times.* In 5? ANS.—$1\frac{2}{3}$ *times.* In 6? ANS.—2 *times.* In 7? ANS.—$2\frac{1}{3}$ *times.* In 8? ANS.—$2\frac{2}{3}$ *times.*

3. How many times is 4 contained in 1? In 2? In 3? In 4? In 5? In 6? In 7? In 8? In 9? In 10?

4. How many times is 5 contained in 1? In 2? In 3? In 4? In 5? In 6? In 7? In 8? In 9? In 10? In 11?

5. How many times is 6 contained in 1? In 2? In 3? In 4? In 5? In 6? In 7? In 8? In 9? In 10? In 11?

6. How many times is 7 contained in 1? In 5? In 7? In 13? In 46? In 38? In 25? In 11? In 2? In 3?

7. How many times is 8 contained in 5? In 2? In 3? In 1? In 8? In 11? In 17? In 23? In 24? In 36?

8. How many times is 9 contained in 11? In 6? In 1? In 0? In 14? In 31? In 100? In 105? In 3? In 7?

9. How many times is 10 contained in 3? In 7? In 0? In 8? In 19? In 16? In 72? In 119? In 112? In 89?

10. How many times is 12 contained in 7? In 43? In 1? In 4? In 9? In 68? In 146? In 113? In 78? In 64?

LESSON VIII.

1. Divide 1 into 2 parts, or what is $\frac{1}{2}$ of 1? ANS.—$\frac{1}{2}$.
2. Divide 2 into 2 parts, or what is $\frac{1}{2}$ of 2? ANS.—1.
3. Divide 3 into 2 parts, or what is $\frac{1}{2}$ of 3? ANS.—$1\frac{1}{2}$.
4. Divide 4 into 2 parts, or what is $\frac{1}{2}$ of 4? ANS.—2.
5. Divide 5 into 3 parts, or what is $\frac{1}{3}$ of 5? ANS.—$1\frac{2}{3}$.
6. Divide 6 into 4 parts, or what is $\frac{1}{4}$ of 6? ANS.—$1\frac{2}{4}$.
7. Divide 3 into 6 parts, or what is $\frac{1}{6}$ of 3? ANS.—$\frac{3}{6}$.
8. Divide 9 into 5 parts, or what is $\frac{1}{5}$ of 9? ANS.—$1\frac{4}{5}$.
9. Divide 11 into 3 parts, or what is $\frac{1}{3}$ of 11? ANS.—$3\frac{2}{3}$.
10. Divide 17 into 5 parts, or what is $\frac{1}{5}$ of 17? ANS.—$3\frac{2}{5}$.
11. What is $\frac{1}{3}$ of 3? Of 4? Of 5? Of 5? Of 7?
12. What is $\frac{1}{4}$ of 1? Of 2? Of 3? Of 4? Of 5?
13. What is $\frac{1}{5}$ of 6? Of 8? Of 9? Of 11? Of 18?
14. What is $\frac{1}{6}$ of 4? Of 8? Of 9? Of 12? Of 6?
15. What is $\frac{1}{7}$ of 9? Of 8? Of 41? Of 16? Of 23?

16. What is $\frac{1}{8}$ of 26? Of 37? Of 46? Of 84? Of 37?

17. What is $\frac{1}{9}$ of 47? Of 87? Of 86? Of 37? Of 43?

18. What is $\frac{1}{10}$ of 67? Of 100? Of 13? Of 17?

19. What is $\frac{1}{11}$ of 97? Of 46? Of 84? Of 37?

20. What is $\frac{1}{12}$ of 36? Of 46? Of 83? Of 74?

NOTE.—It must not be thought that these exercises are sufficient to give skill in mental division, unless they are practised until the pupils become quick and accurate.

LESSON IX.

MENTAL EXERCISES—CONTINUED.

1. If you should divide 6 cents equally among 3 boys, how many cents would you give each boy?

FORMULA.—If you give 6 cents to 3 boys, you will give 1 boy one-third of 6 cents, which is 2 cents; therefore you will give 2 cents to each boy.

2. If a stick is 9 feet long, and you cut it into three equal parts, how long will each part be?

3. A boy wished to divide 12 marbles into 3 equal piles; how many marbles must he put in each pile? Divide 18 marbles in the same way? 15 marbles? 21 marbles?

4. If I give 18 apples to 3 boys, how many will each boy have? If I give 27 apples? 36 apples? 9 apples? 12 apples?

5. If I solve 36 examples in 6 days, how many can I solve in 1 day? If I solve 18 examples? If 24 examples? If 42 examples?

6. How many oranges at 6 cents apiece can you buy for 42 cents?

FORMULA.—If 1 orange costs 6 cents, you can buy as many oranges for 42 cents as 6 cents are contained times in 42 cents, which are 7 times; therefore you can buy 7 oranges for 42 cents.

7. At 4 cents apiece, how many pine-apples can you buy for 48 cents? At 8 cents? At 3 cents? At 12 cents?

8. A farmer pays 28 dollars for 7 sheep; how much is that apiece? If he pays 42 dollars? If he pays $35?

QUESTIONS.—Three boys should have how many times as many cents as 1 boy? One boy should have how many times as many cents as 3 boys? Can the 7th problem be solved by subtraction? Which number could be used as the minuend? Which as the subtrahend? Would there be any remainder?

LESSON X.

PRACTICAL PROBLEMS,

Which contain but one elementary question.

Two men own equal shares in a farm worth $237861; what is the value of each man's share?

MODEL OPERATION.

No. of men. 2)$237861
$118930½

FORMULA.[*]—If two shares of a farm are worth $237861, 1 share is worth one-half of $237861, which is $118930½; therefore one share is worth $118930½.

1. In three miles there are 190080 inches; how many inches in 1 mile?

2. In 4 hogsheads of wine there are 8064 gills; how many gills in 1 hogshead?

3. How many pounds can 1 car carry, if 5 cars can carry 157625 pounds?

4. How many times will a clock tick in 1 day, if it ticks 259200 times in 3 days?

5. In a cornfield containing 4 acres there are 19360 hills of corn; how many hills are there in 1 acre?

6. How many ounces are there in one ton, if there are 224000 in 7 tons?

7. If the earth moves in its orbit 408000 miles in 6 hours, how far does it move in 1 hour?

8. If there are 12587 grains of wheat in 7 pints, how many grains are there in 1 pint?

9. In 8 pounds there are 7680 farthings; how many farthings in 1 pound?

10. How many square feet in 1 square mile, if there are 250905600 square feet in 9 square miles?

---

[*] The following formula has its advantages, and may be used if preferred:—
FORMULA.—*One share is worth one-half as much as* 2 *shares;* 2 *shares are worth* $237861; *therefore* 1 *share is worth one-half of* $237861, *which is* $118930½.

QUESTIONS.—Two apples are worth how many times as much as 1 apple? Three pears are worth how many times as much as 1 pear? Nine hats are worth how many times as much as 1 hat? Forty cows are worth how many times as much as 1 cow.

LESSON XI.

11. How many solid inches are there in 1 solid yard, if there are 373248 solid inches in 8 solid yards?

12. Light moves 1,540,000 miles in 8 seconds; how far does it move in 1 second?

13. How many families are there in a city of 180000 inhabitants, if there are 6 persons in a family?

14. If there are 4096000 drams in 8 tons, how many drams are there in 1 ton?

15. What is the capital of a bank, if $18972756 is nine times its value?

16. Sound moves 8960 feet in 8 seconds; what number of feet does it move in 1 second?

17. If a train of cars moves 21120 feet in 6 minutes, how far does it move in 1 minute?

18. In 7 equal regiments there are 8050 men; what number in each?

19. Eight equal cargoes of iron cost $912568; required the cost of one?

20. The expenses of the government of the United States for 1861 were $7671160 for 1 week; what was the expenditure for one day?

21. 2160000 miles are 9 times the distance from the earth to the moon; what is that distance?

22. What is the diameter of the earth, if 55384 miles are 7 times the diameter?

QUESTIONS.—One orange is worth how many times as much as 6 oranges? *Ans. One-sixth of once as much.* How many times as much as 4 oranges? *Ans. One-fourth as much.* How many times as much as 13 oranges? How many times as much as 11 oranges? One sheep is worth how many times as much as 5 sheep? As 23 sheep? As 13 sheep? As 22 sheep?

REVIEW.—What is a unit? (1.) What is a number? (2.) What is the difference between a concrete and an abstract number? (3.) (4.) Of what does arithmetic treat? What is the difference between notation and numeration? (6.) (12.) What is the difference between the Roman and Arabic notations? (7.) (8.) Whch is used in arithmetic? What is the other used for?

LESSON XII.

23. How many cubic inches in 1 bushel, if there are 10752 cubic inches in 5 bushels?

24. 12973254 is five times what number?

25. A tract of land containing 207360 acres was divided into 9 equal townships; how many acres in each?

26. The national debt of Great Britain is about $4,000,000,000; if it should be paid in 7 equal instalments, what would be the amount of each?

27. Four men own in equal shares a vessel and cargo worth $154326; what is the share of each?

28. If Hoe's patent printing press can print 79158 impressions in 3 hours, how many can it print in 1 hour?

29. In 7 equal piles of lumber there are 1276489 feet; how many are there in each?

30. What will be the cost of supporting 1 regiment for a year, if 9 regiments cost $2713981 for the same time?

31. If 1 man can perform a piece of work in 1728 hours, how long will it take 9 men to do the same?

32. If one man can build a house in 203780 hours, in how many hours can 8 men build it?

LESSON XIII.

33. If a body of 9936 men were divided into 9 regiments? how many men would there be in each?

34. If 9 men eat 216378 ounces of bread in a given time, how much will 1 man eat in the same time?

35. A gentleman left his estate valued at $13120 to be divided equally among his wife and 3 children; how much did each receive?

36. An estate valued at $250628 is divided equally among 4 sons and 5 daughters; what will each receive?

37. The population of New York in 1855 was about 654972, or four times the population of Boston; what was the population of Boston?

38. The population of New York at the same time was about

six times the population of Brooklyn and Williamsburgh together; what was their population?

39. The income from a certain farm in 1 year is 137520 cents, which is six times as great as the income from a certain dairy, what is the income of the dairy?

40. The dairy consisted of 8 cows; what was the income from 1 cow?

41. A locomotive can run 1293600 feet in 7 hours; how far can it run in one hour?

42. In 9 square miles there are 921600 square rods; how many square rods in one mile?

Questions.—In 9 regiments how many times the number of men in 1 regiment? Into how many parts must the dividend be divided in the 35th problem? Which number is the divisor in the 36th problem? Which number is the divisor in the 37th problem? Which, in the 38th problem?

LESSON XIV.

43. There are 24,780,800 square yards in 8 square miles; how many square yards in 1 square mile?

44. In 7 square miles there are 195,148,800 square feet; how many square feet in 1 square mile?

45. In 6 square miles there are 24,086,937,600 square inches; how many square inches in 1 square mile?

46. In 7 cords of wood there are 1,548,288 cubic inches; how many cubic inches in 1 cord?

47. If $138764 are divided into 7 equal parts, how many dollars in each part?

48. If an estate of $38414 is divided into 7 equal shares, what is the value of each share?

49. A man dying left an estate valued at $3874, to be equally divided among his wife and four children; how much did each receive?

Review.—What is the difference between the simple and local values of a figure? (9.) (10.) What is the use of the cipher? (11.) What is to be done with vacant periods? (15.) What with vacant places in a period? (14.) What is a rule? (16.) What is the rule for notation? (17.)

50. Divide the number 387,186,434 into 9 parts; what is one part?

51. The sum of the 8 equal parts of a number is 387,183,464; what is one part?

52. Six times a certain number is 38,718,641; what is the number?

LESSON XV.

At $2 per bushel how many bushels of wheat can be bought for $2243?

MODEL OPERATION.

$$\begin{array}{r} \$2)\$2243 \\ \hline 1121\frac{1}{2} \text{ no. bushels.} \end{array}$$

FORMULA.—If 1 bushel of wheat can be bought for $2, as many bushels can be bought for $2243 as $2 is contained times in $2243, which are 1121½ times; therefore 1121½ bushels can be bought for $2243.

53. How many oranges can be purchased for 68324 cents, if 1 orange costs 3 cents?

54. When pine apples are 9 cents each, how many can be purchased for 871384 cents?

55. How many pounds of flour at 5 cents per pound can be bought for 8,713,643 cents?

56. How many pounds of sugar at 7 cents per pound can be bought for 8,371,864 cents?

57. At 8 cents per pound how many pounds of iron can be bought for 13,786,713 cents?

58. How many loads of hay can be purchased for $2933, if 1 load costs $9?

59. At 9 cents per pound how many pounds of beef can be bought for 28,736,463 cents?

60. If 48240 men are arranged in files of 4 men each, how many files will there be?

REVIEW.—What is the sum of two numbers? (18.) What is addition? (19.) What is the sign of addition? (20.) What is the sign of equality? (21.) What is the dollar sign? (22.) What is an axiom? (23.)

61. If $127860 is invested in coal at $3 per ton, how many tons will be bought?

QUESTIONS.—Can the 53d problem be solved by subtraction? How many times can the divisor be subtracted from the dividend?

LESSON XVI.

62. If 4 farthings make a penny, how many pence in 3871834 farthings?

63. If 8 furlongs make a mile, how many miles in 1528846 furlongs?

64. If there are 8 shillings in 1 dollar, how many dollars in 38716783 shillings?

65. How many square yards in 9837864 square feet, if there are 9 square feet in 1 square yard?

66. How many gross of pens at 6 dimes a gross can be bought for 372464 dimes?

67. If there are 3 feet in 1 yard, how many yards can be cut from a piece of cloth 1234 feet long?

68. How many times will a wagon-wheel turn round in going 1,346,783 feet, if the wheel is 9 feet in circumference?

69. How many times will a wagon wheel 12 feet in circumference turn round in going 378004 feet?

70. There is a field 52840 feet in circumference; how many trees 8 feet apart can be planted around it?

71. How many posts 9 feet apart are required to build a fence 18522 feet in length?

LESSON XVII.

72. How many ties in a railroad track 3,874,136,484 feet long, if they are laid 4 feet apart?

73. If a cannon can be fired once in 9 minutes, how many times can it be fired during a battle of 729 minutes?

74. A body of 12735 men is divided into 9 regiments; how many men in each?

75. In 9 tons there are 288000 ounces; how many ounces in 1 ton?

76. At $5 per ton how many tons of coal may be purchased for $275650?

77. An army of 12864 men is drawn up in files of 4 men; how many files are there?

78. How much iron at $7 per ton can be purchased for $394571?

79. When sugar is $9 per hundred weight, how much can be bought for 352737 dollars?

80. Charles is running after Henry, and gains on him 8 rods in a minute; how long will it take him to catch Henry, if he is 784 rods behind him?

81. If a locomotive can run 5 rods in a second, how many seconds will it take to run 158,713,416 rods?

QUESTIONS.—What is meant by the "ties" of a railroad track? Can the 74th problem be solved by subtraction?

LESSON XVIII.

82. 87,938,640 nuts are divided into 6 equal piles; how many nuts in each pile?

83. A manufacturer expends 1,378,544 cents in iron at 8 cents per pound; how many pounds does he buy?

84. A merchant sold a quantity of sugar for 3 cents per pound more than he gave for it, gaining 136710 cents by the operation; how many pounds did he sell?

85. A dealer sold a quantity of butter for 9 cents per pound more than he gave for it, and gained the sum of 326734 cents; how many pounds did he sell?

86. A drover sold a drove of cattle for 6 dollars a head less than he gave for them, thus losing $21888; how many head did he sell?

87. A speculator sold a tract of land at a loss of 1768 dollars; if he lost $7 on each acre, how much did he sell?

88. A flour merchant sold a quantity of flour for $9 per

REVIEW.—Illustrate each of the axioms used in addition. (23., *a.*) What is a problem? (24.) What is a simple problem? (25.) What is a complex problem? (26.) What is an analysis? (27.) What is an analytical step? (28.)

barrel, which was \$3 more than he gave for it; he gained \$528; how many barrels did he buy?

89. A cotton broker purchased a cargo of cotton, charging a commission of 12 cents per bale, which amounted to 91836 cents; how many bales did he purchase?

90. A produce merchant sold 84578 barrels of produce; he received 8 cents per barrel for selling; his bill amounted to 387864 cents; how many barrels did he sell?

LESSON XIX.

62. ANALYSIS OF LONG DIVISION.

(*a.*) **Problem.**—Divide 26306241 by 483.

MODEL OPERATION.

$$\begin{array}{l}
\text{Divisor.}\quad\text{Dividend.}\quad\text{Quotient.}\\
483)26306241(54464\frac{129}{483}\\
\hphantom{483)}2415\\
\hphantom{483)}\underline{\hphantom{2415}}\\
\hphantom{483)2}2156\\
\hphantom{483)2}\underline{1932}\\
\hphantom{483)26}2242\\
\hphantom{483)26}\underline{1932}\\
\hphantom{483)263}3104\\
\hphantom{483)263}\underline{2898}\\
\hphantom{483)2630}2061\\
\hphantom{483)2630}\underline{1932}\\
\hphantom{483)26306}129\ \text{Rem.}\div 483\ \text{Divis.}
\end{array}$$

PROOF.

$$\begin{array}{rl}
\text{Quotient.} & \\
54464\frac{129}{483} & \\
\underline{483} & \text{Divisor.}\\
163392 & \\
435712\hphantom{0} & \\
\underline{217856\hphantom{00}} & \\
26306112 & \\
\underline{129} & \\
26306241 &
\end{array}$$

(*b.*) **Analysis.**—**1.** For convenience,* write the divisor at the left, and the quotient when obtained at the right of the dividend.

2. 483 is contained in 2630 tens of thousands 5 ten-thousands times, with a remainder of 215 tens of thousands equal to 2150 thousands; write the 5 in the place of tens of thousands in the quotient, and to the 2150 thousands add the 6 thousands of the dividend, which makes 2156 thousands.

---

* NOTE.—It is perhaps more convenient to place the divisor at the right of the dividend and the quotient under it.

3. 483 is contained in 2156 thousand 4 thousand times, with a remainder of 224 thousand equal to 2240 hundreds; write the 4 in the place of thousands in the quotient, and to the 2240 hundreds add the 2 hundreds of the dividend, which makes 2242 hundreds.

4. 483 is contained in 2242 hundreds 4 hundred times with a remainder of 310 hundreds equal to 3100 tens; write the 4 in hundreds' place in the quotient, and to the 3100 tens add the 4 tens of the dividend, which makes 8104 tens.

5. 483 is contained in 3104 tens 6 ten times with a remainder of 206 tens equal to 2060 units; write the 6 in the tens' place in the quotient, and to the 2060 units add the 1 unit of the dividend, which makes 2061 units.

6. 483 is contained in 2061 units 4 times with a remainder of 129 units; write the 4 in the units' place in the quotient, and divide the remainder of 129 units by 483, giving $\frac{129}{483}$ of a unit to be annexed to the quotient.

(*c.*) **Concl.**—Therefore 26,306,241 divided by 483 equals 54,464$\frac{129}{483}$.

(*d.*) **Rule for Long Division.**—*I. Write the divisor at the left of the dividend as in Short Division.*

II. Find how many times the divisor is contained in the fewest left hand figures of the dividend that will contain it, and place the quotient at the right of the dividend.

III. Multiply the divisor by this quotient figure, subtract the product from the partial dividend, and to the remainder annex the next figure of the dividend.

IV. Divide as before, until all the figures of the dividend have been annexed and divided.

V. If any partial dividend does not contain the divisor, place a cipher in the quotient, annex the next figure of the dividend, and divide as before.

VI. If there is a remainder after dividing all the figures of the dividend, it may be changed to a FRACTIONAL QUOTIENT *by writing the* DIVISOR *underneath it.*

REVIEW.—Name the order of solving problems. (29.) Repeat the rule. (29., *e.*) In addition how is a problem proved? (29., *f.*) What is subtraction? (31.) What is the remainder? (34.) What is the difference? (35.) What is the sign of subtraction? (36.) Illustrate the axioms of subtraction. (37.) Give the complete analysis of an example in subtraction? (38., *b.*)

(*e.*) **Proof.**—*The proof of Long Division is the same as that of Short Division.*

Notes.—1. The first step in Long Division is to find the quotient figure of the highest denomination.

2. Multiply the divisor by that figure and write the product under the first partial dividend, then *observe* that if the *product* is *greater* than the *partial dividend* the *quotient figure* is TOO LARGE.

3. Subtract that product from the first partial dividend, and *observe* that if the REMAINDER is *equal to* or GREATER than the DIVISOR, the *quotient figure* is TOO SMALL.

4. Change the remainder to the next lower denomination, and add to it the corresponding denomination in the dividend.

LESSON XXI.

MODEL OPERATION.

```
Dividend.
41371413|4  Divisor.*
4       |10342853¼  Quotient.
—
 13
 12
 —
  17
  16
  —
   11
    8
   —
    34
    32
    —
     21
     20
     —
      13
      12
      —
       1 rem.÷4=¼.
```

STEPS.

1. Find how many times.
2. Multiply.
3. Subtract.
4. Bring down.

---

* It is considered better by some to place the divisor at the right and the quotient under it.

Review.—How may a problem in subtraction be proved? (38., *e.*) In arithmetic, what does the parenthesis denote? (40.) What is the multiplicand? (42.)

(91.)
2)41878376(

(92.)
2)37183434(

(93.)
2)41878643(

(94.)
2)6137834(

(95.)
2)418783701(

(96.)
3)41837415(

(97.)
3)41870341(

(98.)
3)9183716(

(99.)
3)41871346(

(100.)
3)41837834(

(101.)
4)4183713416(

(102.)
4)4183718341(

(103.)
4)391871864(

(104.)
4)538783712(

(105.)
4)6783718371(

LESSON XXII.

(106.)
418734164|5

(107.)
4187183741|5

(108.)
4837186412|5

(109.)
487183786|5

(110.)
437837186|5

(111.)
678389137|6

(112.)
418378437|6

(113.)
9183780013|6

(114.)
898378378|6

(115.)
4878378413|6

(116.)
4183784134|7

(117.)
4183786413|7

(118.)
91837041371|7

(119.)
41300684134|8

(120.)
418301864134|8

QUESTIONS.—*Less. XIX.*—At which hand is the divisor usually written in division? At which the quotient? How many figures are taken as the first partial dividend? What is the first step in the rule for Long Division? The second? The third? The fourth? The fifth? The sixth? When the product of the divisor and quotient figure is larger than the partial dividend, what does it show? When any remainder is larger than the divisor what does it show? When the partial dividend is less than the divisor, what is to be done?

REVIEW.—What is multiplication? (41.) What is the multiplier? (43.) What is the product? (44.) Which are sometimes called factors? What is the sign of multiplication? (46.)

LESSON XXIII.

121. 418378346÷15=what?
122. 837834167÷15= “
123. 40370413÷15= “
124. 4878378÷15= “
125. 4183783÷15= “
126. 87841374÷16= “
127. 91838413÷16= “
128. 9013783÷17= “
129. 9183841÷17= “
130. 418964÷17= “
131. 8783713÷18= what?
132. 4837834÷18= “
133. 6030713÷18= “
134. 6418378÷18= “
135. 4671837÷18= “
136. 9183783÷18= “
137. 6783413÷18= “
138. 6783718÷19= “
139. 41837864÷19= “
140. 6837834÷19= “

LESSON XXIV.

141. 418371834÷20=what?
142. 6784183713÷20= “
143. 418378371÷20= “
144. 41800307÷21= “
145. 4603701÷21= “
146. 191803701÷21= “
147. 418367834÷22= “
148. 4183718032÷22= “
149. 4183783412÷23= “
150. 896418378÷24= “
151. 418378346÷27= what?
152. 41837641834÷26= “
153. 671837183413÷25= “
154. 9183841678÷27= “
155. 418378390÷27= “
156. 418389646÷28= “
157. 641834671÷28= “
158. 418378678÷29= “
159. 37837113÷29= “
160. 468371834÷29= “

LESSON XXV.

Problem.—If a clock ticks 31,536,000 times in a year, how many times does it tick in a day, allowing 365 days to a year?

Review.—Recite the axioms used in multiplication. (47.) Illustrate them. Give the complete analysis of a problem in multiplication. (48., *b.*) What is the rule for multiplication? How may a problem in multiplication be proved? (49., *e.*) What is division? (50.) What is the dividend? (51.) What is the divisor? (52.) What is the quotient? (53.) How is Short Division done? (54.) How is Long Division done? (55.) What is the remainder in division? (56.)

MODEL OPERATION.

```
Days.  Times in 1 year.
365)31536000(86400 times in 1 day.
    2920
    ----
     2336
     2190
     ----
      1460
      1460
```

PROOF.

```
    86400
      365
   ------
   432000
  518400
 259200
 --------
 31536000
```

Formula.—If a clock ticks 31,636,000 times in 365 days, in 1 day, it ticks *one*-365th part of 31,536,000 times which is 86400 times.

Conclusion.—Therefore if a clock ticks 31,536,000 times in 365 days, it ticks 86400 times in 1 day.

161. A man bought 114 acres of land for $4104; what was the price per acre?

162. $417836 was paid to 175 workmen; how much did each receive?

163. The yearly expenses in a certain factory are $46783; how much do the daily expenses amount to, there being 365 days in a year?

164. Nine thousand men are entitled to $347839; how much is that for 1 man?

165. A body of 8786 men receive at one time $400000; how much is that for each man?

166. In one hogshead there are 63 gallons; how many hogsheads are there in 6615 gallons?

167. If a man can count 8677 in an hour, how many hours will it take him to count 38,369,694?

168. The Memphis and Charleston railroad is 287 miles in length, and cost $5,572,470; what was the cost per mile?

Review.—What does the sign of division show? (57.) What is the fractional sign? (58.) Illustrate each of the axioms used in division. (59.) Give the analysis of a problem in Short Division. (60., *b.*) Repeat the rule. (60., *d.*) How may a problem in division be proved? (60., *e.*)

169. A garrison of 10000 men consumed 10712 barrels of flour in 107 days; how much is that per day?

Questions.—Which is the greater, a gallon or a hogshead? What part of a hogshead is a gallon? A hogshead is how many times as great as a gallon? What is meant by a "garrison"? Which numbers are necessary to the solution of the 169th problem?

LESSON XXVI. 8 4

170. How long will it take a vessel to sail from Boston to Canada; the distance is 9036 miles, and the ship sails 155 miles per day?

171. If a planter wants to pack 42864 pounds of cotton in 94 bales, how many pounds must be put in a bale?

172. A farmer owned a field containing 142 acres, which produced 2648 bushels of wheat; what is the number of bushels per acre?

173. In what time can a vat containing 109440 gallons be emptied by a pump discharging 608 gallons per hour?

174. The product of two numbers is 1,893,312; the multiplier is 912; what is the multiplicand?

175. The product of two numbers is 833382; the multiplier is 207; what is the multiplicand?

176. How long could 27 men live on a stock of provisions that would last 1 man 3456 days?

177. If a barrel holds 264 apples, how many barrels will 33432 apples make?

178. There are 100 cents in one dollar; how many dollars in 38,741,600 cents?

179. How many dollars in 34,678,413 cents?

Questions.—How many terms in multiplication? Which two do we multiply together to make the other? What are the multiplier and multiplicand sometimes called? What are the factors of ten? Of fifteen? Of twenty-one?

Review.—What is a unit? (1.) What is the difference between an abstract and a concrete number? (4.) (3.) What is a number? (2.) What is the difference between the Roman and Arabic notations? (7.) (8.) How many characters are used in each? What is the difference between the simple and local values of a figure? (9.) (10.)

LESSON XXV. 25

180. There are 1000 mills in 1 dollar; how many dollars in 47,834,000 mills?

181. How many dollars in 678,300,413 mills?

182. 246 men incur an expense of $175152; what is each man's share?

183. Twelve men own a bridge from which they receive $2352 a year in toll; what is each man's share?

184. If 7776 peach trees are planted in 108 rows, how many trees in a row?

185. The dividend is 3,841,671, the divisor is 4167; what is the quotient?

186. The dividend is 3,718,643, the divisor is 372; what is the remainder?

187. In 437850 yards of cloth how many rolls of 75 yards each?

188. A merchant bought a car load of salt; he paid 85 cents per barrel, freight; how many barrels did he buy if his bill amounted to 7800 cents?

189. A man bought 98746 barrels of flour at $9 per barrel; he sold it so as to gain $2 per barrel; how many barrels must he sell to gain $41374?

QUESTIONS.—Which numbers are essential in the 189th problem? Which numbers might be omitted?

LESSON XXVI. 26

190. 678371341÷30=what?
191. 4876783413÷31= “
192. 418378341÷32= “
193. 41678341÷33= “
194. 491837013÷34= “
195. 398000001÷35= “
196. 418370003÷36= “
197. 148670008÷37= “
198. 48367000÷38= “
199. 418370000÷39= “

200: 418783786÷40 what?
201. 867138480÷51= “
202. 4837864÷62= “
203. 9183718÷73= “
204. 87418378÷84= “
205. 671837183÷95= “
206. 41837834÷106= “
207. 418387838÷107= “
208. 394183786÷108= “
209. 46783783÷109= “

REVIEW.—Which are the digits? (8.) What is a rule? (16.) What is the rule for notation? (17.) For addition? (29., *e.*) For subtraction? (38., *d.*) For multiplication? (48., *d.*) For division? (60., *d.*) What is the sum of two or more numbers? (18.)

LESSON XXVII. 27

210. $\frac{418378643434}{109}=$ what?

211. $\frac{3718378341641}{211}=$ “

212. $\frac{41837183413}{241}=$ “

213. $\frac{387834183712}{263}=$ “

214. $\frac{418371838467}{285}=$ “

215. $\frac{864137838413}{297}=$ “

216. $\frac{83783418673}{378}=$ “

217. $\frac{41839183743}{432}=$ “

218. $\frac{483199000300}{489}=$ “

219. $\frac{387183418364}{497}=$ “

220. $\frac{38411011011103}{1513}=$ what?

221. $\frac{878371867183}{1213}=$ “

222. $\frac{4100100033303}{2130}=$ “

223. $\frac{4018301304130}{3437}=$ “

224. $\frac{91837867834}{4537}=$ “

225. $\frac{418378341783}{5384}=$ “

226. $\frac{56783784137}{6418}=$ “

227. $\frac{878371830413}{7830}=$ “

228. $\frac{91808303080}{8613}=$ “

229. $\frac{898378418384}{9371}=$ “

LESSON XXVIII. 28

230. $\frac{878346734}{4183712}=$ what?

231. $\frac{91871340764}{9183784}=$ “

232. $\frac{918378041}{689783}=$ “

233. $\frac{91837864}{3718341}=$ “

234. $\frac{68300070}{4183713}=$ “

235. $\frac{41837807}{891306}=$ what?

236. $\frac{91837804}{641830}=$ “

237. $\frac{890013783}{4183786}=$ “

238. $\frac{418378346}{418343}=$ “

239. $\frac{983783416}{9137832}=$ “

Review.—What is the answer called in addition? (18.) In subtraction? (34.) In multiplication? (44.) In division? (53.) What is the sign of addition? (20.) Of subtraction? (36.) Of multiplication? (46.) Of division? (57.) In what does the fractional sign of division consist? (58.) What is the dollar sign? (22.) What is the sign of equality? (21.) What does the parenthesis show? (40.)

240. $\frac{6783783418}{4130070}$ = what?

241. $\frac{96418370}{418370}$ = "

242. $\frac{9137864}{67834}$ = "

243. $\frac{418378376}{4183703}$ = "

244. $\frac{9137183702}{8378341}$ = "

245. $\frac{6783718346}{4137834}$ = what?

246. $\frac{9183783416}{41374}$ = "

247. $\frac{41837834830}{678341}$ = "

248. $\frac{678378346}{418371834}$ = "

249. $\frac{9183401101}{30110110}$ = "

LESSON XXIX.

MISCELLANEOUS PRACTICAL PROBLEMS,

Which contain not more than two analytical steps.

PROBLEM.—A farmer has 18 bushels of rye worth 396 shillings; what are 49 bushels worth at the same rate?

MODEL OPERATIONS.

(*a.*)

18)396 sh.
22 "
49 "
198 "
88 "
1078 sh. ANS.

(*b.*)

sh.
$396 \div 18 \times 49 = 1078$ sh. ANS.

Anal. steps.—1. ***Find the cost of* 1 *bushel.***
2. ***Find the cost of* 49 *bushels.***

FORMULAS.—1. 1 bushel will cost 1 *eighteenth* as much as eighteen bushels eighteen bushels cost 396 shillings, therefore 1 bushel will cost 1 *eighteenth* of 396 shillings, which is 22 shillings.

2. Forty-nine bushels will cost 49 times as much as 1 bushel; 1 bushel costs 22 shillings, therefore 49 bushels will cost 49 times 22 shillings, which is 1178 shillings.

250. A grocer has 13 chests of tea which cost him 35490 cents; at that rate what will 47 chests cost him?

251. A merchant purchased 22 yards of cloth for 5258 cents; at that rate what will 78 yards cost?

252. A drover sold 685 cows for $30825; how much did he receive for 13 of them?

253. A farm of 224 acres was purchased for $5600, and afterward sold for $1372 more than the cost; how much was it sold for per acre?

254. Forty-seven acres of land near New York were bought for $32806, and afterward sold for $2467 less than cost; what was it sold for per acre?

255. If 27 oranges cost 162 cents, what will 327 cost?

256. If 29 yards of cloth cost $116, what will 37 yards cost?

257. If 39 sheep are worth $312, what are 37 sheep worth?

258. If I pay $462 for 66 sheep, how much must I pay for 37 sheep?

259. If a man can travel 1560 miles in 130 hours, how far can he travel in 13 hours?

LESSON XXX.

MENT. PROB.—If 6 men can do a piece of work in 1 day, how many men will it take to do it in 3 days?

FORMULA.—3 days will require one-third as many men as one day, 1 day requires 6 men; therefore 3 days will require 1 *third* of 6 men, which is 2 men.

260. If 50 men can cut 300 cords of wood in 30 days, how many cords can 1 man cut in the same time?

261. If it takes 50 men 30 days to cut 300 cords of wood, how long will it take one man to cut the same quantity?

262. If it takes 50 men 30 days to cut 300 cords of wood, how many men will it take to cut it in 1 day?

263. How many men will it take to dig a ditch 30 rods long in 27 days, if 108 men can dig it in 1 day?

REVIEW.—What axioms are used in addition? (23., *a.*) In subtraction? (37.) In multiplication? (47.) In division? (59.) What is the difference between simple and complex problems? (25.) (26.) What is the difference between an analysis and an analytical step? (27.) (28.)

264. How many men can earn $437 in 37 days if 148 men can earn the same amount in 1 day?

265. If 840 men can build a wall 30 feet long, 3 feet thick and 9 feet high, in 1 day; how many men will be required to build it in 12 days?

266. If 42 barrels of flour and 13 barrels of beef will last a garrison of 822 men 1 week, how many men will it support for 6 weeks?

267. If $13688 will support a family of 8 persons 8 years, how many dollars will it cost to support the same family 1 year?

268. If $13688 will support a family of 8 persons 8 years, how long will it support one person?

269. If $13688 will support a family of 8 persons 8 years, how many persons will it support one year?

QUESTIONS.—How many times longer will it take one man to do a certain job than it will take six men to do the same? To do it in one day will take how many times as many men as 6 days?

LESSON XXXI.

MENT. PROB.—If 1 man can do a certain task in 16 days, in what time can 4 men do it?

FORMULA.—4 men will require 1 *fourth* as much time as 1 man, 1 man requires 16 days; therefore 4 men will require 1 *fourth* of 16 days, which is 4 days.

MENT. PROB.—If 3 men can do a certain task in 12 days, how long will it take 9 men to do the same work?

Anal. Steps.—1. *Find how long it will take* 1 *man to do the work.*
2. *Find how long it will take* 9 *men to do the same work.*

270. If 27 men can build a fort in 40 days, how many days will it take 9 men?

271. How many days will it take 18 men to do as much work as 54 men can do in 12 days?

REVIEW.—How do you prove a problem in addition? (29., *f.*) In subtraction? (38., *e.*) In multiplication? (48., *e.*) In division? (60., *e.*) Define the terms used in addition. (18.) In subtraction. (32.) (33.) (34.) (35.) In multiplication. (42.) (43.) (44.) (45.) In division. (51.) (52.) (53.) (56.)

272. If 32 horses eat 4 tons of hay in 6 weeks, how long will it last 8 horses?

273. If 32 horses eat 4 tons of hay in 6 weeks, how many horses will it last for 8 weeks?

274. If 32 horses eat 4 tons of hay in 6 weeks, how many horses will 8 tons last for the same time?

275. If it took 48 men 37 weeks to build a row of houses, how long would it have taken 4 men?

276. A farmer sold 344 barrels of flour at $5 per barrel, and took his pay in cows at $43 a head; how many cows did he get?

277. How much honey at 18 cents per pound must be given for 467 pounds of butter at 36 cents per pound?

278. A merchant gave 26 boxes of lemons for 7 firkins of butter worth $52 per firkin; what was their value per box?

279. A man owed me $4786 and paid me only $2374; I agreed to take the rest in flour at $9 per barrel; how many barrels should I receive?

QUESTIONS.—What is the difference between the 272d and 273d problems? Between the 273d and 274th? In the 275th problem, will 144 men take more or less time than 48 men? How many steps in the problem? What is the rule for the first? For the second?

LESSON XXXII.

280. Divide 346 billions 80 thousand 328 by 3.
281. Divide 34 billions three hundred and sixty-nine by 5.
282. Divide 28 billions 650 millions and forty-five by 3.
283. Divide 66 billions 5 hundred thousand 605 by 2.
284. Divide 276 millions 528 thousand and sixty-nine by 4.
285. Divide fifty millions 60 thousand six hundred by 8.
286. Divide 426 billions 584 millions 728 thousand by 2.
287. Divide thirty-four millions 586 thousand and 6 by 9.
288. Divide 2 billions 220 millions 48 thousand by 6.

REVIEW.—Give the rule for notation. (17.) For addition. (29., *e.*) For subtraction. (38., *d.*) For multiplication. (49., *d.*) For short division. (60., *d.*) For long division. (62., *d.*)

289. Divide 360 millions 528 thousand and seventy by 3.
290. Divide 36 billions 42 thousand and sixty-four by 7.
291. Divide 428 millions 368 thousand 436 by 8.
292. Divide thirty-five millions 699 thousand and 2 by 5.
293. Divide 755 billions 455 millions 55 thousand by 3.
294. Divide 7 billions 5 millions 6 thousand and 5 by 8.
295. Divide 236 millions four hundred and twenty by 4.
296. Divide 28 billions 460 millions and 3 by 6.
297. Divide 42 millions 3 thousand and 60 by 8.
298. Divide 286 billions 66 millions and thirty by 3.
299. Divide 142 millions 361 thousand and 56 by 9.

LESSON XXXIII.

Perform the operations indicated in the following examples:—

300. $8467\div(56+7)$.
301. $8467\div(56-7)$.
302. $8467\div(56\times 7)$.
303. $8467\div(56\div 7)$.
304. $8467\times(56+7)$.
305. $8467\times(56-7)$.
306. $8467\times(56\times 7)$.
307. $8467\times(56\div 7)$.
308. $8467-(56+7)$.
309. $8467-(56-7)$.
310. $8467-(56\times 7)$.
311. $8467-(56\div 7)$.
312. $8467+(56+7)$.
313. $8467+(56-7)$.
314. $8467+(56\times 7)$.
315. $8467+(56\div 7)$.
316. $134+36-47\div 31$.
317. $467-47+36\div 4$.
318. $327\times 2-67\div 8$.
319. $413+14-78\div 21$.
320. $673-413\times 80\div 31$.
321. $(467+3+2)\times 2\div 61$.
322. $1571-(34\times 6)\div 2$.
323. $867-(413-86)\times 4$.
324. $34+(413+86)\times 4$.
325. $162\div(81\div 9)\times 47$.
326. $162\div 81\times 9\times 47$.
327. $37\times(84\div 12)+36$.
328. $37\times 84\div 12+36$.
329. $37\times 84\times 12+36$.
330. $41\times 83+67-380$.
331. $(416\div 4)\times 72+60$.

LESSON XXXIV.

COMPOSITION OF PROBLEMS.

NOTE.—If more problems are needed than are given in the book, let them be composed in the same way as in addition and multiplication.

To the Teacher.—The subjects of multiplication and division are intended to complete the year's study in the fundamental rules. The given work averages about *ten* problems for the daily lesson, which beside the weekly review, allows several weeks for a stated review at the close of each half-year term.

It is very important that before leaving these rules the pupil should be subjected to a searching examination. A standard of about *eighty-five* per cent should be required to entitle the pupil to proceed. The object of the examination should be to test the pupils with regard to the following points* :—

1. *Rapidity and accuracy in abstract operations.*

2. *Familiarity with the terms and definitions, and knowledge of their meaning.*

3. *Ability to analyze problems of not more than two elementary questions.*

TEST QUESTIONS FOR EXAMINATION.

41. What is multiplication? Multiplication is a short method of what? Can all problems in addition be done by multiplication? Can all problems in multiplication be done by addition? What is the difference between a problem that can be done only by addition, and one that can be more easily done by multiplication? ANS.—*In one, the numbers to be added are equal; in the other, they are unequal.* What is the answer called in addition? What is it called in multiplication?

42. What is the multiplicand? Has the multiplicand any corresponding number in addition? Add 184 to itself. What is the 184 called in addition? Can the problem be done by multiplication? What is the 184 called in multiplication?

43. What is the multiplier? What does the multiplier show? What do we call the number that shows how many times the 184 is taken?

44. What is the product? To what in addition does the product correspond? [The teacher should show at the blackboard the real difference between addition and multiplication.] What are the multiplier and multiplicand sometimes called?

45. Which are called factors? What is the meaning of "factor"? How many terms in multiplication? Which two do we multiply together to make the third? Which two terms must be given in order to find the third? If the multiplicand and multiplier be given, how can the product be found? If the product and multiplier be given, how can the multiplicand be found? If the product and multiplicand be given, how can the multiplier be found?

46. What is the sign of multiplication? How is it used? Should the multiplier be placed before or after the sign? Where should the product be placed? What sign should be placed between the factors and the product?

47. What are the axioms of multiplication? Illustrate them. What are the axioms of addition? Can unlike things be added together? Can things be subtracted from unlike things? Give an illustration. If 12 be added to equal numbers, will the sums be equal? If 12 be subtracted from equal numbers, will the remainders be equal? When the parts of a number are given, how can the number be found? If 6 be added to a number and then subtracted from the sum, will the remainder be greater or less than the first number? If 6 be added to both minuend and subtrahend will it change the difference? If 6 be subtracted from both the minuend and subtrahend, will it change the difference? If two equal numbers be multiplied by 9 will the products be equal? [The teacher should see that the principles are illustrated by examples at the blackboard.]

---

* NOTE.—The pupil must not be expected to understand the principles of notation or to explain multiplication and division. The task is too severe for the mind of children. They readily commit to memory, but do not readily understand principles. Better leave these explanations until the mind is more matured.

48. How should the numbers be written in multiplication? Must the multiplier be written under the multiplicand? Is the less number always the multiplier? Does it make any difference in the result, which number is used as the multiplier? Where should the products of the figures be placed? Why? What is a rule? What is the rule for multiplication? How may you prove an answer in multiplication?

49. [The scholar's ability to add rapidly and accurately should be well tested.]

50. What is division? Is there any other way of finding how many times one number contains another? ANS. *By subtraction* Illustrate.

51. What is the dividend?

52. What is the divisor?

53. What is the quotient? To what term in division does *minuend* correspond? To what does the *subtrahend* correspond? How many times is 6 contained in 18? How many times can 6 be subtracted from 18? Has the quotient 3 any corresponding number in subtraction? In subtraction, how many times is the subtrahend taken from the minuend? In division, how many times is the divisor contained in the dividend? To what does the remainder in subtraction correspond in division? What is the remainder in division? What is the remainder in subtraction?

54. What is the difference between short division and long division?

55. What is the first step in long division? What is the second?

56. What is the remainder in division? How can a remainder be changed to a fractional quotient? What is the answer called in division? What is to be done with the fractional quotient?

57. What is the sign of division? What is a sign? Illustrate the use of the sign of division. Is the sign placed before or after the divisor? What sign is placed between the divisor and quotient, when it is to be shown that 18 divided by 6 is 3?

58. What is the fractional sign of division? How are the numbers written? Which is the dividend? Which is the divisor? Illustrate the use of both signs, and show their difference.

59. If two numbers are divided by the same number, will the quotients be equal? Illustrate at the blackboard. If the divisor and dividend be multiplied by the same number, will the quotient be changed? If both divisor and dividend be divided by the same number, will the quotient be changed? What two numbers multiplied together will make the dividend? If the divisor and dividend be given, how can the quotient be found? If the quotient and dividend be given, how can the divisor be found? If the divisor and dividend be given, how can the remainder be found? If the remainder, divisor, and quotient be given, how can the dividend be found?

60. Give the complete analysis of a problem in division. Repeat the rule. State the method of proof.

61. [The pupil should be examined in the blackboard exercises, to test his readiness in giving quotients and remainders.]

62. What is the method of long division? Repeat the rule for long division: What is the first step in long division? The second? The third? The fourth? When the next figure is brought down to the remainder and it will not then contain the divisor, what is to be done? If the product of the divisor and the quotient figure is more than the partial dividend, what does it show? If any remainder equals or exceeds the divisor, what does it show? What is the opposite of division? How can division be proved? How can multiplication be proved? How does multiplying the divisor affect the quotient? How does multiplying the dividend affect the quotient? How does dividing the divisor affect the quotient?

SECTION VI.

LESSON I.

UNITED STATES MONEY.

63. ***United States,*** or ***Federal Money*** is a decimal currency.

(*a.*) TABLE.

10 mills (m.) make 1 cent. . . ct.
10 cents " 1 dime. . . d.
10 dollars " 1 eagle. . . E.

(*b.*) NUMERATION TABLE.

dollars.		cents.		mills.
eagles.	**dollars.**	dimes.	cents.	mills.
0	0.	0	0	0

(*c.*) TABLE OF ALIQUOT PARTS.

½ dol.=50 cts.
¼ " =25 "
⅕ " =20 "
⅛ " =12½ "
⅒ " =10 "
¾ " =75 "
½ ct. = 5 mills.

¼ ct. =2½ mills.
½ m. =5 tenths of a mill.
⅓ dol. =333+mills.
⅙ " =166+ "
⅐ " =142+ "
⅑ " =111+ "
$\frac{1}{12}$ " = 83+ "

64. ***Coins.****—(*a.*) **The Gold coins** are the *double-eagle*, *eagle*, *half-eagle*, *quarter eagle*, *three dollar piece*, and *dollar.*

---

* NOTES.—1. Gold coins contain 9 parts gold and 1 part of an alloy of silver and copper.

2. Silver coins contain 9 parts silver and 1 part copper, except the three-cent piece, which is 3 parts silver and 1 part copper.

3. The two-cent and cent pieces are made of nickel and copper.

4 The nickel contains 88 parts copper and 12 parts nickel.

5. The fifty-dollar piece is not a legal coin. The copper half-cent is no longer coined. The mill is not a coin.

(*b.*) **The Silver coins** are the *dollar*, *half-dollar*, *quarter dollar*, *dime*, *half-dime*, and *three-cent piece.*

(*c.*) **The Nickel coins** are the *cent*, and the new *three-cent piece.*

(*d.*) **The Copper coins** are the *cent*, and *two-cent piece.*

65. The decimal point (.) is used to separate dollars from cents and mills; thus, $46.873, equals 46 dollars 87 cents and 3 mills.

(*a.*) NOTES.—1. Since the places in Federal Money increase from right to left in a ten-fold ratio like simple numbers, it may be added, subtracted, multiplied and divided, according to the same rules.

2. Eagles are read as tens of dollars, and dimes as tens of cents; thus, 4 eagles 9 dollars 5 dimes 6 cents and three mills are read 49 dollars 56 cents and three mills.

QUESTIONS.—What is meant by "currency"? What is meant by "decimal currency"? What are the denominations of U. S. money? A mill is what part of a cent? How many mills make a cent? Which is more, a cent or a mill?

LESSON II.

MENTAL EXERCISES.

1. How many mills in 8 cents?

FORMULA.—Since in 1 cent there are 10 mills, in 8 cents there are 8 times 10 mills, which are 80 mills; therefore in 8 cents there are 80 mills.

NOTE.—In the same way analyze the following questions.

2. How many mills in 3 cents? In 8 cents? In 12 cents?

3. How many mills in 11 cents? In 24 cents? In 9 cents?

4. How many cents in 3 dimes? In 4 dimes? In 9 dimes?

5. How many cents in 40 mills?

FORMULA.—Since in 10 mills there is 1 cent, in 40 mills there are as many cents as 10 mills are contained times in 40 mills, which are 4 times; therefore in 40 mills there are 4 cents.

6. How many dimes in 30 cents? In 40 cts? In 70 cts.?

7. How many dollars in 100 cents? In 800 cts.?

REVIEW.—What is a unit? (1.) What is the difference between abstract and concrete numbers? (3.) (4.) What is a number? (2.) Of what does arithmetic treat? (5.) What is notation? (6.) What is the use of the Roman method of notation? (7.)

8. How many cents and mills in 14 mills? In 17 mills?

9. How many dollars and cents in 384 cents? In 413 cts.? In 916 cts.? In 696 cts.? In 389 cts.? In 471 cts.?

LESSON III.

REDUCTION.

66. ***Reduction*** of U. S. Money is changing a number of one of its denominations to that of another **without altering its value.**

MODEL OPERATION.

```
 $75
 100 cents.
────
7500 cents.
```

FORMULA.—Since there are 100 cents in 1 dollar, in 75 dollars there are 75 times 100 cents, which is 7500 cents; therefore there are 7500 cents in 75 dollars.

(*a.*) **Rule.**—*I. To reduce dollars to cents multiply by* 100; *that is, annex* TWO *ciphers.*

II. To reduce dollars to mills, annex THREE *ciphers.*

III. To reduce cents to mills, annex ONE *cipher.*

NOTE.—To reduce dollars and cents to cents; dollars, cents, and mills to mills, remove the decimal point and the dollar sign.

EXAMPLES FOR PRACTICE.

10. Reduce $24 to cents.
11. Reduce $36 to cents.
12. Reduce $100 to cents.
13. Reduce 66 cents to mills.
14. Reduce $4.36 to cents.
15. Reduce $3.73 to cents.
16. Reduce $14 to mills.
17. Reduce $102 to mills.
18. Reduce 7 cents to mills.
19. Reduce $5.05 to mills.
20. Reduce $6.374 to mills.
21. Reduce $487.411 to mills.

How many dollars in 3746 cents?

MODEL OPERATION.

```
    cts.
100)3746 cents.
    ───────
    $37.46
```

Formula.—Since in 100 cents there is 1 dollar, in 3746 cents there are as many dollars as 100 cents are contained times in 3746 cents, which is 37 with a remainder of 46 cents. Therefore in 3746 cents there are $37.46.

(b.) **Rule.**—*I. To reduce cents to dollars, divide by* 100; *that is, point off* TWO *figures from the right.*

II. To reduce mills to dollars, point off THREE *figures.*

III. To reduce mills to cents, point off ONE *figure.*

22. Reduce 972 cents to dollars.
23. Reduce 34174 cents to dollars.
24. Reduce 39460 cents to dollars.
25. Reduce 37134 mills to cents.
26. Reduce 43714 mills to dollars.
27. Reduce 37146 cents to dollars.
28. Reduce 41674 mills to dollars.
29. In 37164 mills how many dollars?
30. In 4163 cents how many dollars?
31. In 374641 mills how many dollars?

Questions.—Repeat the formula for reducing dollars to cents. How many mills in a dollar?

LESSON IV.

EXAMPLES FOR PRACTICE.

Copy the following expressions, and read them in dollars cents and mills.

MODEL OPERATION.

cts. m.
3743 mills = $3.743.
4403 cents = 44.03.
7084 mills = 7.084.
8780 cents = 87.80.
503 dimes = 50.30.
6078 mills = 6.078.

Review.—Name the letters used in the Roman notation. (7., *a.*) What is the effect of repeating a letter? (7., *c.*) What is the effect of placing a letter of less value before one of a greater? (7., *d.*) Of placing a letter of less value after one of a greater? (7., *e.*)

(32.)
87803 cents.
41700 mills.
34176 mills.
41307 cents.
87603 mills.
41670 mills.
30721 mills.
46870 mills.

(33.)
167 mills.
41 dimes.
13 dollars.
86 mills.
47 cents.
186 cents.
32 dollars.
120 dimes.

(34.)
4137 mills.
4876 dollars.
4137 dimes.
4138 cents.
6783 mills.
4138 mills.
5882 cents.
4031 dollars.

(35.)
16783 mills.
41364 dimes.
41307 dimes.
41307 mills.
41307 cents.
41307 dollars.
14562 dimes.
54208 dollars.

(36.)
1413 cents.
1867 mills.
1413 mills.
6713 dollars.
4137 cents.
4136 mills.
3687 cents.
4208 mills.

(37.)
878 mills.
41 cents.
67 mills.
413 mills.
60 dollars.
81 dimes.
56 cents.
38 dollars.

LESSON V.

67. ADDITION OF UNITED STATES MONEY.

MODEL OPERATION.

Add
37 dollars 37 cents 8 mills = $37.378
9 " 14 " 1 " = 9.141
108 " 0 " 0 " = 108.000
59 " 0 " 3 " = 59.003
1 " 1 " 1 " = 1.011

$214.533 sum total.

(*a.*) **Analysis.**—1. For convenience, write the numbers so that mills shall stand in the column of mills, cents in the column of cents, and dollars in the column of dollars.

2. For convenience, begin at the right hand column to add, carrying as in simple numbers, always placing the decimal point between dollars and cents.

1. Add

Fourteen cents; = $0.14
One hundred and ninety-eight dollars; =198.00
One dollar and one mill; = 1.001
Three mills; = 0.003
Sixteen dollars three cents and two mills. = 16.032
sum total.

2. Add

One dollar 13 cents and 2 mills;
Fifty-six cents and 5 mills;
Four dollars and 1 mill;
Four mills;
Three dollars and 42 cents;
Six cents and 3 mills.

3. Add

Twenty-four cents;
Five dollars and 4 mills;
One cent and 1 mill;
Two dollars and 15 cents;
Nine dollars 5 cents and 4 mills;
One dollar 1 cent and 1 mill.

4. Add

Twenty cents and 3 mills;
Five dollars and 4 mills;
Ninety-nine cents;
Twenty-six cents and 1 mill;
Four dollars 4 cents and 4 mills;
One dollar and 5 cents.

5. Add

Five mills;
Nine dollars and 2 mills;
Thirty-five cents;
Twenty-seven cents;
Six dimes and 3 mills;
Twenty cents and 1 mill.

6. Add

Five dimes 4 cents and 2 mills;
Six dollars and 2 dimes;
Fifty cents;
Six mills;
Eight dollars and 7 mills;
Thirty cents and 4 mills.

7. Add

Twenty-five cents and 1 mill;
Four dollars and 3 mills;
Three dollars 5 cents and 7 mills;
Nine mills;
Ten dimes;
Five cents and 2 mills.

8. Add

Sixty cents;
One dollar and 2 dimes;
Five dollars and four cents;
Twelve cents and 3 mills;
Six dollars;
Eight mills.

9. Add

Three dollars and 2 mills;
Eighteen cents;
Five cents and 1 mill;
One dollar and 2 cents;
Five dimes;
Eleven cents and 7 mills.

REVIEW.—What is the effect of placing a dash over a letter or combination of letters? (7., *f*.) What is the difference between the Roman and Arabic notations? (7, 8.) In what is the Arabic notation chiefly used? (8.) What is the difference between the simple and local values of a number? (9.) (10.) What is the use of the cipher? (11.)

10. Add
One dime and 2 mills;
Twenty-four cents;
Three dollars and 25 cents:
Five cents and 3 mills;
Six dollars 4 dimes;
Four cents and 1 mill.

11. Add
Six dimes and 5 cents;
One dollar and 7 cents;
Four mills;
Twenty-one cents;
Five dollars and 2 mills;
One cent and 8 mills.

12. Add
220½ dollars;
Five dollars and 3 quarters;
36½ dollars;
Ten and three quarter dollars;
Fifty-five dollars;
Five hundred and 1 quarter dollars.

13. Add
Twenty-seven dollars;
50½ dollars;
17⅛ dollars;
350¼ dollars;
Six and one half dollars;
Forty and one fifth dollars.

14. Add
Sixty and one half dollars;
200¾ dollars;
Seventy-seven dollars;
Four dollars;
Seven and one quarter dollars;
Five hundred and 50 dollars.

15. Add
Thirty-five and one half dollars;
Seventy-six and one quarter dollars;
Five hundred and 80 dollars;
Four and one quarter dollars;
Sixty and three quarter dollars;
Seventy-five dollars.

16. Add
Four and one half dollars;
50¾ dollars;
Eight hundred and 90 dollars;
Seven and one quarter dollars;
Fifty-three dollars;
Eight dollars and a quarter.

17. Add
87 dollars 4 cents and 8 mills;
Twelve dollars 3 cents and 5 mills;
16 dollars 14 cents and six mills;
4 thousand 7 hundred and 83 dollars;
Twelve dollars 2 dimes and 3 mills.

18. Add
9 hundred dollars and 6 mills;
4878 dollars and 6 dimes;
Fifty-eight cents and 3 mills;
Forty-six mills;
Four hundred and 28 mills;
Three hundred and 82 mills.

19. Add
3 dollars and 12 and one-half cents;
Forty-nine and one-half dollars;
Four hundred and 16 mills;
Nine hundred and 74 dollars;
7 thousand dollars and 12½ cents.

20. Add
516 dollars and 48½ cents;
$9878 three cents and 1 mill;
Nine thousand 400 and 48 mills;
5976837 mills;
9187683 cents.

21. Add
Two mills;
Five thousand mills;
Five thousand cents;
Five thousand dollars;
One dollar 1 cent and 1 mill.

22. Add
98718764 dollars;
187837864 cents;
41871396 mills;
49 dollars 13 cents and 2 mills;
878 dollars 46 cents and 9 mills.

23. Add
One half a cent;
One half a dollar;
One half a dime;
One quarter of a dollar;
Fifty-five and three-quarter dollars.

24. Add
9786 dollars 39 cents and 4 mills;
9186 cents and 5 mills;
38718¾ dollars;
418786½ cents;
4918713¾ dollars.

25. Add
4987643½ dollars;
9187834½ cents;
4187867 mills;
Three quarters of a dollar;
One quarter of a dollar;
Thirty-seven and one-half cents.

26. Add
Six dollars 12½ cents;
Five dollars 3 mills;
89 dollars 87 and one-half cents;
Forty-six dollars;
Fifty-nine mills.

27. Add
374 dollars 6 cents and 2 mills;
67 dollars 20 cents and 8 mills;
47 dollars 23 cents and 4 mills;
473 dollars 90 cents;
672 dollars 40 cents 8 mills.

QUESTIONS.—How are numbers written in addition of United States money? Why? Can cents be added to mills? Why? What axiom would be violated? What must be done with dollars before adding them to mills To cents before adding them to dollars? What is reducing numbers from one denomination to another without changing their value?

LESSON VI.

Write the following amounts in mills and add.—

MODEL OPERATION.

$878.371 = 878371 mills.
$591.37 = 591370 "
$918.32 = 918320 "
Ninety-one dollars and 13 cents = 91130 "
Eight hundred and forty-nine dollars = 849000 "

3328191 mills

REVIEW.—What is the difference between numeration and notation? (12.) (6.) Repeat the numeration table by periods from right to left. (13., *b*.) From left to right. What is the rule for notation? (17.) What is a rule? (16.) What is meant by "concise"?

28. Write and add in mills,
$49;
876 dollars 13 cents;
Five cents;
Three and one-half cents.

29. Write and add in mills,
Fifty-nine dollars 13½ cents;
Eighty-nine and ¾ dollars;
Twelve and one-half cents.
9 millions 9 thousand and $9;

30. Write and add in mills,
$4876;
8718¾ dollars;
87134¼ dollars;
41878½ cents;
418387 mills.

31. Write and add in cents,
$4187834;
918346¼ dollars;
Three dollars and 3 cents;
Fifty-nine dollars and 7 cents;
$0.86;
$0.09.

32. Write and add in mills,
$384;
$964.80;
$.93½;
$.04½:
$.903;

33. Write and add in cents,
$876.04,
$389.
$700;
$.87;
$94.20;
$46¾;
$55¼.

34. Write and add in dollars,
9871673 mills;
41878376 cents;
8718371 dollars;
One dollar and 2 mills;
Seventeen cents;
49 dollars, 13 cents, 1 mill;
Six eagles and 5 dimes;
46 eagles and 13 dimes;
Four dollars and 27 dimes.

35. Add the costs of the following items.—
Bought 1 box of candles for 6 dollars and 89 cents.
Bought 3 boxes of raisins for 13 dollars and 41 cents.
Bought 5 boxes of cheese for 12 dollars and 2 mills.
Bought 4 casks of wine for 193 dollars.
Bought 3 gallons of molasses for 1 dollar and 2 cents.

36. Add the costs of the following items.—

9 cows	costing	198 dollars;
55 calves	"	325 dollars;
14 oxen	"	900 dollars;
3 hens	"	37½ cents;
9 ducks	"	1 dollar 87½ cents.

REVIEW.—What are the terms of addition? (18.) Of subtraction? (32.) (33.) (34.) (35.) Of multiplication? (42.) (43.) (44.) (45.) Of division? (51.) (52.) (53.) (56.) Define them.

LESSON VII.

PRACTICAL PROBLEMS.

37. What is the total cost of the following items:—

9 wagons for 312 dollars six and one-half cents,
4 horses for 987 dollars five mills,
1 wheelbarrow for 3 dollars and 3 mills,
9 plows for 34 dollars 4 cents,
1 pig for 1 dollar 1 cent and 1 mill.

38. What is the cost of the following items:—

1 gallon of molasses for 37½ cents,
1 pound of cheese for 12½ cents,
3 pounds of butter for 87½ cents,
41 pounds of coffee for 20¾ dollars,
19 gallons of vinegar for 5¼ dollars and 3 mills.

39. A grocer bought a box of candles for 6 dollars 89 cents, a box of cheese for 25 dollars four cents and three mills, a keg of raisins for 1 dollar 12½ cents, and a cask of wine for 40 dollars 37 cents and 8 mills; what did the whole cost him?

40. A farmer bought a cow, for which he paid 30 dollars and 4 mills, a horse for which he paid 104 dollars 60 cents and 1 mill, a wagon for which he paid 85 dollars and 9 mills; how much did the whole cost?

41. Add together 98 dollars, 347 cents, 384 mills, 4786¾ dollars, 387167 cents, and 418713 mills.

42. Mr. Jones sold farmer Sykes 6 chests of tea for 175 dollars 81 cents, 9 yards of broadcloth for $37.878, a plow for 9 dollars 1 mill, a set of harness for $19 and 4 mills; what is the amount of the bill?

43. A grocer sold Mr. Williams 18 hams for $26.30, a bag of coffee for 17 dollars and three mills, a chest of tea for 27 dollars and 4 cents and 7 mills, a firkin of butter for $28.137; what is the amount of the bill?

REVIEW. What are the signs used in arithmetic? (20.) (21.) (22.) (36.) (40.) (46.) (57.) (58.) Describe their uses. What is the answer called in addition? In subtraction? In multiplication? In division? What is a problem? (24.) A simple problem? (25.) A complex problem? (26.)

44. A father bought a suit of clothes for each of his four boys; the suit for the eldest cost 15 dollars 16 cents and 7 mills, that for the second $13.407, that for the third $12\frac{3}{4}$ dollars and that for the youngest 11 dollars four cents and 7 mills; how much did he pay for all?

45. A man is indebted to A $630.49, to B $25, to C $87\frac{1}{2}$ cents, to D four mills; how much does he owe?

46. A father has six children; to the first two he gave $375.416 each, to each of the second two 287 dollars 55 cents, to each of the third two 259 dollars four mills; how much did he give them all?

QUESTIONS.—How may halves and quarters of dollars be added? ANS.—*By reducing them to cents.* How many cents in 3 *fourths* of a dollar? How many mills in 1 *half* of a cent? How many cents in 1 *fifth* of a dollar? In 2 *fifths* of a dollar? ANS.—*Twice the number of cents in* 1 *fifth of a dollar.* In 3 *fifths* of a dollar? In 4 *fifths* of a dollar?

LESSON VIII.

47. Bought a gallon of molasses for 28 cents, a half pound of tea for 78 cents, a piece of flannel for 12 dollars six cents and three mills, a plow for 8 dollars 1 cent and 1 mill, a pair of shoes for 1 dollar and twenty cents; what did the whole cost?

48. Bought 6 pounds of coffee for 1 dollar $12\frac{1}{2}$ cents, a wash-tub for 25 cents and six mills, a tray for 126 cents and 9 mills, a broom for 27 cents, a box of soap for $2, and a cheese for 2 dollars $87\frac{1}{2}$ cents and 3 mills; what is the whole cost?

49. What is the entire cost of the following articles:—2 gallons of molasses, 57 cents; $\frac{1}{2}$ pound of tea, $37\frac{1}{2}$ cents; 2 yards of broadcloth, \$$3.37\frac{1}{2}$; 8 yards of flannel, $9.875; 2 skeins of silk, $12\frac{1}{2}$ cents; and 4 sticks of twist, $8\frac{1}{2}$ cents.

50. Bought a coat for $17.81, a vest for $3.75, a pair of pants for $3.87, and a pair of boots for $5.18; what was the amount of the bill?

51. Sold a load of wood for 7 dollars and six cents, five bushels of corn for four dollars and seventy-five cents, and seven bushels of potatoes for two dollars and six cents; what was received for the whole?

52. Bought a barrel of flour for $6.50, a box of sugar for 9 dollars and 87 cents, a ton of coal for $12.77, and a box of raisins for $2.50; what was paid for the things?

53. Bought of J. H. Johnston a china tea-set for 10 dollars eighty-two cents, a dinner-set for 9 dollars and 62 cents and 5 mills, a lamp for ten dollars and fifty cents, a pair of vases for 4 dollars 62 cents and 4 mills, and a set of silver spoons for 13 dollars; what did all cost?

54. Paid $4.62 for a hat, $9.75 for a coat, $5.75 for a pair of boots, and 1 dollar 50 cents for an umbrella; what was paid for the whole?

55. A grocer sold a pound of tea for $.625, 4 pounds of butter for $.75, 4 doz. lemons for 87½ cents, 9 pounds of sugar for $.80, and 3 pounds of dates for 37½ cents; what was the amount of the bill?

56. A student bought a Latin grammar for $.75, a Virgil for $3.003, a Greek lexicon for $4.703, an English dictionary for $3.074, and a Greek Testament for $.745; what was the amount of the bill?

57. What is the amount due for the following items; sewing silk $0.04, gloves $0.20, suspenders $1.00, cravat $0.50, rubber shoes $1.42, stockings $0.459, hat 3½ dollars and 6 mills?

LESSON IX.

68. SUBTRACTION OF U. S. MONEY.

Subtract $189.437 from $200.

MODEL OPERATION.

$200.000	minuend.
189.437	subtrahend.
$10.563	remainder.

NOTE.—The analysis and proof are the same as in simple subtraction. Remember to place the decimal point between dollars and cents.

REVIEW.—What are the axioms of addition? (23.) Illustrate them. What are the axioms of subtraction? (37.) Illustrate them. What are the axioms of multiplication? (47.) Illustrate them. What are the axioms of division? (59.) Illustrate them.

From $900.000
Take 94.721
Remainder $805.279

From $964.34
Take 3.008
Remainder $961.332

From $800.
Take 0.092
Remainder $799.908

From $389.002
Take 9.14
Remainder $379.862

58. From $897 take $896.01.
59. From $100 take $.78.
60. From $389 take $.009.
61. From $943.86 take $.009.
62. From $50 take $.089.
63. From $95 take 75 cents.
64. From $1000 take 1000 cents.
65. From $385 take 385 mills.
66. From $897 take 1 mill.
67. From 1 cent take 1 mill.
68. From 13 cents take 13 mills.
69. From $25 take 25 dimes 25 cents and 25 mills.
70. From $13.133 take 13 dimes 13 cents and 13 mills.
71. $896.473—$91.36=what?
72. $476.31 —$19.03=what?
73. 478 dollars—14 cents=what?
74. $767—$38.463=what?
75. 387½ dollars—$41.37½=what?
76. $8713.437—473¾ dollars=what?
77. $4783.413—18 cents=what?
78. $3000—4 mills=what?
79. $400—400 mills=what?
80. $10000—1 cent=what?
81. $8713.003—3878 dollars=what
82. $46.978—$38.978=what?

QUESTIONS.—How are numbers written in subtraction of United States money? Which is the subtrahend? Which the minuend? Where should the decimal point be placed? What is at the right of the point? What is at the left? How many places do cents and mills occupy?

LESSON X.

PRACTICAL PROBLEMS,

Which contain not more than two analytical steps.

83. A man's income is $3000 a year; he spends $1187.50; how much does he lay up?

NOTE.—Analyze the following examples the same as in simple subtraction.

84. A man purchased a yoke of oxen for $78, a cow for 26 dollars and 3 mills; how much more did he give for the oxen than for the cow?

85. A man buys a horse for $97.50 and gives a hundred-dollar bill; how much should he receive back?

86. How much must be added to $60.039 to make the sum one thousand and five dollars?

87. How much must be added to thirty-eight dollars seventy-nine cents to make the sum 100 dollars and three mills?

88. A man sold his house for $3005; this sum was $98.039 more than he gave for it; what did it cost him?

89. A man bought a pair of oxen for $100, and sold them for $75.37½; did he make or lose, and how much?

90. A man starts on a journey with a hundred dollars; he spends 87 dollars and 87½ cents; how much has he left?

91. How much must you add to $40.173 to make 100 dollars?

92. A farmer purchased a pair of horses for $450, but finding one of them injured, the seller agreed to deduct $106.325; what had he to pay?

93. A farmer had a horse worth $147.49, and traded him for a colt worth but 35 dollars 4 cents and 8 mills; how much should the farmer receive in money?

REVIEW.—What is the proof of addition? (29., *f.*) Of subtraction? (38., *e*) Of multiplication? (49, *e.*) Of division? (60., *e.*) What is the difference between long division and short division? (55.) (54.) What two numbers multiplied together make the dividend? (52.) (53.)

LESSON XI.

94. My house is worth $8975.034, my barn $695.87; what is the difference of their values?

95. What is the difference between nine hundred and sixty-nine dollars $.801, and $36 nine cents and nine mills?

96. A man had $343.68; he bought a horse for 98 dollars, a carriage for $100.971; how much had he left?

97. A boy on the Fourth of July had $.89; he bought a bunch of fire-crackers for 6½ cents, 2 rockets for 12½ cents, and some powder for half a dollar; how much money had he left?

98. From 100 dollars there were paid to one man 17 dollars and 9 cents, to another $23.08, to another 23 dollars and 6 mills; how much remained?

99. A lady went shopping with $50; she purchased a dress for $15.07, a shawl for $11.10, a bonnet for $7.09, a pair of shoes for 2 dollars; how much money had she remaining?

100. From 10 dollars take 9 mills.

101. From 100 dollars there were at one time paid out, thirty-one dollars three cents and 5 mills, at another time seven dollars four cents and seven mills, at another time five dollars and five cents, at another time twenty dollars twenty cents and two mills; how much was left?

102. Bought a house for $3718.40, 92200 acres of land for $416891, a carriage and horses for $597; I paid $216378.416 down; how much did I still owe?

103. A farmer bought of a grocer the following articles; 4 pounds of coffee for 93 cents, 8 pounds of tea for $6.13, 14 pounds of sugar for $1.40, 13 yards of calico for $1.62½. He gave in return 5 bushels of corn for $4.87½, 9 bushels of oats for

---

REVIEW.—What is the currency of the United States sometimes called? (63.) Repeat the table. (68., *a.*) How many places do cents occupy? (63., *b.*) Mills? (63., *b.*) Tenths of mills? (63., *b.*) How many cents in a half dollar? (63, *c.*)

5 dollars, and 3 pounds of butter for 50 cents; what was the difference of the amounts?

104. Bought 3 cows for $150.304, and 19 horses for $4240.37; I sold them for $4960; did I make or lose, and how much?

LESSON XII.

69. MULTIPLICATION OF U. S. MONEY.

PRACTICAL PROBLEMS

Which contain not more than three analytical steps.

Problem.—If one bushel of corn costs $1.252, how much will 29 bushels cost?

MODEL OPERATION.

```
$1.252 cost of 1 bushel.
    29 number of bushels.
 ------
 11.268
 25.04
 ------
$36.308 cost of 29 bushels.
```

Anal. Steps.*—**1.** *Reduce* $1.252 *to mills.*
2. *Find cost of* 29 *bushels.*

FORMULA.—Since 1 bushel of corn costs 1252 mills, 29 bushels will cost 29 times 1252 mills, which is 36308 mills equal to $36.308; therefore 29 bushels will cost $36.308.

105. If 1 bushel of rye costs $1.25, what will 34 bushels cost?

NOTE.—Change the $1.25 to cents and analyze as before.

106. If one bushel of oats cost 65 cents, what will 98 bushels cost?

107. If one geography costs $.87½, what will 389 cost?

108. If I buy a horse for $128.371, how much must I pay for 327 horses at the same rate?

---

* The reduction to cents or mills, as the problem may require, can be omitted after the pupil thoroughly understands the process.

REVIEW.—How many cents in a quarter of a dollar? (63., *c.*) In 1 eighth of a dollar? In 1 tenth? In 3 fourths? In 3 tenths? How many mills in 1 half of a cent? (63., *c.*) In 1 fourth of a cent?

109. What will 9687 sheep cost at $3.12½ apiece? At $2.37? At $5?

110. What will 37 doz. of eggs come to at 13½ cents per doz.? At 15 cents? At 11 cents?

111. What will 146 kites cost at $.06 apiece? At 9 cents 3 mills? At a half cent each?

112. What will be the cost of 19 inkstands at 12½ cents each? At $.15? At 63 cents?

113. What will be the cost of 98764 hats at $3.97½ each? At 4½ dollars? At 3¾ dollars?

114. What will 5876 yards of cloth come to at 37 cents a yard? At $.043? At $2?

LESSON XIII.

115. What will 38783 bushels of wheat cost at $1.12½ per bushel? At $1.87½? At $2.005?

116. What will 9783 pounds of tea cost at $1.37½ per pound? At $1.15? At 50 cents?

117. What will a firkin of butter containing 90 pounds cost at 25½ cents per pound? At 30 cents? At $0.87?

118. What cost 39787 pairs of shoes at $1.87 per pair? At 99 cents? At a quarter of a dollar?

119. What is the cost of a cask of wine containing 39 gallons at $.75 per gallon? At $3? At $2½?

120. A bale of cloth contains 95 pieces costing $38.42 each; what is the entire cost of the bale?

121. What cost 98076 oranges at 3½ cents each? At $.005? At $.02½? At $.009?

122. A man bought 356 sheep at $4.67 each; at another time he bought 387 sheep at 5 dollars each; how much did they all cost him?

123. A man sold at one time 3876 bushels of apples at 87

REVIEW.—Which are the United States gold coins? (64., *a.*) Which are the silver coins? (64., *b.*) Which are the nickel coins? (64., *c.*) Which are the copper coins? (64., *d.*) What is the use of the decimal point? (65.)

cents per bushel, at another 4678 bushels at 47 cents, at another time 9876 bushels at $.347; how much did he receive for his apples?

124. Bought 864 yards of linen at 68½ cents per yard; how much did it cost? At $.34? At $.23½?

125. A man bought 4 barrels of cider, each barrel containing 31 gallons, at 14 cents per gallon; how much did the cider cost him?

126. A farmer had an orchard containing 43 rows of apple trees; each row contained 28 trees, and each tree yielded 17 bushels of apples, which he sold for $.87½ per bushel; how much did the farmer receive for his apples?

127. Two men bought a number of horses; one bought 38 at $137.25 a head; the other 57 at $139.36 a head; how much more did the second pay for his horses than the first?

128. Bought a farm containing 9783 acres for $39.46 per acre, and sold it for $59 per acre; how much did I gain by the operation?

QUESTIONS.—Six bushels of corn contain how many times as much as 1 bushel? Seven pounds of tea cost how many times as much as 1 pound? How many analytical steps in the 115th problem? How many steps in the 116th problem? How many different formulas are used in the solution of the 119th problem? How many in the 120th?

LESSON XIV.

70. DIVISION OF UNITED STATES MONEY.

Problem.—Divide $3786 by 7.

MODEL OPERATION.

Divisor 7)$3786.000 Dividend.

$540.857+ Quotient.

Proof.

$540.857+ quotient.

7 divisor.

$3785.999 equal to the dividend lacking 1 mill.

(*a.*) **Anal. Steps.—1.** *Reduce* $3786 *to mills.*
2. *Find* 1 *seventh of* 3786000 *mills.*

Formula.—One seventh of 3786000 mills is 540857 mills, equal to $540.857+ (with a remainder); therefore, $3786 divided by 7 equals $540.857+.

Note.—Division of United States currency may be proved as simple division.

129. Divide $4783.413 by 19.
130. Divide $37818.14 by 487.
131. Divide $378 by 6783.
132. Divide $83 by 674.
133. Divide $1 by 376.
134. Divide $3 by 741.
135. Divide 87 cents by 98.
136. Divide 40 cents by 87.
137. Divide 7 cents by 9.
138. $418÷374=what?
139. $341.874÷9876=what?
140. $41.837÷413=what?
141. $896÷991=what?
142. $374.40÷4837=what?
143. $.87÷670=what?
144. What is 1 ninth of $3?
145. What is 1 three-hundred-and-eightieth of $97?
146. What is 1 five-thousand-six-hundred-and-fifth of $93?
147. What is 1 eleventh of 2 cents?
148. What is 1 forty-ninth of $37.18?

Questions.—Why are dollars reduced to mills before dividing? Ans.—*So that if there is any remainder it will be a remainder of mills, and consequently of little value.* What is meant by the sign *plus* after a quotient?

LESSON XV.

If 9 pounds of sugar cost $1.12½, what will one pound cost?

MODEL OPERATION.

No. lbs. 9)$1.125 Cost 9 lb.

$0.125 Cost 1 lb.

FORMULA.—1. $1.125 are equal to 1125 mills.

2. If 9 pounds cost 1125 mills, 1 pound will cost 1 ninth of 1125 mills, which is 125 mills equal to $.125; therefore 1 pound will cost $.125.

NOTE.—If there is any remainder in dividing mills put the sign plus after the answer; for an example, see next problem. Such problems will not prove exactly unless the remainders are kept and added, as in simple division.

If 27 pounds of cheese cost $13.13, what will 1 pound cost?

MODEL OPERATION.

No. lbs. 27) $13.130 Cost 27 lbs.

$0.486+Cost 1 lb.

149. What costs 1 pound of tea, if 38 lbs. cost $18.31?

150. What costs one pound of coffee, if 946 lbs. cost $129.37?

151. What costs one pound of spice, if 29 lbs. cost $6.383?

152. What costs one gallon of vinegar, if 378 gallons cost 4 dollars?

153. What costs one gallon of molasses, if 3783 gallons cost $1876.963?

154. What costs 1 quart of milk, if 37 quarts cost $1.85? If $2.13? If $0.99?

155. What costs 1 sheep, if 42 sheep cost $112? If they cost $113.50? If $100? If $99.372?

156. What costs 1 duck if 987 ducks cost $413.25? If they cost $389.41? If $400.001?

LESSON XVI.

157. What costs 1 bushel of wheat, if 91876 bushels cost $93478? If they cost $100436?

158. What costs 1 book, if an edition of 3934 cost $513.78? If they cost $554.14? If $500?

REVIEW.—What is the reduction of United States money? (66.) What is the rule for reducing dollars to lower denominations? (66., *a.*) How are dollars reduced to mills? Dollars to cents? Which is more, a cent or a mill? A cent or a dime?

159. If I give $5.85 for 9 pounds of tea, what will one pound cost?

160. Paid $29.68 for 14 barrels of apples; what was the price per barrel?

161. If 27 bushels of potatoes cost $10.125, what is the price of one bushel?

162. If a man receives $29.25 for a month's work, how much is that a day, allowing 26 working days to the month?

163. A produce dealer paid $63 dollars for 3 barrels of eggs, each barrel containing 150 dozen; how much did he pay per dozen?

164. A man bought 13 pieces of cloth, for which he paid $88.30; each piece contained 46 yards; how much did he pay a yard?

165. I have three pastures; in the first I have 347 sheep, in the second 386, in the third 416; I sold them all for $3604.20; how much did I receive apiece?

166. What will be the cost of one cow if a herd of 380 sells for $6873? For $7041?

167. A farmer purchased a farm containing 725 acres, for which he paid $18306.25; what did it cost him an acre?

QUESTIONS.—One bushel costs what part of the cost of 6 bushels? What part of the cost of 12 bushels? What part of the cost of 30 bushels? One orange costs what part of the cost of 56 oranges? Of 30 oranges? Of 99 oranges? Of 397 oranges?

LESSON XVII.

168. A drover pays $1250 for 500 sheep; for how much must he sell them apiece, so that he may neither make nor lose by the bargain?

169. A drover pays $1360 for 537 sheep; for what shall he sell them apiece, so that he can make $150 by the bargain?

170. The dairy of a farmer produces $600; he has 25 cows; how much does he make from each cow?

REVIEW.—What is the rule for changing lower denominations to dollars? (66., *b*.) How can mills be changed to dollars? Cents to dollars? Mills to cents? A cent is what part of a dollar? A mill is what part of a cent? Of a dollar?

171. A farmer receives $1840 for the wool of 1400 sheep; how much does each sheep produce him?

172. A merchant buys a piece of goods containing 105 yards, for which he pays $262.50; he wishes to sell it, so as to make $52.50; how much must he ask per yard

173. A farmer buys 136 calves, for which he pays $1000; he wishes to make $95; how much must he sell them for apiece?

174. A boy had 376 marbles for which he paid 2 dollars; for how much must he sell each one to make $.87½?

175. A merchant bought 386 yards of silk for which he paid $648.39; how much must he sell it for per yard to make $100?

LESSON XVIII.

If I pay $4.50 a ton for coal, how many tons can I buy for $67.50?

MODEL OPERATION.

Cost per ton $4.50)$67.50 Amount to spend.

15 No. of tons bought.

Formula.—1. In $67.50 there are 6750 cents; in $4.50 there are 450 cents.

2. Since 1 ton of coal costs 450 cents, I can buy as many tons for 6750 cents as 450 cents are contained times in 6750 cents, which are 15 times; therefore I can buy 15 tons for $67.50.

Note.—The dividend and divisor must always be reduced to the lowest denomination mentioned in either; thus, if the divisor contain only dollars, and the dividend contain mills, the divisor must be changed to mills.

176. How many pounds of tea at 75 cents per pound, can be bought for $6.75?

177. How many barrels of apples at $2.73½ a barrel, can be bought for $49.23?

178. At 34 cents a bushel, how many barrels of apples can I buy for $13.60, allowing 2 bushels to the barrel?

179. At 39 cents a bushel, how many barrels of potatoes can be bought for $14.04, allowing 2 bushels to the barrel?

180. If 1 acre of land costs $28.75, how much can be bought for $3220?

181. Paid $40.50 for a pile of wood at $3.37½ a cord; how many cords in the pile?

182. How many sheep can be bought for $132, at $1.37½ per head?

183. At $4.25 per yard, how many yards of cloth can be bought for $68?

184. At $1.12½ a day, how long would it take a person to earn $157.50?

185. If I spend $150 for coal at $6 per ton, how many tons can I buy?

186. At $5 a ream, how much paper can be bought for $175?

187. At $7.50 per barrel, how many barrels of flour can be bought for $217.50?

188. At $75 per ton, how many tons of iron can be bought for $4875?

QUESTIONS.—What must be done before cents can be taken from dollars? ANS.—*Reduce the dollars to cents.* How can mills be taken from cents? From dollars? What must be done before you can find how many times the divisor can be subtracted from the dividend? ANS.—*Reduce them to the same denomination.* In the 182d problem, to what should the dividend be changed? Why? To what in the 185th problem? Why?

LESSON XIX.

MISCELLANEOUS PROBLEMS, IN U. S. MONEY.

189. A man bought cows at $11.41 per head; how many could he buy for $775.88?

190. A drover bought oxen at $44.064 each; how many could he buy for $12602.304? For $12646.368? For $25292.736?

191. At $41.373 per acre, how many acres can a man buy for $1903.158? For $1944.531? For $2027.277?

192. At $.032 apiece, how many oranges can one buy for $159.584? For $319.424? For $638.848?

193. A man sold six cows for $31.50 apiece; how many sheep can he buy with the money at $2.25 each?

194. A farmer sold 16 horses at $102.125 each; how many barrels of flour can he buy with the money, at $5.37½ per barrel?

195. If 1 yard of cloth cost 3½ dollars, what will 8 yards cost? 16 yards? 87164 yards?

196. If 1 ton of hay cost 14½ dollars, what will 9 tons cost? 13 tons? 18764 tons?

197. If a calf costs 4¼ dollars, what will 12 calves cost? 136 calves? 418764 calves?

198. Mr. Jones bought 250 bushels of oats for which he paid $156.25; how much did they cost him a bushel?

199. If 12 tons of hay cost $138.50, what will 1 ton cost? 3 tons? 67 tons? 1837 tons?

2

LESSON XX.

200. If 9 dozen spelling books cost $7.87½, what will 1 dozen cost? 6 dozen? 387 dozen?

201. If 75 bushels of wheat cost $131.25, what will 1 bushel cost? 38 bushels? 9876 bushels?

202. If 320 pounds of coffee cost $44.80, how much will 1 pound cost? 386 pounds? 978413 pounds?

203. Mr. Charles Smith bought 9 barrels of sugar weighing 216 pounds each, for which he paid $116.64; how much did he pay a pound?

204. If 40 tons of hay cost $580, how much is that per ton? What would 37 tons cost at the same rate? 40 tons? 37864 tons?

205. If Mr. Wilson pays $120 for his winter's wood, and wood is $4 per cord, how many cords does he buy?

206. At 6 dollars a yard, how many yards of cloth can be bought for $24? For $36? For $144?

207. A farmer sold a yoke of oxen for $80.75, 6 cows for $29 each, 30 sheep at $2.50 each, and 3 colts at $33 each; what did he receive for all?

208. A merchant buys 6 bales of goods; each bale contains

36 pieces; the whole costs him $5660; how much does it cost him apiece?

209. A person sells 3 cows at $25 each, and a yoke of oxen at $65; he agrees to take in payment 60 sheep; how much do his sheep cost him a head?

210. A man left an estate of $33463.78 to be divided equally among his four children after his wife had taken her third; what did each child receive?

QUESTIONS.—If a boy has 10 cents, how many times can he subtract 2 cents from them? ANS.—*As many times as* 2 *cents are contained in* 10 *cents.* How many times are 2 cents contained in 10 cents? ANS.—5 *times.* What is the difference between subtraction and this form of division? ANS.—*Division is subtracting the divisor from the dividend as many times as there are units in the quotient, and subtraction is subtracting the divisor from the dividend but once.* Illustrate it at the blackboard. Which in this case is the abstract number? ANS.—*The quotient.*

LESSON XXI.

211. A man settling with his butcher finds that he is charged with 156 pounds of veal at 11 cents a pound, and 37 pounds of beef at 14 cents a pound; how much does he owe?

212. A farmer agrees to furnish a merchant 40 bushels of rye at 62 cents per bushel, and to take his pay in coffee at 16 cents per pound; how much coffee will he receive?

213. A farmer has 6 ten-acre lots, in each of which he pastures 6 cows; each cow produces 112 pounds of butter, for which he receives $.18½ per pound; how much does he receive from his dairy?

214. Bought a farm of Mr. W. Smith for $2346.12, a span of horses for $367.20, 6 cows at 36 dollars each, and paid him $2786.437; how much do I still owe?

215. What cost
15 pounds of tea @ 37 cents,
19 pounds of coffee @ 13½ cents,
46 gallons of molasses @ $.37½,
37 gallons of vinegar @ 11 cents?

216. What cost
37 chests of tea @ $37.46,
37 bags of coffee, 97 pounds each @ $.12½,

49 casks of rice, @ $42,
60 barrels of oil, 38 gallons each @ $1.08?

217. What cost
49748 feet of boards at $.03 per foot,
18764 shingles at $\frac{1}{2}$ cent each,
987864 square feet of boards at 3 cents per foot,
49187 lath at 3$\frac{1}{2}$ cents each?

218. What cost
46 bushels of turnips @ $.36,
97 bushels of carrots @ $.25,
47 bushels of beets @ 63 cents,
138 bushels of potatoes @ $.87$\frac{1}{2}$?

QUESTIONS.—What is meant by a third of anything? How many steps in the 211th problem? Write on the blackboard the steps of the 212th problem. [Until the pupil becomes expert, he should write out the steps of every problem, when he studies his lesson.]

LESSON XXII.

BILLS.

71. ***A Bill or Invoice of goods*** is a written statement of the goods purchased given by the seller to the buyer.

(*a.*) **The Debtor** is the party who owes.

(*b.*) **The Creditor** is the party who is owed.

(*c.*) **An Account** is a bill which contains both debit and credit items.

(*d.*) When a bill amounting to $20 or more is receipted, a U. S. revenue stamp must be put on it and cancelled.

Find the costs of the items and the amounts of the following bills.

REVIEW.—Define a unit. (1.) A number. (2.) A concrete number. (3.) An abstract number. (4.) Notation. (6.) Numeration. (12.) A rule. (16.) Sum. (18.) Addition. (19.) An axiom. (23.) A problem. (24.) A simple problem. (25.) A complex problem. (26.) Subtraction. (31.) Minuend. (32.) Subtrahend. (33.) Remainder. (34.) Difference. (35.) Multiplication. (41.) Multiplicand. (42.) Multiplier. (43.) Product. (44.) Division. (50.) Dividend. (51.) Divisor. (52.) Quotient. (53.) Remainder in division. (56.) Recite the rule for notation. (17.) For addition. (29., *e.*) For subtraction. (38., *d.*)

219. MODEL OPERATION.

New York, July 1, 1866.

Schermerhorn, Bancroft & Co.,

To Chas. Scribner & Co., Dr.

1866.					
April 10.	To 25 Sheldon's Ob. Less. @ 1.50	$37	50		
" 16.	" 200 First Lessons @ 20c	40			
May 10.	" 500 Primary Arith. @ 25c	125			
		202	50		
	Less 10 %.	20	25	182	25
	Cr.				
April 18.	By 12 doz. Potter & Ham. writ. books @ 1.80	21	60		
May 25.	" School apparatus	137			
		158	60		
	Less 10 %.	15	86	142	74
	Balance due C. S. & Co.			$39	51

[Stamp.] *Received Payment,*

Chas. Scribner & Co.,
* per Derby.

220. Brooklyn, L. I., Sept. 3, 1866.

Mr. Simeon Church,

Bo't of S. H. Clarke,

36 pounds of sugar @ $.09½,
3 hhd. molasses 63 gallons each @ $.27 per gal.,
5 casks of rice 285 pounds each @ $.05 per pound,
2 chests of tea 86 pounds each @ $.96 per pound,
4 barrels of oil 31 gallons each @ $.87½ per gal.

Received Payment, $

S. H. Clarke.

221. Manchester, N. H., June 17, 1866.

Mr. Hiram Fish,

Bo't of William Pratt,

69 pieces of carpeting 47 yards each @ $1.13 per yd.
87 " of Brussels do. 35 " @ $1.87½ "
147 " of sheeting 47 " @ $ .05½ "
378 " of calico 39 " @ $ .08 "

Received Payment, $

William Pratt.

---

* When a clerk or agent receipts a bill, he writes the signature of the firm, and attaches his own as factor or agent.

222. BUTTERNUTS, N. Y., May 3, 1866.

Mr. LYMAN A. BURR,

Bo't of HENRY FISK,

69 chests of tea	@ $55.65
126 bags of coffee 100 lbs. each	@ $ .12½,
167 boxes of raisins	@ $ 2.75,
800 bags of almonds	@ $18.50,
9004 bbl. of shad	@ $ 7.50,

$

Received Payment,
HENRY FISK.

223. NEW YORK, Oct. 13, 1866.

Dr. J. R. WHITE,

Bo't of HOLMES, REED & Co.

82 gallons	of Domestic Wine	@$	.75,
99 do.	Port	@	.92,
47 do.	Brandy,	@	.87½,
84 bottles	Madeira Wine	@	1.04,
37 do.	Champagne	@	1.00,

$

Received Payment,
HOLMES, REED & Co.,
per WILLIAMS.

224. TRENTON, N. J., Dec. 14, 1861.

Mr. LEVI WEBSTER,

Bo't of JAMES WILLIAMS,

6 lb. Chocolate	@ .18,
12 lb. Flour	@ .20,
6 pairs Shoes	@1.80,
30 Candles,	@ .005

$

225. NEWARK, N. J., Nov. 17, 1866.

Dr. L. DENNIS,

Bo't of SIMON GRAY,

14	oz. of	Gum Camphor	@$ .63,
12	"	Laudanum	@ .88,
23	"	Gum Arabic	@ .62,
16	"	Tartar Emetic	@ 1.27,
17.	"	Cantharides	@ 2.25,

$

Received Payment,
SIMON GRAY,
per OSGOOD.

226. SOUTHAMPTON, MASS., Mar. 3, 1866.

HENRY ROSE & Co.,

Bo't of MOSES, ATWOOD & Co.

86 Shovels	@ $0.50,
96 Spades	@ .86,
18 Plows	@ 11.00,
23 Handsaws	@ 3.50,
14 Hammers	@ .62.

$

Received Payment,
MOSES, ATWOOD & Co.

227. NEWARK, N. J., Sept. 29, 1866.

Mr. J. F. HUNTSMAN,

Bo't of A. WARD & SON.

37 chests of Green Tea	@ $23.75,
42 chests of Black do.	@ 17.50,
43 casks of Wine	@ 99.00,
12 crates Liverpool Ware	@ 75.00,
19 bbls. Genesee Flour	@ 7.00,
23 bushels Rye	@ 1.52.

$

228.

Troy, N. Y., Oct. 13, 1866.

Mr. Samuel Jones,

To Dr. M. Felter, Dr.

To 3 Professional visits in June	@ $1.50,
" 15 do. " " July	@ 1.00,
" 5 do. " " Aug.	@ 1.25,
" Medicine for family	4.37½,
" 9 Professional visits to child in Sept.	@ 2.00,
" 5 do. " " Oct.	@ 5.00.

$

Received Payment,
M. Felter.

229.

New York, May 13, 1866.

Mr. E. Lewis,

Bo't of Chas. Woodbridge & Co.,

80	pairs of	Silk Hose	@ $1.20,
17	"	Boots	@ 3.00,
19	"	Shoes	@ 1.08,
23	"	Silk Gloves	@ .75½,
186	"	Cotton Gloves	12.86,
37	"	Kid Gloves	35.00.

$

Received Payment,
Chas. Woodbridge & Co.,
per H. Smith.

230.

Albany, Aug. 29, 1866.

Mr. Samuel Green,

Bo't of Wm. Wilde,

27 Young Readers,	@	$.20,
10 Greek Lexicons,	@	3.90,
7 Ainsworth's Dictionaries,	@	2.37½,
20 Testaments,	@	.37½,
19 Bibles,	@	3.75,

$

LESSON XXIII.

MISCELLANEOUS PROBLEMS IN U. S. MONEY

Which contain not more than three elementary questions.

Problem.—A man bought 19 cords of wood at $3 per cord, and 27 tons of coal at $5 per ton; what did they both cost?

MODEL OPERATION.

(1.)	(2.)	(3.)
19 cords of wood.	27 tons of coal.	$57 cost of wood.
$3 per cord.	$5 per ton.	135 cost of coal.
$57 cost of wood.	$135 cost of coal.	$192 cost of both.

Anal. steps.—**1.** *Find the cost of the wood.*

Formula.—If 1 cord of wood costs $3, 19 cords will cost 19 times $3, which is $57.

2. *Find the cost of the coal.*

Form.—If 1 ton of coal costs $5, 27 tons of coal will cost 27 times $5, which is $135.

3. *Find the cost of both.*

Form.—If the wood costs $57 and the coal $135, they both cost the sum of $57 and $135, which is $192.

231. A man sold 38 horses for $248 each, and 24 oxen for $38 each; how much did he receive for them?

232. A drover bought 138 cows at $19 per head, and 437 oxen at $25 per head; how much more did he give for the oxen than for the cows?

233. A farmer has two orchards: in one he has 83 rows of apple trees, and there are 38 trees in each row; in the other he has 93 rows, and there are 105 trees in each row; how many trees in both orchards?

234. There are two books: the first has 998 pages, and on each page 3849 letters; the second has 913 pages, and on each page 4003 letters: how many letters in both books?

Review.—What are bills? (71.) What is a debtor? (71., *a.*) What is a creditor? (71., *b.*) When is a bill called an account? (71., *c.*) When should a U. S. revenue stamp be attached? (71., *d.*)

There are 5 piles of boards: each pile contains 943 boards, and each board 19 feet, and each foot costs 3 cents; how much do the boards cost?

MODEL OPERATION.

$$
\begin{array}{ll}
(1.) & \left\{\begin{array}{r}
943 \text{ boards.} \\
5 \text{ piles.} \\
\hline
4715 \text{ boards.}
\end{array}\right. \\
(2.) & \left.\begin{array}{r}
4715 \text{ boards.} \\
19 \text{ feet.} \\
\hline
42435 \\
4715 \\
\hline
89585 \text{ feet.}
\end{array}\right\} \\
(3.) & \left\{\begin{array}{r}
89585 \text{ feet.} \\
3 \text{ cents.} \\
\hline
\$2687.55
\end{array}\right.
\end{array}
$$

Anal. steps.—1. *Find the number of boards in the five piles.*
2. *Find the number of feet.*
3. *Find the cost of the boards.*

235. A man sold 43 piles of boards, each pile containing 878 boards, and each board containing 21 square feet; he received 7 cents for each foot; how much did he receive for his boards?

236. A merchant bought 46 firkins of butter, each firkin containing 119 pounds; for each pound he gave 28 cents; how much did his butter cost him?

237. A merchant bought 39 pieces of silk, each piece containing 48 yards, and each yard costing 98 cents; how much did the silk cost?

238. A merchant sold 47 boxes of raisins; each box contained 37 pounds, and for each pound he received 28 cents; he gave away $6.20 to a poor woman; how much money had he left?

LESSON XXIV.

239. A miser had 4 chests of money; in each chest there were 29 bags, and in each bag there were $898; while counting it, he had $399 stolen; how much had he left?

240. A sugar merchant sold 4 hogsheads of molasses, each hogshead containing 63 gallons, at 39 cents a gallon; also 13

hogsheads of sugar, each hogshead containing 598 pounds, at 5 cents per pound; how much did he receive for all?

241. A man bought at one time 187 cows at $28 a head, at another time 416 sheep at $3 per head, at another 19 horses at $89 apiece; how much did they all cost him?

242. A boy went to the store and bought 3 books at 29 cents each, 5 slates at 19 cents each, and 41 copy books at 9 cents each; how much money did he spend?

243. A lady went shopping; she purchased 29 yards of lace at 7 cents a yard, 46 yards of calico at 13 cents a yard, 12 yards of silk at 98 cents a yard; how much was her bill?

244. A man owns three farms: the first contains 486 acres worth $198 per acre; the second contains 378 acres worth $431 per acre; the third contains 698 acres worth $99 per acre; how much are the three farms worth?

245. In 1 pound English money there are 20 shillings; how many shillings in 39 pounds?

246. How many pence in 398 pounds, if there are 20 shillings in 1 pound and 12 pence in 1 shilling?

247. How many pence in 467 pounds, if there are 20 shillings in 1 pound and 12 pence in 1 shilling?

248. How many farthings in 864 pounds, if 20 shillings make 1 pound, 12 pence make a shilling, and 4 farthings make a penny?

249. How many farthings in 413 pounds, if 20 shillings make a pound, 12 pence a shilling, and 4 farthings a penny?

250. In 1 bushel there are 4 pecks, how many pecks in 839 bushels?

251. How many pecks in 9876 bushels, if there are 4 pecks in 1 bushel?

252. How many quarts in 478 bushels, if there are 4 pecks in 1 bushel and 8 quarts in 1 peck?

253. How much would 974 bushels of peanuts cost at 4 cents per quart?

254. Bought 14 cows at $23 apiece, and sold them for $28 apiece; how much did I gain by the operation?

255. Bought 98 sheep at $3 a head, and sold them for $389; did I make or lose, and how much?

256. Bought a farm containing 978 acres at $89 per acre, and sold it at $87 per acre; did I make or lose, and how much?

257. Bought 5896 bales of cotton at 6 cents a pound, each bale containing 576 pounds; I sold it for 7 cents per pound; did I make or lose, and how much?

258. Bought 46 hogsheads of sugar, of 478 pounds each at 4 cents per pound; I sold it all for $1264.48; did I make or lose by the operation, and how much?

259. Bought 26 cords of wood at $9 per cord, and 105 tons of coal at $8 per ton; how much more did the coal cost than the wood?

QUESTIONS.—How many steps in the 245th problem? How many steps in the 248th problem? State the steps in the 249th problem.

LESSON XXV.

260. A grocer bought 6 tubs of butter containing 64 pounds each at 18 cents per pound; 7 cheeses each weighing 46 pounds at 9 cents a pound; how much did the whole cost?

261. If a clerk receives $5403 salary, and spends $1347 for board, $317 for clothing, $341 for books, and $397 for other expenses, how much has he left at the close of the year?

262. A farmer having $2150 bought 536 sheep at $2 a head, and 26 cows at $23 a head; how much money had he left?

263. A man sold three horses; for the first he received $275, for the second $87 less than for the first, and for the third as much as for the other two; how much did he receive for the third?

264. B has 174 sheep, C has three times as many, and D has as many as B and C together; how many sheep has D?

265. Bought 76 hogs each weighing 416 pounds at 7 cents per pound, and sold the same at 9 cents a pound; how much was gained?

266. A man bought 14 cows at $26 each, 4 horses at $112 each, and 130 sheep for $384; he then sold the whole for $1237; did he gain or lose, and how much?

267. There are 37 tubs of butter each weighing 109 pounds; the tubs which contain the butter weigh 19 pounds each; the butter was sold at 23 cents per pound; how much was received?

268. A man paid $9137 for building a house, and for his farm 5 times as much as the cost of his house; what did they both cost?

269. A farmer bought 30 sheep at $4 apiece, and a number of cows for which he paid 8 times as much as for his sheep; how much did they both cost?

270. A merchant bought 9 barrels of sugar at $37 per barrel, and sold them for $39 per barrel; how much did he gain?

271. Two men, A and B, travel from the same place in opposite directions; A travels at the rate of 28 miles a day, and B at the rate of 35 miles a day; how far apart will they be at the end of 9 days?

272. Two men, A and B, travel from the same place in the same direction, one travelling at the rate of 35 miles a day, and the other at the rate of 28 miles a day; how far apart will they be at the end of 9 days?

273. Two men travel from the same place in the same direction; one starts on Monday morning, and travels at the rate of 27 miles a day; the other starts on Thursday morning, and travels at the rate of 30 miles a day; how far will they be apart 15 days from the time the first one started?

274. Two men, A and B, start from places 15 miles apart at the same time, and travel in the same direction; A travels at the rate of 15 miles a day, and B follows, travelling twice as fast; how far will they be apart at the end of 6 days?

275. A dog pursues a fox which has 417 rods the start; the fox runs at the rate of 17 rods a minute, the dog follows at the rate of 19 rods a minute; how far will they be apart at the end of 132 minutes?

276. If a man buys 40 acres of land at $36 an acre and 47 acres at $28 per acre, and sells the whole for $36 per acre, does he gain or lose, and how much?

277. In an orchard, 80 apple trees yield 18 bushels each; how much would the apples be worth at 30 cents a bushel?

278. A man bought two farms, one of 436 acres at $34 an acre, and another of 137 at $38 an acre; he paid on the debt $5875; how much does he still owe?

LESSON XXVI.

A farmer bought 15 oxen for $1050, and 13 cows for $325; how much did he give for 1 ox and 1 cow?

MODEL OPERATION.

(1.)
Oxen.
15)$1050 cost of 15 oxen.
70 cost of 1 ox.

(2.)
Cows.
13)$325 cost of 13 cows.
25 cost of 1 cow.

(3.)
$70 cost of 1 ox.
25 cost of 1 cow.
$95 cost of both.

Analysis.—**1.** *Find the cost of* 1 *ox.*

FORMULA.—If 15 oxen cost $1050, one ox will cost one *fifteenth* of $1050, which is $70.

2. *Find the cost of* 1 *cow.*

FORMULA.—If 13 cows cost $325, one cow will cost one *thirteenth* of $325, which is $25.

3. *Find the cost of* 1 *ox and* 1 *cow.*

FORMULA.—If 1 ox cost $70, and one cow cost $25, they both cost the sum of $70 and $25, which is $95.

Conclusion.—Therefore, he gave $95 for one ox and one cow.

279. A farmer has 18 bushels of wheat worth 396 shillings, and 16 bushels of rye worth 128 shillings; he sold 1 bushel of each; how much did he receive for them?

280. A butcher purchased 3 sheep, one from a flock of 45, valued at 1080 shillings, one from a flock of 112 valued at 3584 shillings, and one from a flock of 350 valued at 11200 shillings; how much did they cost him?

281. A grocer has 2 boxes of tea, each containing 12 pounds; one box is valued at $6, the other at $7.80; he sells one pound of each; how much does he receive?

282. A merchant purchased 13 yards of cloth for $65, and sold it for 39 dollars more than it cost him; for how much did he sell it per yard?

283. A drover purchased 864 sheep for $3456; he sold them for $864 more than the cost; how much did he receive for one sheep?

284. A grocer purchased a barrel of sugar for $23, and sold it so as to gain $11; for how much did he sell it per pound if the barrel contained 250 pounds?

285. A farm of 224 acres was purchased for $5600, and sold for $1120 more than it cost; for how much per acre was it sold?

286. Twelve men and 16 boys together do a certain piece of work; the men receive $172.80, the boys $124.80; how much do one man and one boy receive?

287. A farmer having 19 cows worth $655, and an ox worth $67 sold the ox and one cow; how much did he receive for them?

288. A farm of 17 acres was purchased for $425, and sold for $85 more than it cost; for how much per acre was it sold?

289. A man bought 17 sheep for 459 shillings, and sold them so as to gain 31 shillings; for how much did he sell each one?

290. A grocer had 15 barrels of sugar worth $843.75; after selling one barrel, what was the value of the remainder?

291. A man purchased 18 tons of hay for $378; he afterward sold one ton for 3 dollars less than its cost; how much did he receive for it?

292. There are 15 cars loaded with 986415 lb. of wheat, and 17 cars loaded with 442000 lb. of corn; how much heavier is a car-load of wheat than a car-load of corn?

293. In 18 hogsheads of sugar there are 11430 lb., and in

12 barrels of sugar there are 3624 lb.; how many more pounds are there in a hogshead than in a barrel?

294. A grocer has 6248 oranges in 11 boxes, and 7704 lemons in 18 boxes; how many more oranges than lemons are there in a box?

295. A farmer bought 17 oxen for $1445, and sold them for $187 less than they cost; how much did he get for each?

296. A farm of 19 acres was bought for $1425, and sold for $114 less than it cost; what was the selling price per acre?

297. Fifteen men in one week can earn $157.50, and 13 boys can earn $66.95 in the same time; how much less does a boy earn in one week than a man?

LESSON XXVII.

298. A drover bought 13 oxen for $1183, and 17 cows for $544; how much more did an ox cost than a cow?

299. A farmer bought 19 acres of land for $1425, and sold it for $1539; how much did he gain per acre?

300. A merchant purchased 12 yards of cassimere for $42.60, and sold it for $39.72; how much did he lose per yard?

301. A gentleman owned an estate valued at $325250; he left $48130 to his wife, and the remainder to be equally divided among his 9 children; how much did each child receive?

302. In a field of 14 acres were raised 35728 ears of corn, and in a field of 12 acres were raised 32772; how many ears grew on two acres, one of each field?

303. How many more ears grew upon an acre of the first field than upon an acre of the second?

304. In 18 miles the hind wheels of a carriage turn round 7920 times, and the fore wheels 8640; how many more times do the fore wheels turn round in 1 mile than the hind wheels?

305. In an army there are 14909 infantry and 4352 cavalry; how many more men in one-seventeenth of the infantry than in one-seventeenth of the cavalry?

306. How many men in one-seventeenth of the army?

307. A horse and 17 cows are worth $744; the horse is valued at $132; what is the value of one cow?

308. 17 sheep and 13 lambs are worth 834 shillings; the sheep are valued at 561 shillings; what is the value of one lamb?

309. A butcher purchased one sheep and one lamb; how much did he pay for both?

310. 12 bushels of wheat and 19 bushels of corn are worth $40.11; the value of the wheat is $22.44; what is the value of 1 bushel of corn?

311. What is the value of a bushel of the corn and a bushel of the wheat?

312. A grocer has 9 hhd. of molasses and 15 hhd. of sugar; the value of the whole is $9195.47; the value of the molasses is $1315.27; required the value of one hhd. of sugar?

313. The grocer sells one hogshead of each; how much does he receive for them?

314. There are two hogsheads of sugar; the value of the sugar in the 1st hogshead at 11 cents per pound is $68.75, and the value of the sugar in the 2d hogshead at 12 cents per pound is $86.76; how many pounds in both hogsheads?

315. There are two droves of cattle; the value of the first drove at $19 per head is $4864; the value of the second drove, at $18 per head is $5778; how many head of cattle in both droves?

316. There are two flocks of sheep; the value of the first flock at 12 shillings per head, is 15084 shillings; the value of the second flock at 15 shillings per head, is 35580 shillings; how many more sheep in the second flock than in the first?

317. The fore and hind wheels of a carriage together make 16564 revolutions in 18 miles; the fore wheels alone make 8640 revolutions; how many revolutions do the hind wheels make in one mile?

LESSON XXVIII.

318. 15 tons of iron were purchased for $2700, and sold for $2100; what was the loss per ton?

319. A drover purchased 18 oxen for $1520, and sold them for $1387; how much did he lose per head?

320. A house and farm of 17 acres are valued at $7975; the house is worth $2725; what is the value of one acre of land?

321. The value of a house and 15 acres of land is $5795; the value of the house is $2240; what is the value of the house and 1 acre of land?

322. The population of a certain city is 273586; eleven years ago it was 243621; what is the average rate of increase per year?

323. A train consisting of a locomotive and 14 cars weighs 106225 lbs.; the locomotive weighs 62125 lbs.; what is the weight of each car?

324. A man in 13 days can hoe 35789 hills of corn, and a boy in the same time can hoe 17173 hills; how many hills can a man and a boy hoe in one day?

325. A vessel and cargo valued at $1138633 are captured by a privateer; the captain retains the vessel, which is valued at $390089; the cargo is divided among the crew, which consists of 17 persons; how much will each receive?

326. There are two locomotives; the first can run 2782560 feet in 17 hours; the second can run 4029204 feet in the same time; how much farther can the second run per hour than the first?

327. A merchant invested 256788 shillings in grain; with one half of his money he purchased wheat at 9 shillings per bushel, and with the remainder he bought rye at 7 shillings per bushel; how much grain did he purchase?

328. A merchant bought 328 pounds of tea at 56 cents per pound, and sold it for $200.08; how much did he gain per pound?

329. A farmer bought 156 acres of land for $75 per acre; he sold the land for $16180; how much did he gain per acre?

330. Thirteen hundred and twenty-four bushels of corn were bought at sixty-two cents per bushel, and the whole number of bushels was sold for ninety-nine thousand three hundred cents; how much was gained on one bushel?

331. A farmer owes a merchant eleven thousand five hundred and forty-seven cents; he pays one hundred and fifty-seven pounds of butter at twenty-three cents per pound, and the balance in corn at sixty-two cents per bushel; how many bushels does he pay?

332. A farmer owes a merchant eight thousand seventy cents; he pays one hundred and sixty-seven bushels of oats, at thirty-five cents per bushel, and for the balance he pays eighty-nine pounds of butter; how much per pound was the butter?

333. A merchant purchased a hogshead of molasses containing one hundred and twenty-nine gallons, for thirty-nine cents per gallon; but thirty-one gallons having leaked out, he sold the balance for four thousand one hundred and sixteen cents; for how much did he sell it per gallon?

334. The sum of three numbers is five million seven hundred and seventy-eight thousand nine hundred and forty-four; the first number divided by eight hundred and sixty-four equals five thousand two hundred and seventy-one; the second equals ninety-nine millions eight hundred and sixteen thousand six hundred divided by one thousand and five; what is the third number?

335. If the population of Illinois was 125323 in 1830, and 1723663 in 1860, what was the average increase per year?

336. What would have been the population in 1850 at the average rate of increase?

337. There are two numbers; the first equals 356 multiplied by 507; the second equals 1955085 divided by 85; what is the sum of the numbers?

LESSON XXIX.

Ment. Problems.—If 7 pounds of sugar cost 42 cents, what will one pound cost? If 8 pounds of rice cost 72 cents, what will one pound cost? If 6 pounds of sugar cost 48 cents, what will 2 pounds cost? If 5 ounces of alum cost 25 cents, what will 3 ounces cost?

338. If 17 pounds of sugar cost $4.38, what will 1 pound cost?

339. If 23 pounds of coffee cost \$13.37½, what will one pound cost?

340. If 89 pounds of butter is worth \$48, what are 11 pounds worth?

MODEL OPERATION.

lbs.
89)\$48.0000
.5393=cost 1 lb.
11
\$5.9323=cost 11 lb.

Anal. Steps.—1. *Find the cost of* 1 *lb.*
2. *Find the cost of* 11 *lb.*

NOTE.—Require the pupil in these problems to change the dollars, cents, or mills, to *tenths* of a mill.

341. If 7 pounds of pepper are worth \$6.37½, what are 3 pounds worth?

342. If 13 bushels of potatoes are worth \$10, what are 5 bushels worth?

343. If 11 bushels of turnips are sold for \$8, at that rate what are 4 bushels worth?

344. If 8 eggs are worth 25 cents, what are 3 worth?

345. If 11 eggs are worth 25 cents, how much are 6 worth?

346. If 7 eggs are worth 25 cents, how much are 4 worth?

347. If 7 pounds of sugar cost \$1.10, what is the cost of 3 pounds?

348. If six spoons are worth \$12.37½, what are 4 worth?

349. If a dozen candles are worth 38 cents, what should I give for 3 candles? For 7 candles? For 9 candles?

350. If 8 pounds of beef are bought for 98 cents, how much is given for 3 pounds?

351. There are 16 ounces in a pound; if a pound of sugar is worth 19 cents, what are 4 ounces worth?

352. If a pound of cheese is worth 22 cents, what are 7 ounces worth?

353. If a pound of coffee is worth 68 cents, what are 6 ounces worth?

354. If a pound of rice is worth 27 cents, what are 11 ounces worth?

355. What must I give for 7 ounces of pepper at 83 cents a pound?

QUESTIONS.—One pound is what part of 11 pounds? The cost of 1 pound is what part of the cost of 11 pounds? If the cost of 11 pounds is given, how is the cost of 1 pound found? Five pounds cost how many times as much as 1 pound? When the cost of 11 pounds is given, how can the cost of 5 pounds be found?

TEST QUESTIONS FOR EXAMINATION.

63. What is the currency of the United States? What is meant by a decimal currency? When was it adopted? ANS.—1786. What are the denominations? In business how are eagles read? How are dimes read? Which denomination of United States money is the unit? How many cents make a dollar? An eagle? How many mills make a cent? A dime? A dollar? How many cents in a half-dollar? In a quarter-dollar? In three-quarter dollars? How many cents in 1 eighth of a dollar? In 1 tenth? In 1 fifth? In 2 fifths? A mill is what part of a cent? A cent is what part of a dollar? Into how many parts is a cent divided? What are the parts called?

64. What are coins? Of what are coins made? What is the place where they are made? What metals are called precious? Which precious metal is worth the most? What are the gold coins of the United States? What is meant by *alloy?* What part of the gold coins is alloy? What part of the silver coins? What alloy is used in gold coins? What alloy is used in silver coins? What is the silver 3 cent piece made of? Which coins are made mostly of nickel? Which mostly of copper? How many kinds of legal coins are there in circulation?

65. How are dollars separated from cents? What is the decimal point? Show its use?

66. What is the reduction of Federal money? How are dollars reduced to cents? Which of the fundamental rules is used to reduce dollars to cents? Cents to mills? Mills to cents? Mills to dollars? Cents to dollars? Cents to mills? Give the formula for reducing $2 to cents. 500 cents to dollars. 4 cents to mills. 80 mills to cents. 5000 mills to dollars. Recite the rule for reducing dollars to cents. Dollars to mills. Mills to cents. Mills to dollars. Cents to dollars. Cents to mills.

67. How are numbers in Federal money added? Can dollars be added to cents? Cents to mills? Mills to dollars? How are the numbers written? Why? What is the answer called? How may the answer be proved? How many places do mills take? Cents? Where is the decimal point placed in the answer? What sign is placed before dollars?

68. What is subtraction of United States money? How are the numbers written? Which is the subtrahend? Which the minuend? Which the remainder? Can cents be taken from dollars? Mills from cents? Cents from cents? Show at the blackboard how 1000 mills can be taken from a thousand cents. A thousand mills from a thousand dollars. A thousand cents from a thousand dollars. How may subtraction be proved? What is the opposite of subtraction?

69. What is multiplication? What do we call the number to be multiplied? Show at the blackboard how 1 dollar 1 cent and 1 mill can be multiplied by 100. Which is the multiplicand? Which the multiplier? Which the product? How should the product be written? The cost of 2 apples is how many times as much as the cost of 1 apple? 20 cows are how many times as many as 1 cow? The cost of 20 cows is how many times as much as the cost of 1 cow? How may multiplication be proved?

70. What is division? What do we call the number to be divided? What do we call the number by which we divide? What is the answer called? When the dividend is divided into equal parts, of what denomination is the quotient? Is it like or unlike, the dividend? When the dividend is cents, what is the quotient? When the dividend is mills what is the quotient? When we want to find how many times one sum of money is contained in another, what must first be done? Ans.—*If they are unlike reduce them to the same denomination.* If the divisor is mills and the dividend is cents, what must be done? What if the divisor is dollars and cents, and the dividend is dollars? If the divisor is cents and mills, and the dividend is dollars and cents? If the divisor is mills and *tenths* of a mill, and the dividend is dollars? If the dividend is cents and the divisor is dollars? If the dividend is tenths of a mill, and the divisor is cents? How many times are 6 cents contained in 6 dollars? 20 mills in 20 dollars? 5 *tenths* of a mill in 5 cents? 5 dimes in five eagles? 1 mill in 1 eagle? 1 dime in 1 dollar? Divide 20 dollars into 5 equal parts. Into 4 equal parts. Into 3 equal parts? Ans.—$6.666+. Why reduce the dollars to mills before dividing? Ans.—*That the remainder may be mills and so not worth much.* What does the sign *plus* after the answer show? What is 1 *third* of 30 cents? 1 *third* of 10 mills? What is it customary in business to do with the remainder of mills? Ans.—*When it equals or exceeds* 5, *add* ONE *to the mills, if less reject it as of no value.* In the final answer, what is done with the mills? Ans.—*When the mills equal or exceed* 5, *add* ONE *to the cents, if less reject them.*

71. What is a bill or invoice of goods? By whom are bills given? What is their use? What is a debtor? What is a creditor? When is a bill called an account? Write a bill on the blackboard. Write an account. When must a United States revenue stamp be used? What is it to receipt a bill? Is the bill receipted by the debtor or by the creditor? How is a bill receipted by a clerk or agent?

SECTION VII.

COMPOUND DENOMINATE NUMBERS.

LESSON I.

72. *A Simple* number is either abstract or denominate, and of but one denomination; as, 4867859 dollars, 337 mills.

73. *A Compound Denominate* number is a denominate number of two or more different denominations; as, 3 furlongs 19 rods 4 yards, 15 days 6 hours 37 minutes.

74. *The Reduction* of Compound Denominate numbers is the process of changing a number of one denomination to that of another, without altering its value.

75. *Reduction Descending* is the process of changing a number from one denomination to that of a less; as, pounds to farthings.

76. *Reduction Ascending* is the process of changing a number from one denomination to that of a greater; as, farthings to pounds.

LESSON II.

ENGLISH MONEY.

77. *English or Sterling Money,* is the currency of Great Britain.

(*a.*) TABLE.

4 farthings (far. or qr.)	make	1 penny,	marked d.
12 pence	"	1 shilling,	" s.
20 shillings	"	1 pound or sovereign,	£, sov.
21 shillings	"	1 guinea,	marked guin.

78. *Coins.*—(*a.*) **The Gold coins** are the *sovereign,* (1£.) and the *half-sovereign* (10s.)

(*b.*) **The Silver coins** are the *crown* (5s.), the *half-crown* (2s. 6d.), the *florin* (2s.), the *shilling* (12d.), *sixpenny piece* (6d.), and *three-penny piece* (3d.).

(*c.*) **The Bronze coins** are the *penny*, *half-penny*, and *farthing*. Farthings are generally written as fractions of a penny, thus: 1 far.$=\frac{1}{4}$d.; 2 far.$=\frac{2}{4}$ or $\frac{1}{2}$d.; 3 far.$=\frac{3}{4}$d.

(*d.*) **Canadian currency** is decimal, and the denominations are the same as Federal money.

(*e.*) **The Franc** is the unit of the French decimal currency, and is worth $0.186. The denominations are *francs* and *centimes.*

MENTAL EXERCISES.

1. How many farthings in 2d.?

FORMULA.—Since there are 4 farthings in one penny, in 2 pence there are 2 times 4 farthings, which is 8 farthings; therefore in 2 pence there are 8 farthings.

2. How many farthings in 9 pence? In 8d.? In 12d.? In 7d.? In 4d.? In 5d.? In 20d.? In 30d.?

3. How many pence in 4 shillings? Shillings in 4 pounds? Shillings in 2 guineas?

4. How many pence in 15 farthings?

FORMULA.—Since in 4 farthings there is 1 penny, in 15 farthings there are as many pence as 4 farthings are contained times in 15 farthings, which is three times with a remainder of 3 farthings; therefore in 15 farthings there are 3 pence and 3 farthings.

5. How many pence in 13 farthings? In 16 far.? In 33 far.? In 20 far.? In 19 far.? In 37 far.? In 50 far.?

6. How many pounds in 20 shillings? Shillings in 24 pence? Pence in 8 farthings?

LESSON III.

Problem.—Change £46 6s. 8d. to the denomination of farthings.

REVIEW.—What is a unit? (1.) What is the difference between concrete and abstract numbers? (3.) (4.) What is the difference between the Roman and Arabic notations? (7.) (8.) Illustrate at the blackboard the way in which numbers are expressed in the Roman notation. (7.)

MODEL OPERATION.

£. s. d.
46+6+8
20s.
———
1st. 920s.
6
———
926s.
12d.
———
1852d.
926
———
11112d. 2d.
8d.
———
11120d.
4far.
———
3d. 44480

PROOF.
4 far.)44480far.
———
12d.)11120d.
———
20s.)926s.+8d.
———
£46+6s.

Anal. steps.—**1.** *Change £ to s.*
2. *Change s. to d.*
3. *Changed to far.*

Anal. steps.—**1.** *Change the pounds to shillings, and add the 6 shillings.*

FORMULA.—Since in 1 pound there are 20 shillings, in 46 pounds there are 46 times 20 shillings, which is 920 shillings, and 6 shillings added make 926 shillings.

2. *Change shillings to pence, and add the 8 pence.*

FORM.—Since in 1 shilling there are 12 pence, in 926 shillings there are 926 times 12 pence, which is 11112 pence, which with the 8 pence added are 11120 pence.

3. *Change pence to farthings.*

FORM.—Since in 1 penny there are 4 farthings, in 11120 pence there are 11120 times 4 farthings, which is 44480 farthings.

Conclusion.—Therefore, in £46 6s. 8d. there are 44480 farthings.

(*f.*) **Rule for Reduction Descending.**—*I. Multiply the highest denomination of the given compound number by the number of the next lower unit which a unit of this denomination equals, and add to this product the given number, if any, of the lower denomination.*

II. Proceed in the same manner with each lower denomination, until the required result is obtained.

1. Reduce £84 3s. 8d. 3 far. to farthings.
2. Reduce £45 13s. 4d. 2 far. to farthings.
3. Reduce £187 19s. 7d. to pence.
4. Reduce £198 7d. 3 far. to farthings.
5. Reduce 198 guin. 13s. 1 far. to farthings.
6. In 47 guineas, how many pence?
7. In 377 guineas, how many farthings?
8. In £3896, how many shillings?
9. In £83972, how many pence?
10. In 391783s., how many farthings?

Questions.—What is the first step in changing pounds to lower denominations? How many shillings make a pound? Which of the fundamental rules is used in reducing pounds to shillings? How many steps are required to change pounds to pence? How many pence in 1 shilling? One penny is what part of a shilling? A shilling is how many times as much as a penny? A pound is how many times as much as a shilling?

LESSON IV.

Problem.—Change 8416784 far. to pounds.

MODEL OPERATION.

4 far.)8416784 far. } 1st.
2d. { 12 d.)2104196d. } 1st.
2d. { 20s.)175349s.+8d. } 3d.
£8767+9s. } 3d.

PROOF.
£8767 9s. 8d.
20s.
175340s.
9
175349s.
12d.
2104188d.
8d.
2104196d.
4 far.
8416784 "

Anal. steps.—1. *Change far. to d.*
2. *Change d. to s.*
3. *Change s. to £.*

Anal. steps.—1. *Change farthings to pence.*

Formula.—Since in 4 farthings there is 1 penny, in 8416784 farthings there are as many pence as 4 farthings are contained times in 8416784 farthings which are 2104196.

2. *Change pence to shillings.*

Form.—Since in 12 pence there is 1 shilling, in 2104196 pence there are as many shillings as 12 pence are contained times in 2104196 pence, which are 175349 with a remainder of 8 pence.

3. *Change shillings to pounds.*

Form.—Since in 20 shillings there is 1 pound, in 175349 shillings there are as many pounds as 20 shillings are contained times in 175349 shillings which are 8767 with a remainder of 9 shillings.

Conclusion.—Therefore, in 8416784 farthings there are £8767 9s. 8d.

(*g.*) **Rule for Reduction Ascending.**—*I. Divide the given denomination by the number it takes of this denomination to make a unit of the next higher.*

II. Proceed in the same way to divide each quotient thus obtained, until the number is reduced to the denomination required. The last quotient with the several remainders annexed in the order of their values, is the answer.

(*h.*) **Proof.**—*Reduction Descending may be proved by Reduction Ascending.*

11. Change 8874134 farthings to pounds.
12. How many pounds in 871341 farthings?
13. Change 44480 farthings to pounds.
14. Change 371864 farthings to shillings.
15. How many guineas in 3871841 farthings?
16. How many shillings in 871837 farthings?
17. How many pence in 37843 farthings?
18. How many pounds in 387183 pence
19. Change 37874 pence to guineas.
20. Change 34713 shillings to guineas.
21. Change 371864 pence to sovereigns.
22. A man bought 347 boxes of oranges, each box containing 467 oranges, and gave for each orange 3 farthings; how many pounds English money did they cost him?

Review.—What is the difference between the simple and the local value of a figure? (9.) (12.) What is the use of the cipher? (11.) What is the difference between notation and numeration? (6.) (12.) How are the vacant places of a number filled? (14.) What is a rule? (16.) What is the rule for notation? (17.)

23. A man bought a farm of 462 acres, giving £25 18s. 8d. 3 far. for each acre; how many farthings did the farm cost him?

24. Bought 483 bushels of wheat at 8s. 5d. per bushel; what was the cost?

25. Bought 1483 bushels of wheat at 34d. 2 far. per bushel; what was the cost?

26. Bought 9783 pounds of pork at 4d. 2 far. per pound; what did it cost?

Questions.—Which of the fundamental rules is used in changing farthings to higher denominations? What kind of reduction is this called? Which two fundamental rules are used in reduction? What is reduction ascending? What is reduction descending? How many steps are there in changing farthings to pounds? Pounds to farthings? Shillings to pounds? Shillings to farthings? Pence to farthings? Pence to shillings? Guineas to shillings? Guineas to pounds? What is the difference between a pound and a sovereign?

LESSON V.

WEIGHT.

79. Weight is the measure of the quantity of matter a body contains.

TROY WEIGHT.

80. Troy Weight is used in weighing gold, silver, and jewels, and in determining the specific gravity of bodies, &c.

(*a.*) TABLE.

24 grains (gr.)	make	1 pennyweight,	marked	pwt.
20 pwt.	"	1 ounce.	"	oz.
12 oz.	"	1 pound,	"	lb.
$3\frac{1}{5}$ grains	"	1 carat (diamond wt.)		k

(*b.*) Note.—1. A carat is a weight of about 3.2 grains, and is used by jewellers to weigh diamonds. The term *carat* is also used to denote the fineness of gold. When gold contains 18 parts pure gold and 6 parts alloy, which

---

Review.—What is addition? (19.) What is the sum of two or more numbers? (18.) What is the sign of addition? (20.) (Of equality? (21.) The dollar sign? The sign of subtraction? (36.) Of multiplication? (46.) Of division? (57.) (58.) Define the terms of addition? (18.) Of subtraction? (32.) (33.) (34.) (35.) Of multiplication? (42.) (43.) (44.) (45.) Of division? (51.) (52.) (53.) (56.)

is usually silver and copper, it is said to be 18 carats fine. Gold 14 carats fine contains 14 parts pure gold and 10 parts alloy, &c.

MENTAL EXERCISES.

1. How many grains in 5 pennyweights? In 2 pennyweights? In 3 pwt.? In 10 pwt.? In 1 pwt.?
2. How many ounces in 3 pounds? In 7 lb.? In 4 lb.? In 9 lb.? In 2 lb.? In 6 lb.? In 1 lb.?
3. How many pennyweights in 3 ounces? In 6 oz.? In 7 oz.? In 4 oz.? In 5 oz.? In 2 oz.?
4. How many ounces in 40 pwt.? In 80 pwt.? Pounds in 24 oz.? Pwt. in 48 gr.?

LESSON VI.

Problem.—Change 136 lb. 4 oz. 13 pwt. 19 gr. to gr.

MODEL OPERATION.

1st.
136 lb. 4 oz. 13 pwt. 19 gr. to gr.
12 oz.
1632 "
4 "
1636 "

2d.
20 pwt.
32720 "
13 "
32733 "

3d.
24 gr.
130932 "
65466 "
785592 "
19 "
785611 "

PROOF.

1st.
24 gr.)785611 gr.
20 pwt.)32733pwt.+19 gr.

2d.
12 oz.)1636 oz.+13 pwt.

3d.
136 lb.+4 oz.

(c.) Anal. Steps.

Red. Desc.	Red. Asc.
1. *Change lbs. to oz.*	1. *Change gr. to pwt.*
2. *Change oz. to pwt.*	2. *Change pwt. to oz.*
3. *Change pwt. to gr.*	3. *Change oz. to lb.*

Anal. Steps.*—1. *Change pounds to ounces, and add the ounces.*

2. *Change ounces to pennyweights, and add the pennyweights.*

3. *Change pennyweights to grains, and add the grains.*

Concl.—Therefore, in 136 lb. 4 oz. 13 pwt. 19 gr., there are 785611 grains.

Anal. Steps of Proof.—1. *Change grains to pennyweights.*

2. *Change pennyweights to ounces.*

3. *Change ounces to pounds.*

Concl.—Therefore, in 785611 grains there are 136 lb. 4 oz. 13 pwt. 19 grs.

27. Reduce 187689 grains to pounds.
28. Reduce 14786 ounces to pounds.
29. Reduce 418716 grains to pounds.
30. How many ounces in 3786413 pwt. ?
31. How many pounds in 37841113 pwt. ?
32. How many ounces in 3718413 pwt. ?
33. How many ounces in 418783 gr. ?
34. How many grains in 134 lb. 11 oz. 16 pwt. 12 gr. ?
35. How many grains in 84 lb. 3 pwt. ?
36. How many pennyweights in 387 lb. 8 oz. 6 pwt. ?
37. How many pennyweights in 144 lb. 3 oz. 16 pwt. ?
38. How many pennyweights in 378 oz. 13 pwt. ?
39. How many grains in 186 lb. 18 gr. ?
40. How many grains in 7 pwt. 13 gr. ?

QUESTIONS.—How many steps are required to change pounds to grains? How many to change grains to pounds? Which is reduction ascending? Which is reduction descending?

LESSON VII.

41. How many grains in 9 oz. 17 pwt. 3 gr. ?
42. How many ounces in 113 lb. 6 oz. ?
43. How many grains in 374 lb. 9 oz. 13 pwt. 19 gr. ?
44. How many pennyweights in 416 lb. 3 oz. ?
45. How many pounds in 37186413 grains ?
46. How many ounces in 478348 grains ?
47. How many pennyweights in 371837 gr. ?

---

* NOTE.—The pupil should be taught to give the right formula until he can do so readily.

48. How many grains in 371 lb. 13 pwt.?
49. How many ounces in 416713 gr.?
50. How many pounds in 3786434 gr.?

LESSON VIII.

PRACTICAL PROBLEMS.

If 3 lb. of silver are worth $40.37½, what is 1 grain worth?

MODEL OPERATION.

Value 3 lb.

3)$40.3750 *

12)13.4583+=Value of 1 lb.

20)1.1215+= " 1 oz.

24).0560+= " 1 pwt.

.0023+= " 1 gr. Ans.

Anal. Steps.—1. *Find value of* 1 *lb.*

Form.—If 3 lb. of silver are worth $40.37½, 1 lb. is worth *one-third* of 40.375, which is $13.4583.

2. *Find value of* 1 *oz.*

Form.—Since 1 lb., or 12 oz., is worth $13.4583, 1 oz. is worth *one-twelfth* of $13.4583, which is $1.1215.

3. *Find value of* 1 *pwt.*

Form.—Since 1 oz., or 20 pwt., is worth $1.1215, 1 pwt. is worth *one-twentieth* of $1.1215, which is $.0560.

4. *Find value of* 1 *gr.*

Form.—Since 1 pwt., or 24 gr., is worth $.0560, 1 grain is worth *one-twenty-fourth* of $.0560, which is 2 mills and 3-*tenths*; therefore 1 grain is worth 2 and 3 *tenth* mills.

If 1 lb. gold dust cost $200, what is the cost of 1 oz.? What is the cost of 1 pwt.? What is the cost of 1 gr.? What is the cost of all these quantities?

---

* Notes.—1. Change the cost in each of the following problems to *tenths* of a mill, in order to have the answer correct.

2. The formula for each step should be required until the pupil understands it clearly; and then the steps only need be given.

MODEL OPERATION.

12)$200.0000
20)16.6666+ = $16.6666 Cost 1 oz.
24).8333+ = .8333 " 1 pwt.
.0347+ = .0347 " 1 gr.
$17.5346 Total cost.

Anal. Steps.—1. *Find cost of* 1 *oz.*
2. *Find cost of* 1 *pwt.*
3. *Find cost of* 1 *gr.*
4. *Find total cost.*

51. If 1 lb. of gold dust is worth $210, what is the value of 1 oz. 1 pwt. and 1 gr.?

52. If 1 lb. of silver is worth $10, what is the worth of 1 oz. 1 pwt. and 1 gr.?

53. If 1 lb. of silver is worth $8.90, what is the worth of 1 oz. 1 pwt. and 1 gr.?

54. If 2 lb. of silver are worth $19.90, what is ½ lb. worth? What is the worth of 1 lb. 1 oz. and 1 pwt.?

55. If 5 lbs. of silver are worth $49.40, what is the worth of 1 lb. 1 oz. 1 pwt. and 1 gr.?

56. If 7 lb. of gold are worth $1463.87½, what is the worth of 1 lb. 1 oz. 1 pwt. and 1 gr.?

57. What is the value of a silver vessel weighing 1 lb. 1 oz. 1 pwt. 1 gr. at $9 per pound?

58. What is the value of a gold chain weighing 1 oz. 1 pwt. 1 gr. at $18 per ounce?

A man bought 41 oz. 12 pwt. 14 gr. of gold dust at $16 per oz.; what did it cost him?

MODEL OPERATION.

pwt. Cost of 1 oz. oz.
20)$16.0000 × 41 = $656.0000 Cost 41 oz.
grs. pwt.
24).8000 × 12 = 9.6000 " 12 pwt.
grs.
.0333 × 4 = .4662 " 4 gr.
$666.0662 Total.

REVIEW.—Illustrate at the blackboard the axioms of addition. (23., *a.*) Of subtraction. (37.) Of multiplication. (47.) Of division. (59.) What is the difference between simple and complex problems? (25.) (26.)

Anal. Steps.—1. *Change* $16 *to tenths of mills.*

2. *Find the cost of* 41 *oz.*

3. *Find the cost of* 1 *pwt.*

Formula.—Since 20 pwt. or 1 oz. cost 160000 tenths of a mill, 1 pwt. will cost 1-*twentieth of* 160000 tenths, which is 8000 tenths.

4. *Find the cost of* 12 *pwt.*

Form.—Since 1 pwt. costs 8000 tenths, 12 pwt. will cost 12 times 8000 tenths, which is 96000 tenths.

5. *Find the cost of* 1 *gr.*

6. *Find the cost of* 14 *gr.*

7. *Find the total cost of* 41 *oz.* 12 *pwt. and* 14 *gr.*

Conclusion.—Therefore, 41 oz. 12 pwt. and 14 gr. cost $666.0662.

What will 7 oz. 2 pwt. of gold dust come to at $12 per ounce?

MODEL OPERATION.

$$\begin{array}{rrr} 20)\underline{\$12.000} & \times 7 = & \$84.000 \\ .600 & \times 2 = & 1.200 \\ \cline{3-3} & & \$85.200 \end{array}$$

59. What is the cost of a gold chain weighing 17 pwt. 11 gr. at $1.60 per pwt.?

60. What is the cost of a gold locket weighing 12 pwt. 13 gr. at $1.50 per pwt.?

61. What is the cost of a gold bracelet weighing 2 oz. 3 pwt. 11 gr. at $1.37½ per pwt.?

62. What is the cost of a silver plate weighing 8 oz. 9 pwt. 3 gr. at $2.37½ per ounce?

63. What will a pair of napkin-rings cost, each ring weighing 4 oz. 16 pwt. at $.80 per ounce?

Anal. Steps.—1. *Find the cost of* 1 *ring.*
2. *Find cost of both.*

64. What will six silver spoons cost at 75 ct. per ounce, if each spoon weighs 12 pwt. 11 gr.?

Review.—Give rule for notation. (17.) For addition. (29., *e.*) For subtraction. (38., *d.*) For multiplication. (49., *d.*) For division. (60., *d.*) (62., *d.*)

65. I bought a silver pitcher weighing 11 oz. 11 pwt. 14 gr. at $2.87½ ct. per oz.; what was the cost?

66. What will a ring cost, weighing 15 gr. at $1.12½ per pwt.?

LESSON IX.

AVOIRDUPOIS WEIGHT.

81. ***Avoirdupois*** weight is used for weighing all ordinary articles.

(*a.*) TABLE.

16 drams (dr.)	make	1 ounce,	marked	oz.
16 oz.	"	1 pound,	"	lb.
25 lb.	"	1 quarter,	"	qr.
4 qr.	"	1 hundred-weight,	"	cwt.
20 cwt.	"	1 ton,	"	T.

MENTAL EXERCISES.

1. How many drams in 2 oz.

FORMULA.—Since in 1 oz. there are 16 drams, in 2 ounces there are twice 16 drams, which are 32 drams; therefore in 2 oz. there are 32 drams.

2. How many quarters in 3 hundred weight? In 6 cwt.? In 7 cwt.? In 8 cwt.? In 5 cwt.? In 2 cwt.?

3. How many ounces in 3 pounds? In 2 lb.? In 1 lb.?

4. How many hundred weight in 3 tons? In 6 T.?

5. How many pounds in 4 quarters? In 3 qr.? In 8 qr.?

6. How many ounces in 32 drams? In 33 dr.? In 17 dr.?

7. How many qr. in 28 lb.? In 51 lb.? In 99 lb.?

8. How many pounds in 32 ounces? In 37 oz.? In 65 oz.?

LESSON X.

Problem.—Change 6 T. 17 cwt. 3 qr. 19 lb. 8 oz. 11 dr. to drams.

REVIEW.—What is U. S. money? (63.) Repeat the table. (63., *a.*) Repeat the numeration table. (63., *b.*) Repeat the table of aliquot parts. (63., *c.*)

MODEL OPERATION.

```
      ⎧ 6 T. 17 cwt. 3 qr. 19. lb. 8 oz. 11 dr.
1st.  ⎨  20 cwt.
      ⎩ 137  "  ⎫
          4 qr. ⎬ 2d.
      ⎧  551  " ⎭
      ⎪   25 lb.
      ⎪        "
3d.   ⎨  2755
      ⎪ 1102
      ⎩ 13794  "  ⎫
           16 oz. ⎪
          82764 " ⎬ 4th.
         13794  " ⎪
      ⎧  220712 " ⎭
      ⎪      16 dr.
5th.  ⎨ 1324272  "
      ⎪ 220712
      ⎩ 3531403  "
```

PROOF.

```
1st. ⎧ 16 dr.)3531403 dr.
     ⎩ 16 oz.)220712 oz.+11 dr. ⎫ 2d.
3d.  ⎧ 25 lb.)13794 lb.+8 oz.   ⎭
     ⎩ 4 qr.)551 qr.+19 lb.     ⎫ 4th.
5th. ⎧ 20 cwt.)137 cwt.+3 qr.   ⎭
     ⎩           6 T.+17 cwt.
```

Analytical Steps.

1. *Change T. to cwt.*
2. *Change cwt. to qr.*
3. *Change qr. to lb.*
4. *Change lb. to oz.*
5. *Change oz. to dr.*

Analytical Steps to Proof.

1. *Change dr. to oz.*
2. *Change oz. to lb.*
3. *Change lb. to qr.*
4. *Change qr. to cwt.*
5. *Change cwt. to T.*

67. Change 13 T. 9 cwt. 3 qr. 13 lb. 13 oz. 5 dr. to drams.
68. Change 14 T. 17 cwt. 2 qr. 15 lb. 7 oz. 5 dr. to drams.
69. Change 171 T. 15 cwt. 9 lb. 14 oz. 6 dr. to drams.
70. Change 37 T. 11 cwt. 1 qr. 13 oz. to drams.
71. Change 58 lb. 13 dr. to drams.
72. Change 3784167 oz. to tons.
73. Change 8341673 dr. to cwt.

REVIEW.—Name the gold coins. (64., *a.*) Name the silver. (64.. *b.*) Name the nickel. (64., *c.*) Name the copper. (64., *d.*) What is the use of the decimal point? (65.) What is the rule for changing dollars to cents? (66., *a.*) What is the rule for changing mills to dollars? (66, *b.*) How are numbers added in Federal money? (67.) How subtracted? (68.) How multiplied? (69.) How divided? (70)

74. Change 4183741 oz. to cwt.

75. Change 38718641 lb. to tons.

76. Change 41837 dr. to lb.

77. How many quarters in 3871864 drams?

QUESTIONS.—How many steps are required to change tons to drams? Drams to tons? Which is reduction ascending? Which reduction descending? Which of the fundamental rules is used in changing drams to tons? Ounces to pounds? Ounces to drams? Pounds to quarters? Quarters to hundred-weight?

LESSON XI.

78. How many hundred weight in 3718641 oz.?

79. How many ounces in 37841 lb.?

80. How many drams in 418783 oz.?

81. How many ounces in 37841 cwt.?

82. How many drams in 37 T. 14 cwt. 9 lb.?

83. How many ounces in 483 T. 3 qr. 14 lb.?

84. How many tons in 3718674 oz.?

85. How many tons in 913783 qr.?

86. How many pounds in 49 T. 28 cwt. 3 qr.?

87. How many drams in 383 cwt. 3 oz.?

88. How many ounces in 48 T. 38 lb.?

89. How many tons in 3871678346 drams?

90. How many drams in 37 T. 14 cwt. 3 qr. 19 lb. 6 oz. 13 dr.?

LESSON XII.

PRACTICAL PROBLEMS.

Problem.—What is the cost of 5 cwt. 2 qr. 12 lb. of hay at $37.50 per ton?

REVIEW.—What are bills? (71.) Which party is the debtor? (71., *a.*) Which party is the creditor? (71., *b.*) When is a bill called an account? (71., *c.*) When must a bill have a stamp? (71., *d.*) What is a simple number? (72.) What is a compound denominate number? (73.) What is reduction? (74). What is reduction descending? (75.) What is reduction ascending? (76.) What is sterling money? (77.) Which are the gold coins? (78., *a.*) Which are the silver coins? (78., *b.*) Which are the bronze? (78., *c.*) Repeat the table of sterling money? (77., *a.*)

MODEL OPERATION.

```
cwt. 20)$37.5000
     qr. 4)1.8750 ×  5=9.3750 cost 5 cwt.
     lb. 25).4687 ×  2= .9374  "   2 qr.
             .0187 × 12= .2244  "  12 lb.
                       $10.5368 ANS.
```

Anal. Steps.—1. *Find cost of* 1 *cwt. and then of* 5 *cwt.*
2. *Find cost of* 1 *qr. and then of* 2 *qr.*
3. *Find cost of* 1 *lb. and then of* 12 *lb.*
4. *Find cost of* 5 *cwt.* 2 *qr.* 12 *lb.*

91. What is the cost of 4 cwt. 3 qr. of hay at $37.50 per ton?

92. What cost a quarter of coal at $12.50 per ton?

93. What cost 3 cwt. 2 qr. of coal at $13.70 per ton?

94. What cost 9 cwt. 3 qr. of coal at $11 per ton?

95. What must I give for 3 cwt. 2 qr. of feed at $3 per cwt.?

96. What must I give for 37 pounds of flour at $5.30 per cwt.?

97. What must I give for 18 lb. of flour at $4.50 per cwt.?

98. What cost 3 cwt. 12 lb. of flour at $2.30 per cwt.?

99. What cost 3 qr. 16 lb. salt at $1.87 per cwt.?

100. What must I give for 87 lb. of flour at $3 per cwt.?

101. What must I give for 2 qr. 13 lb. beef at $4.87 per cwt.?

102. What will 8 lb. 9 oz. of sugar cost at 19 cents per pound?

103. What will 36 lb. 5 oz. of salt be worth at $2.20 per cwt.?

104. What will 17 lb. of beef be worth at $27 per cwt.?

105. What must be given for 16 lb. 2 oz. of butter at 37½ ct. per pound?

106. What must be given for 11 oz. of cheese at 25 ct. per pound?

107. If 7 lb. of sugar are worth $1.20, what is 1 lb. worth?

108. If 7 lb. of sugar are worth $1.60, what are 5 lb. worth?

109. If 7 lb. of sugar are worth $1.17½, what are 9 lb. 8 oz. worth?

110. If 7 lb. of sugar are worth $1.20, what are 3lb. 8 oz. worth?

LESSON XIII.

111. What cost 46 tons of coal at 2½ cents per pound?

112. What cost 34 tons 13 cwt. 3 qr. of coal at 2d. per pound?

(*b.*) TABLE OF MISCELLANEOUS WEIGHTS.

196 lb.	make	1	barrel of	flour.
200 "	"	1	"	beef, pork, or fish.
280 "	"	1	"	salt at N. Y. S. Salt Works.
32 "	"	1	bushel of	oats.
48 "	"	1	"	barley.
56 "	"	1	"	corn or rye.
60 "	"	1	"	wheat.

NOTES.—1. The gross ton of 2240 lb. was formerly in common use, but is now seldom used except at the United States Custom House, and at the Pennsylvania coal mines.

2. Butter is usually packed for market in firkins which hold from 50 to 100 pounds.

113. What will be the cost of 13 firkins of butter at 18½ ct. per pound, allowing each firkin to hold 56 pounds?

REVIEW.—What is the Canada currency? (78., *d.*) What is the rule for reduction ascending? (78., *h.*) What is the rule for reduction descending? (78., *g.*) What is weight? (79.) What is the use of Troy weight? (80.) What are the denominations of Troy weight? Recite the table. (80., *a.*) What is the use of avoirdupois weight? (81.) Recite the table. (81., *a.*)

114. What will be the cost of 15 barrels of flour at 2 ct. per pound?

115. Bought 148 barrels of pork at 2 ct. per pound; what did it cost?

116. What cost 374 barrels of salt at 1½ ct. per pound?

117. How many pounds in 13 barrels of flour?

118. How many pounds in 376 bushels of oats?

119. How many pounds in 137 bushels of rye?

120. How many pounds in 417 bushels of barley?

121. How many pounds in 371 bushels of wheat?

122. How many pounds in 417 bushels of corn?

QUESTIONS.—How many pounds make a hundred weight in the common ton? How many in the gross ton?

LESSON XIV.

APOTHECARIES' WEIGHT.

82. *Apothecaries' Weight* is used in preparing prescriptions, but drugs and medicines are bought and sold by avoirdupois weight.

(*a*.) TABLE.

20 grains (gr.)	make 1 scruple,	marked	sc. or ℈.
3 scruples	" 1 dram,	"	dr. or ʒ.
8 drams	" 1 ounce,	"	oz. or ℥.
12 ounces	" 1 pound,	"	lb. or ℔.

APOTHECARIES' FLUID MEASURE.

83. *Apothecaries' Fluid Measure* is used for measuring liquids in preparing medical prescriptions.

(*a*.) TABLE.

60 minims (♏)	make 1 fluid drachm	marked	fʒ.
8 fluid drachms	" 1 fluid ounce	"	f℥.
16 fluid ounces	" 1 pint	"	O.
8 pints	" 1 gallon	"	Cong.

NOTE.—One minim equals one drop.

REVIEW.—Repeat the table of the avoirdupois weight. (81., *a*.) For what is it chiefly used? (81.) Recite the table of miscellaneous weights? (81., *b*.)

(b.) A TABLE OF THE COMPARISON OF WEIGHTS.

* 1 lb. Troy=5760 gr. Troy.
1 ℔. Apoth.=5760 gr.
1 lb. Avoir.=7000 gr. Troy.
144 lb. Avoir.=175 lb. "

SCALE OF COMPARISON.

TROY WEIGHT.

lb.	oz.	pwt.	gr. †
1	12	240	5760
	1	20	480
		1	24
		1k.	3⅕

AVOIRDUPOIS WEIGHT

lb.	oz.	dr.	gr. †
1	16	256	7000
	1	16	437½
		1	27 11/32

APOTHECARIES' WEIGHT.

℔	℥	ʒ	℈	gr. †
1	12	96	288	5760
	1	8	24	480
		1	3	60
			1	20

T.	cwt.	qr.	lb.
1	20	80	2000
	1	4	100
		1	25

MENTAL EXERCISES.

1. How many grains in 3 scruples?

FORMULA.—Since in 1 scruple there are 20 grains, in 3 scruples there are 3 times 20 grains which are 60 grains; therefore in 3 scruples there are 60 grains.

2. How many drams in 3 ounces? In 4 oz.? In 7 oz.?
3. How many scruples in 3 drams? In 4 ʒ? In 6 ʒ?
4. How many ounces in 4 lbs? In 3℔? In 9℔?
5. How many drams in 12 oz.? In 6 ℥? In 9 ℥?

---

* The *pound*, *ounce*, and *grain*, of troy and apothecaries' weight are equal, although the intermediate denominations differ.

† The troy grain is the unit of measure in this scale.

REVIEW.—What is the use of apothecaries' weight? (82.) By what weight are medicines bought and sold? Recite the table. (82., *a.*) What is the use of apothecaries' fluid measure? (83.) Recite the table. (83., *a.*) Recite the table of the comparison of weights? (83., *b.*)

LESSON XV.

Change 34℔ 11℥ 5ʒ 2℈ 14 gr. to grains.

MODEL OPERATION.

34℔ 11℥ 5ʒ 2℈ 14 gr.
12℥
———
419 "
8ʒ
———
3357 "
3℈
———
10073 "
20 gr.
———
201474 "

PROOF.

20 gr.)201474 gr.
———
3℈)10073℈+14 gr.
———
8ʒ)3357ʒ+2℈
———
12℥)419℥+5ʒ
———
34℔+11℥

Anal. steps.—1. *Change* ℔. *to* ℥.
2. *Change* ℥ *to* ʒ.
3. *Change* ʒ *to* ℈.
4. *Change* ℈ *to gr.*

Proof.—1. *Change gr. to* ℈.
2. *Change* ℈ *to* ʒ.
3. *Change* ʒ *to* ℥.
4. *Change* ℥ *to* ℔.

123. Change 46℔ 8℥ 7ʒ 2℈ 13 gr. to grains.
124. Change 37℔ 8℥ 4ʒ 1℈ 15 gr. to grains.
125. Change 39℔ 7℥ 7ʒ 1℈ 17 gr. to grains.
126. Change 43℔ 3℥ 2ʒ 2℈ 19 gr. to grains.
127. Change 97℔ 3℥ to grains.
128. Change 9℥ 2℈ to grains.
129. Change 37℔ 1℈ to grains.
130. Change 37456 gr. to ℔.
131. Change 391674℈ to ℥.
132. How many pounds in 478374℈?
133. How many ℥ in 387867 gr.?
134. How many ℔ in 9187641ʒ?

Review.—How many kinds of reduction are there? (75.) (76.) What is the difference between simple and denominate numbers? (72.) (73.) What are the denominations of English money? (77., *a.*) Of troy weight? (80., *a.*) Of avoirdupois weight? (81., *a.*) Of apothecaries' weight? (82., *a.*) Of apothecaries' fluid measure? (83., *a.*) Which is the heavier, a pound apothecaries' weight or a pound troy? A pound troy or a lb. avoirdupois?

135. How many gr. in 3487 ʒ ?

136. How many gr. in 871864 ℈ ?

137. How many doses in 7 ℥ 6 ʒ 2 ℈ of tartar emetic, allowing 20 gr. for each dose?

138. How many pills can be made of 9 ℥ 4 ʒ 2 ℈ 8 gr. of calomel, allowing 12 gr. for each pill?

LESSON XVI.

MEASURES OF SPACE.

84. ***Space.***—Extension in any direction is called space.

85. ***A Line*** has length only.

86. ***A Surface*** has length and breadth.

87. ***A Solid*** has length, breadth, and thickness.

LONG MEASURE.

88. ***Long Measure*** is used for distances, etc.

(*a.*) TABLE.

12 inches (in.)	make 1 foot,	marked ft.
3 ft.	“ 1 yard,	“ yd.
5½ yd.	“ 1 rod,	“ rd.
40 rd.	“ 1 furlong,	“ fur.
8 fur.	“ 1 mile,	“ mi.

SCALE OF COMPARISON.

mi.	fur.	rd.	yd.	ft.	in.
1=	8=	320=	1760 =	5280 =	63360
	1=	40=	220 =	660 =	7920
		1=	5½=	16½=	198
			1 =	3 =	36
				1 =	12

LESSON XVII.

CLOTH MEASURE.

89. ***Cloth Measure*** is used by merchants in the sale of cloth, ribbons, laces, etc.

(*a.*) TABLE.*

2 sixteenths (16th)	make	1 eighth,	marked	8th, $\frac{1}{8}$ yd.
2 eighths	"	1 quarter,	"	qr., $\frac{1}{4}$ yd.
2 quarters	"	1 half,	"	hlf., $\frac{1}{2}$ yd.
4 quarters or 2 halves	"	1 yard,	"	yd.

(*b.*) NOTE.—The old system of measuring cloth is not now used. By it each yard is divided into 4 quarters, and each quarter into 4 nails, a nail being $2\frac{1}{4}$ inches. 3 quarters make a Flemish ell, 5 quarters an English ell, and 6 quarters a French ell.

(*c.*) TABLE OF MISCELLANEOUS LINEAR MEASURE.

4 inches	make	1 hand.	Used in measuring the height of horses at the shoulder.
9 inches	"	1 span.	
3 feet	"	1 pace, or step.	
6 feet	"	1 fathom.	Used in measuring depths at sea.
880 fathom	"	1 mile.	
3 geographical miles	"	1 league.	
60 " "	"	1 degree.	Of latitude.
$69\frac{1}{6}$ statute "			Of longitude on the equator.

NOTE.—A ship's cable is a chain, usually about 120 fathoms or 720 feet long, hence the term "cable-length" in nautical language denotes about that distance.

NOTES.—1. A knot is a nautical or geographical mile. Thus, the phrase, "thirteen knots an hour," means thirteen geographical miles an hour.

2. 1 English mile equals 5280 feet, and 1 nautical, or geographical mile, equals 6086 feet.

LESSON XVIII.

MENTAL EXERCISES.

1. How many yards in 7 rods?

FORMULA.—Since there are $5\frac{1}{2}$ yards in one rod, in 7 rods there are 7 times $5\frac{1}{2}$ yards. 7 times 5 yards equal 35 yards. 7 times 1 half-yard equal 7 half-yards, equal to 3 and 1 half yards, which added to 35 yards makes $38\frac{1}{2}$ yards.

Therefore in 7 rods there are $38\frac{1}{2}$ yards.

2. How many yards in 3 rods? In 4 rd.? In 6 rd.? In 9 rd.? In 11 rd.?

3. How many statute miles in 3 degrees? In 2°? In 10°?

---

* The teacher will doubtless appreciate the new table of cloth measure, for the reason that while it is *practical*, at the same time it can be used to enable the pupil to obtain a clear and definite conception of the meaning and use of fractions, concretely, before being required to study them abstractly.

4. How many rods in 17 yards?

Anal. Steps.—1. *Change 17 yds. to half-yards.*

FORMULA.—Since in 1 yard there are 2 halves, in 17 yd. there are 17 times 2 half-yards, which is 34 half-yards.

2. *Change half-yards to rods.*

FORM.—Since in 11 half-yards there is 1 rod, in 34 half-yards there are as many rods as 11 half-yards are contained times in 34 half-yards; which is 3 times with a remainder of 1 half-yard.

Therefore in 17 yards there are 3 rods and 1 half of a yard over.

5. How many rods in 12 yards? In 18 yd.? In 20 yd.? In 16 yd.? In 14 yd.? In 10 yd.?

LESSON XIX.

Change 775683 mi. 1 fur. 38 rd. 2 yd. 2 ft. 9 in. to inches.

MODEL OPERATION.

775683 mi.
8 fur.

6205464 "
1 "

6205465 "
40 rds.

248218600 "
38 "

2)248218638 "
5½ yds.

124109319 "
1241093190 "

1365202509 "
2 "

1365202511 "
3 feet.

4095607533 "
2 "

4095607535 "
12 in.

49147290420 "
9 "

49147290429 "

PROOF.

12 in.)49147290429 in.

3 ft.)4095607535 ft. + 9 in.

5½ yd.)1365202511 yd. + 2 feet.
2 h. yd. 2 half-yd.

11 h. yd.)2730405022 half-yd.

40 rd.)248218638 rds. + 4 h. yd. = 2 yd.

8 fur.)6205465 fur. + 38 rd.

775683 mi + 1 fur.

Anal. Steps.

Red. Desc.	Red. Asc.
1. *Change mi. to fur.*	1. *Change in. to ft.*
2. *Change fur. to rds.*	2. *Change ft. to yd.*
3. *Change rds. to yd.*	3. *Change yd. to h. yd.*
4. *Change yd. to ft.*	4. *Change h. yd. to rds.*
5. *Change ft. to in.*	5. *Change rds. to fur.*
	6. *Change fur. to mi.*

Anal. Steps.—1. *Change miles to furlongs.*

FORMULA.—Since in 1 mile there are 8 furlongs, in 775,683 miles there are 775,683 times 8 furlongs, which is 6,205,464 furlongs, which with 1 furlong added makes 6,205,465 furlongs.

2. *Change furlongs to rods.*

FORM.—Since n 1 furlong there are 40 rods, in 6,205,465 furlongs there are 6,205,465 times 40 rods, which is 248,218,600 rods, which with 38 rods added makes 248,218,638 rods.

3. *Change rods to yards.*

FORM.—Since in 1 rod there are 5½ yards, in 248,218,638 rods there are 248,218,638 times 5½ yards, which is 1,365,202,509 yards, which with 2 yards added makes 1,365,202,511 yards.

4. *Change yards to feet.*

FORM.—Since in 1 yard there are 3 feet, in 1,365,202,511 yards there are 1,365,202,511 times 3 feet, which is 4,095,607,533 feet, which with 2 feet added makes 4,095,607,535 feet.

5. *Change feet to inches.*

FORM.—Since in 1 foot there are 12 inches, in 4,095,607,535 feet there are 4,095,607,535 times 12 inches, which is 49,147,290,420 inches, which with 9 inches added makes 49,147,290,429 inches.

Concl.—Therefore, in 775683 mi. and 1 fur. 38 rd. 2 yd. 2 ft. 9 in. there are 49,147,290,429 inches.

Anal. Steps of Proof.—1. *Change inches to feet.*

FORM.—Since in 12 inches there is 1 foot, in 49,147,290,429 inches there are as many feet as 12 inches are contained times in 49,147,290,429 inches, which is 4,095,607,535, with a remainder of 9 inches.

2. *Change feet to yards.*

FORM.—Since in 3 feet there is 1 yard, in 4,095,607,535 feet there are as many yards as 3 feet are contained times in 4,095,607,535 feet, which is 1,365,202,511, with a remainder of 2 feet.

3. *As yards cannot be changed directly to rods, change them to half-yards.*

FORM.—Since there are 2 half-yards in 1 whole yard, in 5½ yards, (the number of yards in a rod,) there are 5½ times 2 half-yards, which is 11.

FORM.—Since there are 2 half-yards in one whole yard, in 1,365,202,511 yards there are 1,365,202,511 times 2 half-yards, which are 2,730,405,022.

4. *Change half-yards to rods.*

FORM.—Since in 11 half-yards there is 1 rod, in 2,730,405,022 half-yards there are as many rods as 11 half-yards are contained times in 2,730,405,022 half yards, which is 248,218,638, with a remainder of 4 half-yards, equal to 2 yards.

5. *Change rods to furlongs.*

FORM.—Since in 40 rods there is 1 furlong, in 248,218,638 rods there are as many furlongs as 40 rods are contained times in 248,218,638 rods, which is 6,205,465, with a remainder of 38 rods.

6. *Change furlongs to miles.*

Form.—Since in 8 furlongs there is 1 mile, in 6,205,465 furlongs there are as many miles as 8 furlongs are contained times in 6,205,465 furlongs, which is 775,683, with a remainder of 1 furlong.

Concl.—Therefore, in 49,147,290,429 inches there are 775683 mi. 1 fur. 38 rd. 2 yd. 2 ft. 9 in.

LESSON XX.

139. Change 12 mi. 3 fur. 7 rd. 1 yd. 1 ft. 6 in. to inches.
140. Change 5 mi. 2 fur. 6 rd. 2 yd. 2 ft. 4 in. to inches.
141. Change 11 mi. 2 fur. 3 rd. 2 yd. 2 ft. 3 in. to inches.
142. Change 3 mi. 2 fur. 7 rd. 3 yd. 2 ft. 3 in. to inches.
143. Change 7 mi. 2 fur. 3 rd. 2 yd. 2 ft. 2 in. to inches.
144. Change 346745 in. to miles.
145. Change 171234 in. to miles.
146. Change 186732 in. to miles.
147. Change 186413 ft. to miles.
148. Change 141374 yd. to miles.
149. Change 3467 yd. to rods.
150. Reduce 4176 ft. to miles.
151. Reduce 6713 half-yards to miles.
152. Reduce 4 mi. 6 yd. to feet.
153. Reduce 46 mi. 33 rd. to inches.

Questions.—How many steps are required to change rods to miles? Inches to miles? Rods to yards? Rods to feet? Furlongs to feet? Rods to inches? Yards to inches? Inches to yards? Yards to half-yards? Inches to half-yards? Yards to furlongs? Feet to furlongs? Rods to miles?

LESSON XXI.

154. How many feet in 31864 hands?

Anal. Steps.—1. *Change hands to inches.*
2. *Change inches to feet.*

155. Change 47183 hands to feet.
156. How many hands in 371864 feet?
157. How many hands in 341140 feet?
158. How many feet in 371864 spans?
159. How many feet in 418371 spans?
160. How many spans in 37134 feet?

161. How many leagues in 378413 geographical miles?
162. How many leagues in 41837 geographical miles?
163. How many geographical miles in 378467 leagues?
164. How many geographical miles in 34126 degrees?
165. How many leagues in 371864 degrees?
166. How many yards in a geographical mile?
167. How many yards in a statute mile?
168. How many yards in a league?
169. How many feet longer is a geographical than a statute mile?

LESSON XXII.

PRACTICAL PROBLEMS.

What cost 5 and 3 8*ths* yards of cloth at 37½ cts. per yard?

MODEL OPERATION.

8)$.3750 × 5 = 1.8750 cost 5 yd.

.0468 × 3 = .1404 " ⅜ "

$2.0154 Ans.

Anal. Steps.—1. *Find cost of 5 yd.*
2. *Find cost of ⅛th, then of ⅜ths.*

170. What cost 4 and 3 quarters yards of cloth at $1.75 per yard?

171. What must I give for 17⅜ yds. of lace at $1.25 per yard?

172. What must I give for 11¾ yards of calico at 39½ cts. per yard?

173. What must I give for 2 sixteenths of a yard of silk at $2 per yard?

174. What must I give for 17½ yards of muslin at 38 cts. per yard?

175. What will 9⅜ yards of ribbon come to at 87 cts. per yard?

176. What will 46¾ yards of tape cost at 7 cts. per yard?

177. What will 39⅛ yd. of velvet be worth at $5 per yard?

LESSON XXIII.

MISCELLANEOUS PRACTICAL PROBLEMS.

178. What is the amount of
16½ yd. calico @ 38 cts. per yd.,
14⅜ yd. velvet @ $4 per yd.,
11 5-16ths yd. satin @ $9 per yd.,
41¾ yd muslin @ 53½ cts. per yd.,
16⅝ yd. lawn @ 13cts. per yd. ?

179. What is the amount of the following items?
6 lb. 6 oz. sugar @ 19 cts. per lb.,
12 oz. spice @ 37½ cts. per lb.,
8 lb. 8 oz. raisins @ 30 cts. per lb.,
5 lb. 6 oz. rice @ 22 cts. per lb.,
4 lb. 8 oz. honey @ 38 cts per lb.

180. What is the amount of
12 lb. 13 oz. butter @ 80 cts. per lb.,
13 cwt. 82 lb. flour at $6 per cwt.,
87 lb. beef @ $18 per bbl.,
69 lb. pork @ $30 per bbl.,
47 lb. fish @ $37 per bbl. ?

181. What is the amount of
17 lb. oats @ 87½ cts. per bushel,
37 lb. barley @ 90 cts. per bushel,
39 lb. corn at 95 cts. per bushel,
39 lb. wheat at $2.25 per bushel,
46 lb. rye @ $1.12½ per bushel?

182. What is the amount of
3 cwt. of coal @ $11.50 per ton,
3 qr. of coal @ $13.70 per ton,
2 cwt. 3 qr. of sugar @ $6.30 per cwt.,
5 bags of rice each weighing 98 lb. @ $12 per cwt. ?

183. What cost
7 yd. lace @ 37½ cts. per yd.,
14 yd. muslin @ 40 cts. per yd.,
14⅛ yd. velvet @ $18.70 per yd.,
13¾ yd. silk @ $1.50 per yd.,
19⅝ yd. calico @ 36 cts. per yd.?

184. What cost
3 lb. 6 oz. pepper @ 38 cts. per lb.,
4 lb. 3 oz. spice @ 40 cts. per lb.,
18 lb. 2 oz. cinnamon @ 80 cts. per lb.,
5 lb. 7 oz. cloves @ $1.80 per lb.,
9 lb. 11 oz. dates @ 37½ cts. per lb. ?

185. What must I give for
7 lb. 8 oz. brass @ 42 cts. per lb.,
6 lb. 3 oz. copper wire @ 87 cts. per lb.,
46 lb. nails @ 6½ cts. per lb.,
13lb. 6 oz. iron wire @ 11 cts. per lb.,
6 lb. 5 oz. sheet brass @ 50 cts. per lb. ?

LESSON XXIV.

SURVEYOR'S MEASURE.

90. Gunter's Chain is used by land surveyors. It is 4 rods or 66 feet long, and contains 100 links.

(*a*.) TABLE.

25 links* (li.)	make	1 rod,	rd.
4 rods	"	1 chain,	ch.
80 chains	"	1 mile,	mi.

Prob.—How many miles in 4187864 links?

MODEL OPERATION.

25 links)4187864 links.

4 rd.)167514 rd.+14 links.

80 ch.)41878 chains+2 rods.

523 mi.+38 chains.

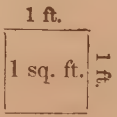
Anal. Steps.	**Steps of Proof.**
1. *Change li. to rd.*	1. *Change mi. to ch.*
2. *Change rd. to ch.*	2. *Change ch. to rd.*
3. *Change ch. to mi.*	3. *Change rd. to li.*

186. Change 320 rd. to miles.
187. How many chains in 674 miles?
188. How many links in 374 mi. 40 ch. 3 rd.?
189. How many rods in 74 mi. 43 ch. 2 rd.?
190. How many rods in 674134 links?
191. How many miles in 37187413 links?
192. How many chains in 4187643 links?
193. How many rods in 374183 links?

Questions.—What is the use of surveyor's chain measure? How many links make a chain? How many chains make a mile? How many inches in a link? Is changing links to chains reduction ascending or descending? Which of the fundamental rules is used in reduction ascending? Which in reduction descending?

LESSON XXV.

SQUARE MEASURE.



91. A Square is a figure having four equal sides and four right angles or corners; a square foot is a square 1 foot in length and 1 foot in breadth.

---

* Notes.—1. A link is 7.92 in. in length.

2. Canal and railroad engineers commonly use the engineer's chain, which consists of 100 links each 1 foot long.

(*a.*) TABLE.

144	square inches (sq. in.)	make	1 square foot,	marked	sq. ft.
9	square feet	"	1 square yard,	"	sq. yd.
30¼	square yards	"	1 square rod,	"	sq. rd., P.
40	square rods	"	1 rood, or qr. acre,	"	R.
4	roods	"	1 acre,	"	A.
640	acres	"	1 square mile or section,	"	sq. m., sec.

(*b.*) SCALE OF COMPARISON.

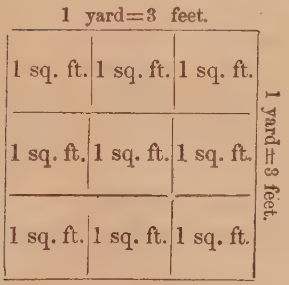
A.	R.	P.	sq. yds.	sq. ft.	sq. in.
1=	4=	160=	4840 =	43560 =	6272640.
	1=	40=	1210 =	10890 =	1568160.
		1=	30¼=	272¼=	39204.
			1 =	9 =	1296.
				1 =	144.

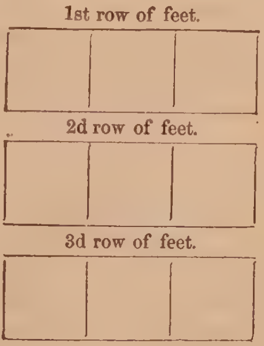
92. ***A Square Yard*** is a square 1 yard or three feet in length, and 1 yard or 3 feet in breadth; therefore a *square yard* will consist of 3 *rows* of feet with 3 feet in each row.

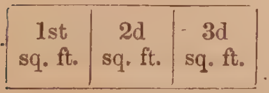
FORM.—Since in 1 row there are 3 square feet, in 3 rows there are 3 times 3 square feet, which are 9 square feet. Therefore in 1 square yard there are 9 square feet. Hence to find the area of any parallelogram, we have the following

(*a.*) **Rule.**—*Multiply the* LENGTH *by the* BREADTH, *and the product will be the surface.*

ILLUSTRATION.







(*b.*) **Square measure** is used in calculating areas or surfaces; as of land, lumber, painting, paving, &c.

(*c.*) **Artificers** usually estimate their work—

1. *By the square foot;* as in glazing and stone-cutting.

2. *By the square yard;* as in painting, plastering, paper-hanging, &c.

3. *By the* 100 *square feet;* as in flooring, roofing, slating, &c.

4. *By the thousand bricks, by the square yard, and* 100 *square feet;* as in bricklaying.

(*d.*) NOTES.—1. The painting of mouldings, cornices, &c., is estimated by measuring the entire surface.

2. When bricklaying is estimated by square measure, the work is understood to be 12 inches thick.

QUESTIONS.—What is meant by a square foot? How long is a square foot on each side? If 3 square feet are placed in a row, how long will the side measure? If 9 square feet are placed in a row, how long will one side measure? If 13 square feet are placed in the same way?

LESSON XXVI.

MENTAL EXERCISES.

1. How many square rods in 50 square yards?

(*e.*) **Anal. Steps.—1.** *As square yards cannot be directly changed to square rods, first change them to quarter square yards.*

FORM.—Since in 1 sq. yd. there are 4 *quarter* sq. yards, in 30¼ sq. yards (the number of square yards in 1 sq. rod) there are 30¼ times 4 quarter sq. yd., which are 121 quarter square yards.

FORM.—Since in 1 sq. yd. there are 4 quar. sq. yd., in 50 sq. yd. there are 50 times 4 quar. sq. yd., which are 200 quar. sq. yd.

2. *Next change quarter sq. yards to sq. rods.*

FORM.—Since in 121 quarter sq. yards there is 1 rod, in 200 quar. sq. yd. there are as many rods as 121 quar. sq. yd. are contained times in 200 quar. sq. yd., which is 1 sq. rd. with a remainder of 79 quar. sq. yd. equal to 19¾ sq. yd.

Concl.—Therefore, in 50 sq. yd. there are 1 sq. rd. 19¾ sq. yd.

2. How many square feet in 299 sq. inches? In 137 sq. in.?

REVIEW.—What are coins? Which are the gold coins? (64., *a.*) Which the silver coins? (64., *b.*) Which are the nickel coins? (64., *c.*) Which the copper? (64., *d.*) What is the use of the decimal point? (65.) How do the places in Federal money increase? (65., *a.*) How are eagles read? (65., *a.*) How are dimes? (65., *a.*)

3. How many square yards in 19 sq. feet? In 28 sq. ft.?
4. How many square yards in 2 sq. rods? In 3 sq. rd.?
5. How many square rods in 2 acres? In 3 A.? In 4 A.?
6. How many acres in 320 sq. rd.? In 329 sq. rd.?
7. How many roods in 3 acres? In 7 A.? In 6 A.?
8. How many roods in 38 sq. rd.? In 43 sq. rd.?
9. How many acres in 2 sq. miles? In 3 sq. mi.?
10. How many square rd. in 60 sq. yd.? In 100 sq. yd.?

LESSON XXVII.

How many square miles in 418783741864 square inches?

MODEL OPERATION.

144 sq. in.)418783741864 sq. in.

9 sq. ft.)2908220429 sq. ft.+88 sq. in.

$30\frac{1}{4}$ sq. yd. 323135603 sq. yd.+2 sq. ft.
4 q. sq. yd. 4 quar. sq. yd.

121 q. sq. yd.)1292542412 "

40 sq. rd.)10682168 sq. rd.+84 q. sq. yd.=21 sq. yd.

4 R.)267054 R.+8 sq. rd.

640 A.)66763 A.+2 R.

104 sq. mi+203 A.

Analytical Steps.	**Anal. Steps of Proof.**
1. *Change sq. in. to sq. ft.*	1. *Change sq. mi. to A.*
2. *Change sq. ft. to sq. yd.*	2. *Change A. to R.*
3. *Change sq. yd. to qr. sq. yd.*	3. *Change R. to sq. rd.*
4. *Change qr. sq. yd. to sq. rd.*	4. *Change sq. rd. to sq. yd.*
5. *Change sq. rd. to R.*	5. *Change sq. yd. to sq. ft.*
6. *Change R. to A.*	6. *Change sq. ft. to sq. in.*
7. *Change A. to sq. mi.*	

REVIEW.—What is U. S. Money? (63.) Recite the table. (63., *a.*) How many cents in 1 eighth of a dollar? In 1 tenth? In 1 fourth? In 3 fourths? In 2 tenths? In 1 fifth? How many mills in 1 half of a cent? In 1 fourth of a cent? In 1 third of a dollar? In 1 sixth of a dollar? In 1 ninth of a dollar?

QUESTIONS.—How many steps are required to change sq. inches to sq. miles? Sq. ft. to sq. yd.? Sq. in. to qr. sq. yd.? Sq. ft. to sq. rd.? How many steps are required to change sq. mi. to sq. in.? Sq. rd. to sq. ft.? Sq. roods to sq. rd.? Give the formula for each step in the model operation?

LESSON XXVIII.

194. How many square inches in 307 sq. mi. 37 A. 2 R. 8 sq. rd. 13 sq. yd. 7 sq. ft. 137 sq. in.?

195. How many square inches in 41 sq. mi. 14 A. 3 R. 14 sq. rd. 19 sq. yd. 6 sq. ft. 97 sq. in.?

196. How many square inches in 1 mi. 1 A. 1 R. 1 sq. rd. 1 sq. yd. 1 sq ft. 1 sq. in.?

197. How many sq. miles in 3718644187 sq. inches?

198. How many sq. miles in 4187184130 sq. inches?

199. How many sq. miles in 487000718 sq. inches?

200. How many sq. miles in 4187183 sq. feet?

201. How many sq. feet in 31 sq. mi. 1 R. 37 sq. yd.?

202. How many sq. miles in 4187134 sq. yards?

203. How many sq. yards in 317 acres 3 R.?

204. How many sq. miles in 4187133 sq. rods?

205. How many sq. miles in 4830004 R.?

206. Change 3743 sq. yards to sq. inches.

207. Change 418378 sq. yards to acres.

208. Change 40000 sq. rd. to sq. miles.

209. Change 83003 acres to sq. yards.

210. Change 378 acres to sq. rods.

211. Change 87186 sq. rods to acres.

212. Change 41876 sq. yards to acres.

213. Change 4183 sq. acres to sq. yards.

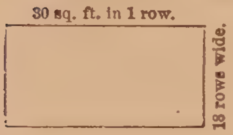
REVIEW.—What is reduction of U. S. money? (66.) What is the rule for changing dollars to cents? (66.) Cents to mills? Mills to *tenths* of a mill? What is the rule for changing mills to cents? (66., *b.*) Mills to dollars? Cents to dollars? What are bills? (71.) What is a debtor? (71., *a.*) What is a creditor? (71., *b.*) What is an account? (71., *c.*)

LESSON XXIX.

PRACTICAL PROBLEMS.

How many yards of carpeting 1 yard wide will it take to cover the floor of a room 30 feet long and 18 ft. wide?

ILLUSTRATION.*



MODEL OPERATION.

30 sq. ft.
18 no. rows.
sq. ft.
9)540 sq. ft.
60 sq. yd. Ans.

Anal. Steps.—**1.** *Find the area of the floor of the room.*
2. *Find the number of yards required.*

Formula.—1. If there are 30 square feet in 1 row of square feet across the room, in 18 rows there are 18 times 30 sq. ft., which are 540 sq. ft.

2. Since in 9 sq. ft. there is 1 sq. yard, in 540 sq. ft. there are as many sq. yards as 9 sq. feet are contained times in 540 sq. ft., which are 60; therefore it will take 60 sq. yards.

214. How many sq. feet in the floor of a room 37 ft. by 40?

215. How many sq. yards in the floor of a room 86 ft. by 30?

216. How many sq. feet in the four walls of a room 9 ft. high, 46 ft. by 30?

217. How many sq. yd. in the four walls of a room 8 ft. high, 37 ft. by 50?

218. How many sq. ft. in the ceiling of a room 46 ft. by 37?

219. How many sq. ft. in the ceiling of a room 93 ft. by 67?

220. How many square feet in the ceiling and walls of a room 11 ft. high, 38 ft. by 67?

221. How many square yards in the ceiling and walls of a room 15 ft. high, 39 ft. by 87?

---

* Note for the Teacher.—The pupil should be required to illustrate problems of this character at the blackboard by diagrams in which the square feet are shown on a reduced scale.

222. What will it cost to paper the walls of a room 11 ft. high, 36 ft. by 24 at \$1.37½ per sq. yard?

223. What will it cost to paint the walls of a room 10 ft. high, 36 ft. by 54 at 87½ cts. per sq. yard?

224. How many square feet in a board 13 feet long 8 inches wide?

LESSON XXX.

MODEL OPERATION.

12 in.
13 no. of feet.
156 sq. in. in 1 row.
8 no. rows.
sq. in.
144)1248 sq. inches.
8 sq. ft. 96 sq. in. Ans.

Anal. Steps.—1. *Find the length of the board in inches.*
2. *Find the number of sq. in. in the board.*
3. *Change the sq. in. to sq. ft.*

225. How many sq. feet in a board 9 feet long 9 in. wide?
226. How many sq. feet in a board 16 feet long 13 in. wide?
227. How many sq. feet in a board 16 feet long 18 in. wide?
228. How many sq. feet in a board 11 feet long 17 in. wide?
229. How many sq. feet in a window 4 ft. by 3?
230. How many sq. feet in a window 6 ft. by 5 ft. 6 in.?

SUGGESTION.—Change the length and width to inches before multiplying.

231. How much will it cost to glaze a window 8 ft. by 5 at \$2.87½ per sq. foot?

232. How much will it cost to glaze a sash 9 ft. by 6 at \$1.39 per sq. foot?

LESSON XXXI.

233. What will it cost to slate a roof 40 ft. by 60 at \$3.80 per 100 sq. feet?

Anal. Steps.—1. *Find cost of* 1 *sq. foot.*
2. *Find the number of sq. feet in the roof.*
3. *Find the cost.*

234. What will it cost to put on a tin roof 48 ft. by 70 at $4.86 per 100 sq. feet?

235. What will it cost to floor a room 80 ft. by 60 at $3.86 per 100 sq. feet?

236. What must I give for slating a roof 80 feet long 47 wide at $3.40 per 100 sq. feet?

237. What must I give to have a tin roof put on a house, each slope being 60 ft. by 37, at $2.27 per 100 feet?

238. How much must I give for a board containing 36 square feet, if 144 sq. in. or 1 sq. foot is worth 6 cents?

239. How much must I give for a board 13 ft. long 18 in. wide at 8 cts. per sq. ft.?

240. How much must I give for paving a walk 300 ft. long 8 ft. wide at $3 per 100 sq. ft.?

241. How much must I give for paving a walk 89 ft. long 9 feet wide at $3.37½ per 100 sq. ft.

QUESTIONS.—What are the dimensions of a square foot? What is the length of 9 sq. feet placed in a row? What is the length of one side of a square containing 9 sq. feet? What is the length of a row of 13 sq. inches? How many square feet will it take to cover the surface of a table 9 feet long and 6 feet wide? Seven feet long and 4 feet wide? How many blocks 1 foot square will it take to cover the floor of a room 10 feet long and 12 feet wide? How many rows of feet will it take? How many blocks in a row?

LESSON XXXII

242. I own a piece of land 7 furlongs 13 rods long, and 15 rods wide; how many square rods does it contain?

243. I own a piece of land 9 furlongs 7 rods long, and 37 rods wide; how many rods of fence will it take to enclose it?

REVIEW.—What is a simple number? (72.) What is a compound number? (73.) What is reduction? (74.) What is reduction descending? (75.) What is reduction ascending? (76.) What is English money? (77.) Recite the table. (77., *a.*) Which are the gold coins? (78., *a.*) Which the silver? (78., *b.*) The bronze? (78., *c.*)

244. I own a piece of land 3 furlongs 3 rods long, and 13 rods wide; how much will the fence enclosing it cost me at 6 cents per foot?

245. I own a piece of land 2 furlongs 5 rods long, and 37 rods wide; how much will the land cost me at 3 cents per square foot?

246. How many square feet in a ceiling 23 feet long and 17 feet wide?

247. How many square feet in the four walls of a room 26 feet long, 19 feet wide, and 9 feet high?

248. How many square feet in the floor of a room 40 feet long and 18 feet wide?

249. How many square feet in the walls, floor, and ceiling of a room 87 feet long, 38 feet wide, and 18 feet high?

250. How much would it cost to paper the walls of a room 19 feet long, 13 feet wide, and 9 feet high, at 9 cents per square foot?

251. How much would it cost to plaster the walls and ceiling of a room 17 feet long, 13 feet wide, and 9 feet high, at 7 cents per square foot?

LESSON XXXIII.

252. What will it cost to paint 1 square foot, if one square yard costs 37 cents?

253. What will it cost to pave 3 square feet, if it costs $1.44 to pave 1 square yard?

254. What will it cost to lay 1 square foot of floor, if 100 square feet cost $2.39?

255. What will it cost to lay 27 square feet of roof, if 100 square feet cost $9.00?

256. What will it cost to roof a house with tin, the length

REVIEW.—What is the Canada currency? (78., *d.*) What is the rule for reduction ascending? (78., *g.*) What is the rule for reduction descending? (78., *f.*) How may reduction descending be proved? (78., *h.*) Of what is reduction ascending the reverse?

of each side being 84 feet and the width 27 feet at $18 per 100 square feet?

257. What will it cost to pave a walk 46 rods long and 8 feet wide at 54 cents per square yard.

258. What will it cost to roof a house with slate, the length of each side being 46 feet and the width 18 feet, at $16 per 100 square feet?

259. What will it cost to paint the outside of a house 54 feet long 33 feet wide 21 feet high, at 75 cents per square yard, making no allowance for windows and doors?

260. What must I give for 41 acres of land at 2 cents per square foot.

261. What will it cost to pave a street 144 rods long 4 rods wide at $12 per square yard.

LESSON XXXIV.

SURVEYOR'S SQUARE MEASURE.

93. ***Surveyor's Square Measure*** is used in finding the area of land.

(*a.*) TABLE.

625 square links (sq. l.)	make	1 sq. rod,	marked	sq. rd.
16 sq. rods	"	1 sq. chain,	"	sq. ch.
10 sq. chains	"	1 acre,	"	A.
640 acres	"	1 sq. mile,	"	sq. mi.
36 sq. miles (six miles square)	"	1 township,	"	Tp.

262. How many square miles in 379786413 sq. l.?

REVIEW.—What is weight? (79.) What is the use of troy weight? (50.) What are the denominations? (80., *a.*) Recite the table. What is the use of avoirdupois weight? (81.) What are the denominations? (81., *a.*) What is the use of the gross ton? (81., *b.*, 1.) Recite the miscellaneous table. (81., *b.*) What is the use of apothecaries' weight? (82.) Recite the table. (82., *a.*) What is the use of apothecaries' fluid measure? (83.) Recite the table. (83., *a.*) Recite the table of the comparison of weights. (83., *b.*)

263. How many sq. rods in 14 sq. mi. 390 A. 9 sq. ch. 15 sq. rd.?

264. How many sq. links, in 13 sq. mi. 11 A. 3 sq. ch.?

265. How many square rods in 4187864 sq. l.?

266. How many acres in 4878346 square links?

267. How many acres in 371834 square rods?

268. How many square miles in 3784137 sq. chains?

LESSON XXXV.

CUBIC MEASURE.

94. ***A Cube*** is a solid, having six equal square sides. Thus, a cubic foot is a body 1 foot in length, 1 foot in breadth, and 1 foot in thickness.

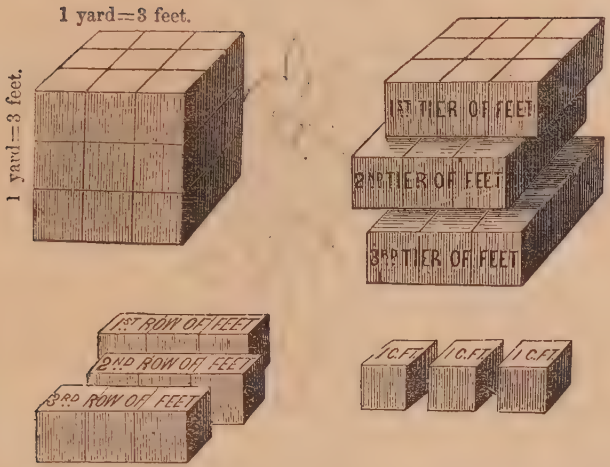
(*a.*) TABLE.

1728 cubic inches (cu. in.)	make 1 cubic foot,	marked cu. ft.
27 cubic feet	" 1 cubic yard,	" cu. yd.
40 cubic ft. of round timber, or 50 cubic feet of hewn timber	" 1 ton or load,	" T.
16 cubic feet	" 1 cord foot,	" cd. ft.
8 cord feet, or 128 cubic feet	" 1 cord of wood,	" Cd.
24¾ cubic feet	" 1 perch of stone, or masonry,	" Pch.

95. ***A Cubic Yard*** is a solid, one yard or three feet in length, 1 yard or 3 feet in width, and 1 yard or three feet in thickness; therefore a cubic yard will consist of three *tiers* of feet, each tier consisting of three rows, and each row consisting of 3 cubic feet.

REVIEW.—Recite the table of English money. (77., *a.*) Recite the table of troy weight. (80., *a.*) Of avoirdupois weight. (81., *a.*) Of apothecaries' weight. (82., *a.*) Of apothecaries' fluid measure. (83., *a.*) Of long measure. (88., *a.*) Of cloth measure. (89., *a.*) Of surveyor's measure. (90., *a.*)

ILLUSTRATION.



(*a.*) **Anal. Steps.—1.** *Find the number of rows of cu. ft. in* 1 *cu. yd.*

Form.—Since there are 3 *rows* of cubic feet in 1 *tier* of feet, in 3 *tiers* there are 3 times 3 *rows*, which are 9 *rows.*

2. *Find the number of cubic feet in* 1 *cu. yard.*

Form.—Since in 1 row there are 3 *cubic feet*, in 9 *rows* there are 9 times 3 cubic feet, which are 27 cubic feet.

Concl.—Therefore, in one cubic yard there are 27 cubic feet. Hence to find the solid contents of a body we have the following

(*b.*) **Rule.**—*Multiply the* LENGTH *by the* BREADTH, *and that product by the* THICKNESS.

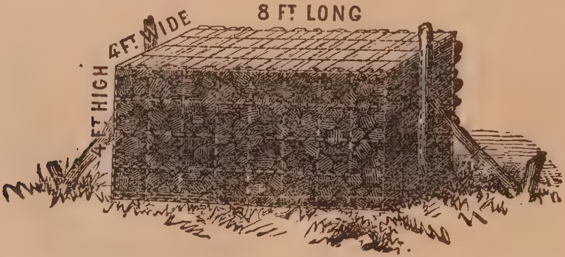
(*c.*) **Cubic Measure** is used in estimating the contents of solids; as wood, stone, capacity of cisterns, &c.

(*d.*) Notes.—1. A load of earth contains a cubic yard.

2. Railroad and transportation companies estimate light freight by the number of cubic feet it occupies; but heavy freight is estimated by weight.

Review.—Recite the table of square measure. (91., *a.*) Of surveyor's square measure. (93., *a.*) Of cubic measure. (94., *a.*) What are the steps to change pounds to shillings? (77., *a.*) Shillings to pence? Farthings to pounds? What is the use of troy weight? (80.) Of avoirdupois weight? (81.)

3. A pile of wood 4 feet wide, and 4 feet high, and 8 feet long contains 1 *cord* ; and a *cord foot* is 1 foot in length of such a pile, thus:



4. A perch of stone or masonry is 16½ feet long, 1½ feet wide, and 1 foot high, and contains 24¾ cubic feet.

5. A brick is usually 8 inches long, 4 inches wide, and 2 inches thick, hence 27 bricks make a cubic foot.

6. Joiners, painters and masons make no allowance for windows, doors, &c. Masons make no allowance for the corners of the walls of houses or of cellars. The size of a cellar is estimated by the measurement of the outside of the wall.

7. Round timber when squared is estimated to lose *one-fifth* ; hence a ton of round timber is said to contain only 40 cubic feet.

269. How many cords of wood in 37846 cubic feet ?
270. How many cubic feet in 41 perch of stone ?
271. How many cubic feet in 37 perch of masonry ?
272. How many perch of stone in 864134 cubic feet ?
273. How many perch in 41878 cubic feet of masonry ?

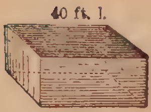
QUESTIONS.—How may every cubic solid be separated ? ANS.—*Into tiers.* How may each tier be separated ? ANS.—*Into rows of cubes.* How may each row be separated ? ANS.—*Into cubes.* [The teacher should with small cubes show the pupils the different shaped piles into which they may be placed. He should also show that the length of the side of a square or edge of a cube simply shows the number of squares or cubes that can be placed in *one* row across the square or pile.]

LESSON XXXVI.

PRACTICAL PROBLEMS.

How many loads of earth must be removed to dig a cellar 40 feet long 18 feet wide and 9 feet deep ?

MODEL OPERATION.



40 cu. ft. 1 row.
18 no. rows of cu. ft.
720 cu. ft. in a tier.
9 no. tiers ft.
cu. ft. 27)6480 cu. ft.
240 loads. ANS.

Anal. Steps.—1. *Find the cu. ft. in* 18 *rows or* 1 *tier of cu. ft.*
2. *Find the cu. ft. in* 9 *tiers of cu. ft.*
3. *Find the number of loads.*

274. How many loads of earth must be removed in digging a cellar 37 by 60 ft., 8 feet deep?

275. What will it cost to dig a cellar 27 by 60 ft., 10 ft. deep, at 75 cts. per load?

276. How many cubic feet in a bin 8 ft. long, 3 ft. wide, and 4 ft. deep?

277. Making no allowance for thickness of boards, how many square feet of boards will it take to make a box and cover, 15 ft. long 8 ft. wide and 4 ft. high?

278. How many cubic feet of stone in a wall 37 ft. long, 3 ft. high, and 2 ft. thick?

279. How many cubic feet of stone are required in the walls of a cellar 37 ft. long, 16 ft. wide, 5 ft. high, and 2 ft. thick?

280. How many perches of stone are required in the walls of a cellar 37 ft. long, 18 ft. wide, 6 ft. high, and 3 ft. thick?

281. How many bricks in a wall, 37 ft. long, 8 ft. high, and 2 ft. thick?

282. What will the brick cost to build a wall 37 ft. long, 6 ft. high, 8 ft. thick, at $3.30 per hundred?

283. I am to build a house the walls of which are as follows:

REVIEW.—What dimensions has a line? (83.) What has space? (84.) What has a surface? (86.) What has a solid? (87.) What is the use of Long Measure? (88.) Recite the table. (88., *a*.)

The front wall 50 ft. high and 30 wide; the rear 40 ft. high and 30 wide; the two side walls each 50 by 120 ft. What will the brick cost me, if the walls are each 1 ft. thick at $0.50 per hundred?

LESSON XXXVII.

284. How many cubic blocks one foot on each side can be sawed from a cube 1 yard on each edge allowing no waste for sawing?

285. How many 1 inch cubic blocks will it take to fill a mortise 1 foot deep, 1 foot wide, and 1 foot long?

286. How many 1 quarter inch cubic blocks will it take to fill a mortise one inch deep, one inch wide, and one inch long?

287. How many 1 inch cubic blocks will it take to fill a mortise 2 feet long, 2 feet wide, and 2 feet deep?

288. Bought 16 loads of hewn timber, at 4 cts. per cubic foot; what was the cost?

289. How many cubic feet in a block of marble 8 feet long, 3 feet wide, and 2 feet thick?

290. How many cubic feet of earth in a pile 46 feet long, 38 feet wide, 17 feet thick?

291. How many cubic yards of earth in a pile 39 feet long, 25 feet wide, and 19 feet thick?

292. How many cubic yards of earth were removed from a cellar 37 feet long, 21 feet wide, and 11 feet deep?

QUESTIONS.—In the 284th problem, how many tier of feet in the block? How many cu. ft. blocks in 1 row of a tier? How many tiers of cu. inch blocks in the 285th problem? How many tiers of quarter inch cubic blocks in the 286th problem?

LESSON XXXVIII.

293. How many cubic feet of masonry in a wall 67 feet long, 12 feet high, and 4 feet thick?

REVIEW.—What is the use of cloth measure? (89.) Recite the table. (89., *a*.) What instrument is used by surveyors in measuring land? (90.) What is its length? Recite the table. (90., *a*.) What is the length of the chain used by civil engineers? (90., *a*, 2.)

294. How many perches of masonry in a wall 49 feet long, 11 feet high, and 5 feet thick?

295. How many perches of stone in the wall of a cellar 14 feet long, 13 feet wide, and 9 feet deep, making allowance for the corners, the wall being 1 foot thick?

296. How many perches of stone in the wall of a cellar 37 feet long, 26 feet wide, and 11 feet deep, making allowance for the corners, the wall being 2 feet thick?

297. How many perches of stone will it take to enclose a lot 38 rods long, 14 rods wide, the wall being 7 feet high and 4 feet thick, making allowance for the corners?

298. How many cubic feet of masonry in a rectangular fort 138 feet long, 97 feet wide, the wall being 28 feet high and 13 feet thick, allowance being made for the corners? How many perches?

299. If 128 cubic feet of wood cost $5.00, what will 1 foot cost?

300. If 128 cubic feet or 1 cord cost $5.888, what will 39 feet cost? 46 feet? 84 feet? 37 feet?

301. Bought a pile of wood 19 feet long, 4 feet wide, and 4 feet high; how many cubic feet?

302. Bought a pile of wood 27 feet long, 4 feet wide, 7 feet high; what was the cost, at $4.992 per cord?

Questions.—How are cubic feet changed to perches? Ans.—1. *Change cu. ft. to quarter cu. ft.* 2. *Change quarter cu. ft. to perches.* In the 295th problem what is meant by *making allowance for the corners?* Ans.—*Since the wall is* 1 *foot thick, the end walls prevent the side walls from extending the entire length of the cellar; but the latter lack* 1 *foot on each end, thus the side walls are each two feet shorter than the cellar.* How much shorter are the side walls in the 296th problem than the end walls? How much shorter are the side walls in the 297th problem than the end walls?

Review.—What is a square? (91.) Recite the table. (91., *a.*) What is a square yard? (92.) Recite the rule for finding the area of surfaces. (92., *a.*) What is the use of square measure? (92., *b.*) How do artificers estimate their work? (92., *c.*) What is the use of surveyor's square measure? (93.) Recite the table. (93., *a.*) What is a cube? (94.) What is the use of apothecaries' weight? (82.) What is the use of apothecaries' fluid measure? (83.) What has space? (84.) What has a line? (85.) What has a surface? (86.) What has a solid? (87.) What is the use of long measure? (88.) What is the use of cloth measure? (89.) What is the use of surveyor's measure? (90.)

LESSON XXXIX.

MEASURES OF CAPACITY.

96. ***Measures of capacity*** are cubic measures, and are the liquid or dry measures.

LIQUID OR WINE MEASURE.

97. ***Liquid measure*** is, of course, used in measuring liquids.

(*a.*) TABLE.

4 gills (gi.)	make	1 pint,	marked	pt.
2 pints	"	1 quart,	"	qt.
4 quarts	"	1 gallon,	"	gal.
31½ gallons	"	1 barrel,	"	bbl.
2 barrels, or 63 gallons	"	1 hogshead	"	hhd.

NOTE.—The denominations barrel and hogshead are used in estimating the capacity of cisterns, reservoirs, vats, &c.

(*b.*) The following denominations are sometimes used:

36 gallons	make	1 barrel of beer.
54 gallons, or 1½ barrels	"	1 hogshead of beer.
42 gallons	"	1 tierce.
2 hogsheads, or 126 gallons	"	1 pipe, or butt.
2 pipes	"	1 tun.

(*c.*) NOTES.—1. The barrel, hogshead, tierce, pipe, butt and tun are the names of casks, which are usually gauged, having the number of gallons they hold marked on them.

2. Ale or beer measure formerly used in measuring beer, ale, and milk, is now seldom used.

3. 1 gallon of pure water weighs nearly 8⅓ lb. avoirdupois; hence a pint weighs about a pound.

REVIEW.—Recite the table. (94., *a.*) What is a cubic yard? (95.) Give the analysis of the cubic yard. (95., *a.*) Recite the rule. (95., *b.*) What is the use of cubic measure? (95., *c.*) How many cubic feet in a load of earth? (95., *d.*) How do R. R. companies estimate heavy freight? (95., *d.*, 2.) How light freight? How large is a cord of wood? (95., *d.*, 3.) What are the dimensions of a perch of masonry? What is the usual size of a brick? (95., *d.*, 5.) How many bricks make a cubic foot? How do carpenters, joiners, and masons estimate their work? (95., *d.*, 6.)

How many gills in 13 hhd. 3 qt. 1 pt. 2 gi.?

MODEL OPERATION.

13 hhd. 3 qt. 1 pt. 2 gi.
63 gal.
819 "
4 qt
3279 "
2 pt.
6559 "
4 gi.
26238 "

PROOF.

4 gi.)26238 gi.
2 pt.)6559 pt.+2 gi.
4 qt.)3279 qt.+1 pt.
63 gal.)819 gal.+3 qt.
13 hhd.

303. How many gills in 43 hhd. 14 gal. 3 qt. 1 pt.?
304. How many pints in 37 hhd. 50 gal. 2 qt.?
305. How many quarts in 39 barrels 14 gal.?
306. How many pints in 16 barrels 12 gal.?
307. How many gallons in 378413 gills?
308. How many barrels in 4178341 gills?
309. How many barrels in 371841 quarts?
310. How many quarts in 374 barrels?

LESSON XL.

PRACTICAL PROBLEMS.

311. What will 24 gallons of domestic wine cost at $3.80 per gallon?

312. What will 34 gal. of molasses cost at 87½ ct. per gallon?

313. What will 27 gallons of vinegar cost at 18½ ct. per gallon?

Review.—What is the simple value of a figure? (9.) What is the local? (10.) What is the use of the cipher? (11.) Define numeration. (12.) What is a rule? (16.) What is the rule for notation? (17.) Define sum. (18.) Addition. (19.) What does the sign of equality show? (21.) What does the dollar sign show? (22.) What are the axioms of addition? (23., *a*.)

What will 4 gal. 3 qt. 1 pt. 2 gi. of molasses cost at \$1.87½ per gallon?

MODEL OPERATION.

$$\begin{array}{rl}
4)\underline{\$1.8750} \times \overset{\text{gal.}}{4} = & 7.5000\\
2)\underline{.4687} \times \overset{\text{qt.}}{3} = & 1.4061\\
4)\underline{.2343} \times \overset{\text{pt.}}{1} = & .2343\\
.0586 \times \overset{\text{gi.}}{2} = & \underline{.1172}\\
 & \$9.2576
\end{array}$$

Anal. Steps.—1. *Find cost of* 4 *gallons.*
2. *Find cost of* 1 *qt. then of* 3 *qt.*
3. *Find cost of* 1 *pt.*
4. *Find cost of* 1 *gi. then of* 2 *gi.*
5. *Find total cost.*

314. What is the cost of 2 gal. 3 qt. of vinegar at 29 ct. per gallon?

315. What cost 3 gal. 2 qt. of vinegar at 37½ ct. per gal.?

316. What cost 3 qt. 1 pt. of cider at 39 ct. per gal.?

317. What must I give for 13 gal. 2 qt. of molasses at \$1.87½ per gallon?

318. What must I give for 3 qt. 3 gi. of molasses at \$.87½ per gallon?

319. What must I give for 7 gallons 2 qt. of milk at 40 ct. per gallon?

320. What must I give for a pint of vinegar at 40 ct. per gallon?

321. What must I give for 3 gills of molasses at \$1.84 per gallon?

322. What must I give for a pint of domestic wine at \$3 per gallon?

Review.—What is a unit? (1.) What is a number? (2.) Define a concrete number. (3.) An abstract number. (4.) Of what does arithmetic treat? (5.) Define notation. (6.) What is the use of the Roman notation? (7.) Of the Arabic? (8.)

323. What must I give for 2 qt. of beer at 63 cts. per gal.?

324. I paid $63 for a hogshead of molasses; how much was that per gallon?

325. I paid $185 for a cask of brandy containing 56 gallons; how much was that per quart?

326. I paid $4 for a gallon of alcohol; how much was that per gill?

327. What is the amount of the following bill:—

3 qt. molasses	@	$1.87½	per gal.
2 gal. milk	@	40 cts.	“
3 gi. kerosene	@	88 cts.	“
1 pt. vinegar	@	37½ cts.	“
3 qt. cider brandy	@	$1.63	“

$——

328. What is the amount of the following items:—

3 gal. 2 qt. of vinegar	@	40 cts.	per gal.
18 gal. of cider	@	$4	per bbl.
11 gal. 3 qt. of molasses	@	$1.87	per gal.
3 gal. 3 pt. of wine	@	$4	per gal.

LESSON XLI.

DRY MEASURE.

98. ***Dry Measure*** is used for things not liquid, as grain, fruit, ashes, &c.

(*a*.) TABLE.

2 pts. (pt.)	make 1 quart,	marked qt.
8 quarts	“ 1 peck,	“ pk.
4 pecks	“ 1 bushel,	“ bu.

NOTE.—The English quarter contains 8 bushels of 70 pounds each, and i used in measuring grain. It is ¼ of the gross ton.

REVIEW.—What is a problem? (24.) What is a simple problem? (25.) Wha is a complex problem? (26.)

(*b.*) TABLE OF COMPARISON OF THE MEASURES OF CAPACITY.

1 gallon or 4 qt.	wine measure contains 231 cubic inches.
½ pk. or 4 qt.	dry measure " 268⅘ "
1 gallon or 4 qt.	beer measure contains 282 cubic inches.
1 bushel	dry measure " 2150¾ "

The bushel is 18½ inches in diameter and 8 inches deep.

One pt. dry measure equals 1.1635 pt. liquid measure.

SCALE OF COMPARISON.

WINE MEASURE.

gal. qt. pt. gi. cu. in.
1=4=8=32=231
1=2= 8= 57¾
1= 4= 28⅞

DRY MEASURE.

bu. pk. qt. pt. cu. in.
1=4=32=64=2150¾ nearly.
1= 8=16= 537½ "
1= 2= 67⅕ "

How many pints in 13 bu. 2 pk. 7 qt. 1 pt.?

MODEL OPERATION.

13 bu. 2 pk. 7 qt. 1 pt.
4 pk.
54 "
8 qt.
439 "
2 pt.
879 "

PROOF.

2 pt.)879 pt.
8 qt.)439 qt.+1 pt.
4)54 pk.+7 qt.
15 bu.+2 pk.

329. How many pints in 14 bu. 2 pk. 7 qt.?
330. How many pints in 25 bu. 3 pk.?
331. How many bushels in 37184 pints?
332. How many bushels in 387134 quarts?

REVIEW.—What is division? (50.) What is the dividend? (51.) What is the divisor? (52.) What is the quotient? (53.) What is the difference between Long division and Short division? (55.) (54,) What is the remainder in division? (56.) What is the sign in division? (57.) What is the fractional sign? (58.)

333. How many pecks in 418371 pints?

334. Bought 46 gal. 3 qt. of vinegar at 4 ct. per quart; how many bushels of apples at 1½ ct. per quart must I give in return to balance the debt?

335. Bought 14 barrels of cider at 18 ct. per gallon to be paid for in potatoes at $1.87 per peck; how many bushels of potatoes must I give?

336. Sold 24 barrels of oil at $1.12½ per gallon, and received corn in pay at 12½ cents per peck; how much did I receive?

337. Sold 12 barrels 14 gal. of burning fluid at 80 cts. per gallon, and took in pay flour at 2 ct. per pound; how much did I receive?

338. Sold 186 bushels of wheat at $1.25 per bushel, and took in pay sugar at 5 cents per pound; how much did I receive?

339. What cost 46 bu. 3 pk. of corn at 87½ ct. per bushel?

340. What must I give for 3 bu. 3 pk. of clover seed at $5.33 per bushel?

341. What must I give for 2 bu. 3 qt. of flaxseed at $2.37½ per bushel?

342. What will 9 qt. of lime come to at 40 ct. per bushel?

343. I bought 2 bu. 4 qt. of corn at $1.40 per bushel; to what did it amount?

344. What must I give for 2 loads of corn at 84 ct. per bushel; the first contains 41 bu. 3 pk.; the second, 32 bu. 1 pk.?

345. What cost 1 bu. 2 qt. peaches at $1.40 per bushel and 3 bu. 2 pk. apples at 75 ct. per bushel?

346. What cost 14 bu. 2 pk. of peas at $2.20 per bushel?

347. I sold 8 bu. 3 pk. of beans at $4 per bushel, and 4 pk. 2 qt. of peas at $3.75 per bushel; how much should I receive?

348. A market woman sold 9 bu. 2 qt. of plums at $6 per bushel, 2 bu. 2 pk. peaches at $3 per bushel, 6 bu. 3 qt. of cherries at $2.75 per bushel, and 11 bu. 3 pk. of peas at $2.87½; how much should she receive for all?

LESSON XLII.

MEASURE OF TIME.

99. ***Time*** is the measure of duration.

(*a.*) TABLE.

60 seconds (sec.)	make	1 minute,	marked	min.
60 minutes	"	1 hour,	"	h.
24 hours	"	1 day,	"	da.
7 days	"	1 week,	"	wk.
4 weeks 2 days, or 30 days,	"	1 month,	"	mo.
365 days, or 52 weeks 1 day, 12 calendar months,	"	1 year,	"	yr.
100 years	"	1 century,	"	C.

(*b.*) The calendar year is divided as follows.

Season.	Months.	No. of Days.	Abbreviations.
Winter	1. January	31	Jan.
	2. February	28 or 29	Feb.
Spring	3. March	31	Mar.
	4. April	30	Apr.
	5. May	31	—
Summer	6. June	30	Jun.
	7. July	31	—
	8. August	31	Aug.
Autumn	9. September	30	Sept.
	10. October	31	Oct.
	11. November	30	Nov.
Winter	12. December	31	Dec.
		365 or 366.	

(*c.*) NOTES.—1. The exact length of the solar year is 365 days 5 h. 48 min. 49 sec.; but, for convenience, it is reckoned 11 min. 11 sec. more than this, or 365 da. 6 h.=365¼ days. This ¼ day in four years makes 1 day, which every fourth year, (called Bissextile or leap year,) is added to the shortest month, giving it 29 days. The numbers denoting leap years are exactly divisible by 4; as, 1856, 1860, 1864; except years whose number can be divided without a remainder by 100, but not by 400.

2. The pupil should carefully commit to memory the names of the months in their order, and the number of days in each. He should associate the name of each month with the number of days it contains, independently, rather than rely on the common rhyme to assist the memory. Many persons through their whole lives are obliged to repeat, "Thirty days hath September," &c., whenever they wish to recall the number of days in any particular month.

3. In most business transactions 30 days are called a month and 52 weeks a year.

4. The centuries are numbered from the commencement of the Christian era; the months from the commencement of the year; the days from the commencement of the month; and the hours from the commencement of the day (12 o'clock midnight), and from mid-day or noon. A. M. denotes time before noon, M., at noon, and P. M., after noon. Thus, 9 o'clock A. M., May 23, 1860, is the end of the ninth hour of the 23d day of the fifth month of the 60th year of the 18th century.

LESSON XLIII.

PROB. How may seconds in 12 yr. 11 mo. 22 da. 12 h. 51 min. 3 sec.?

PROOF. How many years in 403707063 seconds?

MODEL OPERATION.

(*a.*)

12 yr. 11 mo. 22 da. 12h. 51 min. 3 sec.
12 mo.
155 "
30 da.
4672 "
24 h.
112140 "
60 min.
6728451
60 sec.
403707063 "

PROOF.

60 sec.)403707063 sec.
60 min.)6728451 min.+3 sec.
24 h.)112140 h.+51 min.
30 da.)4672 da.+12 h.
12 mo.)155 mo.+22 da.
12 yr.+11 mo.

349. How many years in 40783064 seconds?
350. How many years in 384167 minutes?
351. How many years in 471834 hours?
352. How many weeks in 371834 seconds?
353. How many months in 478364 minutes?
354. How many seconds in 41 years of 365 days?
355. How many seconds in 41 years of 12 months?

LESSON XLIV.

356. How long will it take to walk 3 mi. 7 fur. 30 rd. 8 ft., if it takes 1 second to walk 6 ft.?

357. How long will it take a cannon ball to fly 5 miles, if it flies at the uniform rate of 1500 feet per second?

358. How long will it take a cannon ball to fly 4 mi. 7 fur. 36 rd., if the uniform rate is 1500 feet per second?

359. How long will it take a railroad car to run 138 miles, if it runs 3 rods per second?

360. How long will it take a railroad car to run 397 miles 14 fur. 29 rd., if it runs 2 rods per second?

361. How long will it take a fox to run 39 miles, if he runs 22 feet each second?

LESSON XLV.

CIRCULAR OR ANGULAR MEASURE.

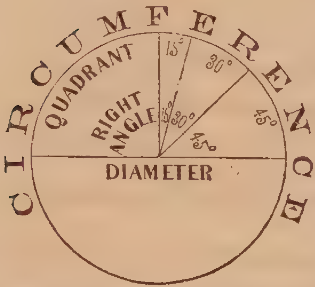
100. ***This Measure*** is used to measure angles or the

Review.—What is an analysis? (27.) What is an analytical step? (28.) What is the order of solving problems? (29.) What is the rule for addition? (29., *e.*) What is the proof? (29., *f.*) Define subtraction. (31.) What is the minuend? (32.) What is the subtrahend? (33.) What is the remainder? (34.) What is the difference in subtraction? (35.) What is the sign of subtraction? (36.) What are the axioms in subtraction? (37.) What is the rule for subtraction? (38., *d.*) How may subtraction be proved? (38., *e.*) What does the parenthesis show? Define multiplication. (41.) Multiplicand. (42.) Multiplier. (43.) Product. (44.) Which are factors? (45.) What is the sign of multiplication? (46.) What are the axioms? (47.) Recite the rule. (49., *d.*) State the method of proof. (49., *e.*)

arcs of circles. It is used in astronomy, geography, navigation, and surveying, and for calculating differences of time.

(*a.*) TABLE.

60 seconds (″) make	1 minute,	marked	′
60 minutes “	1 degree,	“	°
30 degrees “	1 sign,	“	sig.
90 degrees “	1 quadrant, 1 right angle,	“ “	quad. r. a.
4 quadrants or 12 signs “	1 circumference or circle.	“	cir.



(*b.*) NOTES.—1. The greatest distance across a circle is called its *diameter*. The distance around it is called its *circumference*. Any part of the circumference is called an *arc*.

2. If any circumference, whether large or small, be divided into 360 equal arcs, each arc is called a degree. The degree is divided into 60 minutes, and the minute into 60 seconds. The length of a degree, minute, or second, depends on the size of the circle. If the size of the circle is increased or decreased, the length of the degree, minute, or second is also increased or decreased.

3. The greatest circumference of the earth's surface is about 24,930 miles; 1° of that circumference is one 360th of 24,930 miles, which is 69¼ miles.

4. A geographical or nautical mile is equal to 1′ of the earth's greatest circumference, which is found to be a little more than 1 statute mile and 49 rods.

5. Latitude is measured north or south from the equator on any meridian, and is expressed in degrees, minutes, and seconds; thus, 43° 17′ 31″ north lat. denotes a position 43° 17′ 31″ north from the equator.

6. Longitude is measured east or west from any given meridian, Washington being generally used in the United States, Greenwich in England, and Paris in France; thus, 43° 17′ 13″ east long. from Greenwich denotes a position 43° 17′ 13″ east from the meridian which passes through Greenwich.

7. The linear extent of a degree of longitude depends upon the latitude, and diminishes as the latitude increases; thus, at latitude 10° its extent is 359640 feet; at lat. 40° it is 280106 feet; and at lat. 80° it is only 63612 feet.

8. Since the sun apparently goes around the earth once in 24 hours, in 1 hour it must pass over *one* 24th of 360°, which is 15°; hence, if in the sun's

apparent motion 15° of long. require 1 hour, or 60 minutes of time, *one* deg. will require *one* 15th of 60 minutes, which is 4 minutes.

Since 1° or 60′ long. requires 4 min., or 240 sec. of time, 1′ long. will require *one* 60th of 240 sec., which is 4 sec.; therefore, 15° long. require 1 hour of time; 1° long. requires 4 minutes of time; 1′ long. requires 4 seconds of time.

LESSON XLVI.

How many circles in 387864837 seconds?

MODEL OPERATION.

60″)387864837″
60′)6464413′+57″
30°)107740°+13′
12 sig.)3591 sig.+10°
299 cir.+3 sig.

PROOF.

299 cir. 3 sig. 10° 13′ 57″
12 sig.
3591 "
30°
107740°
60′
6464413′
60″
387864837″

362. How many signs in 878374′?

363. How many minutes in 48° 38′?

364. How many minutes of time does the sun require to pass over 3483° of longitude?

365. How many circles in 41837°?

366. How many seconds in 12 cir. 12′ 14″?

367. How many degrees in 4187834″?

368. How much time will the sun require to pass over 13° 4′ of longitude?

MODEL OPERATION.

13° 4′
4
52 min. 16 sec. Ans.

FORMULA.—Since 1′ of long. requires 4 sec. of time, 4′ will require 4 times 4 sec., which are 16 sec.

2. Since 1° of long. requires 4 min. of time, 13° will require 13 times 4 min., which are 52 min.

Therefore, to pass over 13° 4′ of long., the sun requires 52 min. 16 sec. of time; hence the following

(*c.*) **Rule.**—I. *Multiply the difference of longitude by 4 to find difference of time.*

II. *To find the time at places east, add the difference to the given time; to find the time at places west, subtract it from the given time.*

LESSON XLVII.

369. As the earth turns round on its axis from west to east once every 24 hours, the sun *appears* to move from east to west over 360° of longitude in the same time; how many degrees of longitude does it move over in 1 hour?

370. If the sun moves over 15° in 1 hour or 60 minutes, how much time will it require to move over 1 degree?

371. If the sun in passing over 1° or 60′ of long. requires 4 minutes of time, how much time will it require to pass over 1′ or geographic mile?

372. If the sun requires 4 seconds of time to pass over 1′ or 1 nautical mile, how much time will it require to pass over 59′?

373. If the sun requires 4 seconds of time to pass over 1′, how much time will be required to pass over 87° 13′?

374. When the sun is on the meridian of (i. e. perpendicular to) any place it is said to be noon at that place. Washington is 77° west of Greenwich, and it is now noon at Greenwich; how long a time will elapse before it will be noon at Washington?

375. Sacramento City is 44° 17′ west of Washington; when it is noon in Washington, what time will it be in Sacramento City?

376. Boston is 71° 4′ west of Greenwich; if my watch is 12 o'clock M. in Greenwich, what time will my watch show in Greenwich when it is noon in Boston?

377. San Francisco is 51° 17′ west of Boston; when it is noon in San Francisco, how long since it was noon at Boston?

LESSON XLVIII.

378. Iowa City is 14° 50′ west of Washington; what time is it in Washington when it is noon at Iowa City?

379. Greenwich is 91° 50′ east of Iowa City; what is the difference in time of the two places?

380. Eastport in Me. is 5° 51′ east of New Haven; if a man should start from the latter place and visit Eastport, would his watch be slower or faster than the Eastport time, and how much?

381. If the man correct his watch by the Eastport time, on his return to New Haven would it be slower or faster than the New Haven time, and how much?

382. Washington is 76° 53′ west of London; two persons, one in Washington and the other in London, agreed to look at a certain star at 9 o'clock on the evening of a certain day; when the friend in Washington was gazing at the star, what time was it in London?

383. What is the difference in time of two places 39° 41′ long. apart?

384. What is the difference in time of two places 37° 29′ long. apart?

385. What is the difference in time of two places 89° 11′ long. apart?

386. How much sooner does the sun rise in London than in Montgomery, the capital of Alabama, London being 86° 52′ east of Montgomery?

REVIEW.—What are the axioms in division? (59.) Give the rule for short division. (60., *d.*) State the method of proof. (60., *e.*) Give the rule for long division. (62., *d.*) What are the steps in long division? What kind of currency is U. S. money? (63.) What are the gold coins? (64., *a.*) Silver? (64., *b.*) Nickel? (64., *c.*) Copper? (64., *d.*) What is the use of the decimal point? (65.) What is reduction of U. S. money? Give rules. (66., *a.*) (66., *b.*) What are bills? (71.) What is the difference between debtor and creditor?

387. Columbus, O., is 11° 56′ west of Boston; how much sooner does the sun set in Boston than in Columbus?

LESSON XLIX.

101. MISCELLANEOUS TABLES.

(*a.*) PARTICULARS.

12 units or things	make	1 dozen.
12 dozen	"	1 gross.
12 gross	"	1 great gross.
20 units	"	1 score.

(*b.*) PAPER.

24 sheets	make	1 quire.
20 quires	"	1 ream.
2 reams	"	1 bundle.
5 bundles	"	1 bale.

(*c.*) BOOKS.

A sheet	folded in	2	leaves	is called	a folio.
"	"	4	"	"	a quarto or 4to.
"	"	8	"	"	an octavo or 8vo.
"	"	12	"	"	a 12mo or duodecimo.
"	"	16	"	"	a 16mo.
"	"	18	"	"	an 18 mo.
"	"	24	"	"	a 24 mo.
"	"	32	"	"	a 32 mo.

(*d.*) COPYING.

NOTE.—A folio varies in different states and countries, and usually contains from 75 to 100 words.

LESSON L.

102. ADDITION OF COMPOUND NUMBERS.

What is the sum of 36 bu. 2 pk. 6 qt. 1 pt., 25 bu. 1 pk. 2 pt., 18 bu. 3. pk. 7 qt. 1 pt., 9 bu. 2 qt. 1 pt.?

MODEL OPERATION.

36 bu.	2 pk.	6 qt.	1 pt.
25 "	1 "	0 "	2 "
18 "	3 "	7 "	1 "
9 "	0 "	2 "	1 "
90 "	0 "	1 "	1 "

(*a.*) **Anal. Steps.**—**1.** For convenience, write the denominations so that pints may stand in the column of pints, quarts in the column of quarts, pecks in the column of pecks, &c.

2. For convenience begin to add with the column of pints.

3. 1, 2, 4, 5; 5 pt. equal 2 quarts and 1 pint; write the 1 pint under the column of pints, and add the 2 quarts to the sum of the quarts.

4. 2, 9, 15, 17; 17 quarts are equal to 2 pk. and 1 quart; write the 1 quart under the column of quarts, and add the 2 pecks to the sum of the pecks.

5. 3, 4, 6, 8; 8 pecks are equal to 2 bushels and no pecks; add the 2 bushels to the sum of the bushels.

6. 9, 27, 52, 88, 90; write the 90 bushels under the column of bushels.

(*b.*) **Concl.**—Therefore, the sum of 36 bu. 2 pk. 6 qt. 1 pt., 25 bu. 1 pk. 2 pt., 18 bu. 3 pk. 7 qt. 1 pt., 9 bu. 2 qt. 1 pt., is 90 bu. 1 qt. 1 pt.

(*c.*) **Rule.**—I. *Write the numbers so that those of the same denomination shall stand in the same column.*

II. *Add the lowest denomination as simple numbers, carrying one for as many units as it takes of the denomination added to make one of the next higher.*

(*d.*) **Proof.**—*The same as in simple numbers.*

LESSON LI.

(388.)

£47	10s.	9d.	1far.
25	6	4	3
36	18	0	2
12	0	10	0
8	7	3	1

(389.)

10℔	10℥	4ʒ	1℈	12 gr.
0	9	5	2	10
14	4	0	0	16
6	0	7	1	00
0	6	3	2	15

REVIEW.—What is a simple number? (72.) What is a compound number? (73.) What is reduction? (74.) What is reduction descending? (75.) What is reduction ascending? (76.) What is English money? (77.)

(390.)

24 hhd.	21 gal.	3 qt.	1 pt.
102	42	2	0
38	9	0	1
42	50	1	0

(391.)

3 T.	12 cwt.	15 lb.	10 oz.	11 dr.
0	16	20	7	9
5	9	6	0	12
0	18	17	14	00

(392.)

27 da.	14 h.	40 min.	36 sec.
106	20	14	25
16	12	50	45
52	16	39	18

(393.)

16 lb.	11 oz.	18 pwt.	21 gr.
26	9	15	10
11	10	0	8
4	6	12	0

(394.)

2 mi.	5 fur.	25 rd.	4 yd.	1 ft.	10 in.
1	3	30	1	2	7
4	0	16	5	0	4
10	6	8	2	2	11

(395.)

12 sq. rd.	20 sq. yd.	5 sq. ft.
9	15	6
15	10	7
20	26	3

396. What is the sum of 28 sig. 12° 40′ 25″, 5 sig. 9° 27′ 38″, 16° 10′ 50″, 1 sig. 16°?

397. What is the sum of 44 A. 2 R. 24 sq. rd., 10 A. 20 sq. rd., 25 A. 1 R., 6 A. 36 sq. rd.?

398. What is the sum of 25 rd. 12 ft. 5 in., 28 rd. 9 ft. 10 in., 18 rd. 10 ft., 12 rd. 14 ft. 9 in.?

399. What is the sum of 5 cd. 6cd. ft. 9 cu. ft., 4 cd. 3 cd. ft. 12 cu. ft., 10 cd. 14 cu. ft., 2 cd. 7 cd. ft.?

LESSON LII.

103. SUBTRACTION OF COMPOUND NUMBERS.

Prob. From 24 lb. 6 oz. 5 pwt. 12 gr. take 14 lb. 9 oz. 10 pwt. 7 gr.

MODEL OPERATION.

24 lb.	6 oz.	5 pwt.	12 gr.
14 "	9 "	10 "	7 "
9 "	8 "	15 "	5 "

(*a.*) **Anal. Steps.—1.** For convenience, write the subtrahend under the minuend, so that grains may stand under grains, pennyweights under pennyweights, ounces under ounces, &c.

2. For convenience, begin with the lowest denomination to subtract.

3. 7 gr. from 12 gr. leave 5 gr.; write the 5 gr. under the column of grains.

4. As 10 pwt. cannot be subtracted from 5 pwt., take 1 oz. from 6 oz.; 1 oz. is equal to 20 pwt., which added to 5 pwt. make 25 pwt.; then 10 pwt. taken from 25 pwt. leave 15 pwt., which is written under the column of pwt.

5. As 9 oz. cannot be taken from 5 oz., take 1 lb. from 24 lb.; 1 lb. is equal to 12 oz., which added to 5 oz. make 17 oz.; then 9 oz. taken from 17 oz. leave 8 oz., which is written under the column of ounces.

6. 14 lb. taken from 23 lb. leave 9 lb., which is written under the column of pounds.

Concl.—Therefore 14 lb. 9 oz. 10 pwt. 7 gr. taken from 24 lb. 6 oz. 5 pwt. 12 gr. leave 9 lb. 8 oz. 15 pwt. 5 gr.

(*b.*) **Rule.**—I. *Write the subtrahend under the minuend so that numbers of the same denomination shall stand under each other.*

II. *Begin with the lowest and subtract each denomination of the subtrahend from the corresponding denomination in the minuend, as in simple numbers.*

III. *If any denomination in the minuend is less than the same denomination in the subtrahend, add to it as many units as it takes to make one of the next higher denomination, and subtract; then regard the next higher denomination of the minuend as one less. Proceed in the same manner with each denomination.*

(*c.*) **Proof.**—*Prove as in simple subtraction.*

			(400.)					(401.)	
From	18 cwt.	1 qr.	14 lb.	9 oz.	8 dr.	7 hhd.	28 gal.	2 qt.	1 pt.
Take	5	2	20	6	10	3	42	3	0
Rem.									

Review.—How many dimensions has space? (84.) How many dimensions has a line? (85.) How many has a surface? (86.) How many has a solid? What is long measure? (88.) Repeat the table. (88., *a.*) What is the use of cloth measure? (89.) Recite the table. (89., *a.*) What instrument is used by surveyors? (90.) Recite the table. (90., *a.*)

(402.)
12℔ 7℥ 3ʒ 1℈ 11 gr.
8 5 4 2 15

(403.)
104 bu. 2 pk. 6 qt.
56 3 4 1 pt.

(404.)
40 mi. 5 fur. 30 rd. 3 yd. 2 ft. 10 in.
14 6 15 4 1 10

(405.)
400 A. 2 R. 25 sq. rd.
325 1 30

(406.)
10 wk. 4 da. 16 hr. 40 m. 22 sec.
4 5 12 45 50

(407.)
6 sig. 25° 45′ 38″
4 28° 40′ 50″

(408.)
14 T. 5 cwt. 2 qr. 18 lb. 9 oz.
10 14 3 12 14

(409.)
120 cd. 4 cd. ft. 6 cu. ft. 520 in.
94 7 12 1500

LESSON LIII.

410. From £78 11s. 5d. 2 far., take £41 13s. 3d. 3 far.

411. From 15 lb. 3 oz. 12 pwt. 14 gr., take 9 lb. 11 oz. 17 pwt. 21 gr.

412. From 15℔ 7℥ 1ʒ 2℈ 15 gr., take 11℔ 9℥ 7ʒ 1℈ 19 gr.

413. From 16 cwt. 1 qr. 5 lb. 14 dr., take 13 cwt. 3 qr. 17 lb. 1oz. 15 dr.

414. From 3 mi. 7 fur. 31 rd. 1 yd. 1 ft. 3 in., take 1 mi. 1 fur. 39 rd. 1 yd. 2 ft. 7 in.

415. From 19 mi. 1 fur. 1 rd. 3 ft. 7 in., take 8 mi. 7 fur. 1 rd. 16 ft. 9 in.

416. From 11 sig. 7° 13′ 15″, take 9 sig. 29° 17′ 36″.

REVIEW.—What is a square? (91.) Recite the table of square measure. (91., *a.*) What is a square yard? (92.) Recite the rule for finding the area of a square. (92.,*a.*) What is the use of square measure? (92., *b.*) How do artificers estimate their work? (91., *c.*) For what is surveyor's square measure used? (93.) Recite the table. (93., *a.*) Recite the miscellaneous table. (93., *b.*)

What is the difference of time between Oct. 16th, 1852, and August 9th, 1854?

MODEL OPERATION.

1854 yr.	8 mo.	9 da.
1852 "	10 "	16 "
1 "	9 "	23 "

417. What is the time from March 21st, 1853, to Jan. 6th, 1857?

418. A note was given Nov. 15th, 1852, and paid April 25, 1857; how long was it on interest?

419. John Quincy Adams was born at Braintree, Mass., July 11, 1767, and died at Washington, D. C., Feb. 23, 1848; to what age did he live?

420. Andrew Jackson was born at Waxar, S. C., March 15, 1767, and died at Nashville, Tenn., June 8, 1845; at what age did he die?

421. What is the difference of time between Jan. 1, 1837, and Nov. 3, 1852.

422. What is the sum of the differences of time between the following dates:

Jan. 13, 1843, and Sept. 8, 1854;
Aug. 11, 1833, and Nov. 23, 1831;
Feb. 27, 1762, and July 4, 1861;
Apr. 19, 1861, and May 21, 1852?

LESSON LIV.

423. What is the difference in the longitude of two places, one being 48° 37′ 44″ east, and the other 37° 17′ 39″ west?

REVIEW.—What is a cube? (94.) Recite the table. (94., *a.*) What is a cubic yard? (95.) Recite the rule for finding the contents of a cube. (95., *b.*) What is the use of cubic measure? (95., *c.*) How is light freight estimated? (95., *c.*) How heavy freight? What is a cord of wood? How many cu. ft. in a perch of masonry? What is the size of a brick? How many in a cubic foot? What allowance do carpenters, joiners, and masons make for windows and doors? Why is a ton of round timber said to contain only 40 feet? Do masons make any allowance for the corners of the walls of a cellar or house?

424. What is the difference in latitude of two places, one being 37° 41′ 25″ south, and the other 16° 32′ 41″ north?

425. What is the difference in the latitude of two places, one being 41° 37′ 22″ south, and the other 37° 12′ 43″ north?

426. What is the difference in the time of two places, one being situated in longitude 44° 37′ west, and the other 97° 25′ west?

427. What is the difference in the time of two places, one being situated in long. 39° 41′ east, and the other 71° 41′ east?

428. What is the difference in the time of two places, one being situated in long. 47° 31′ east, and the other 33° 27′ west?

429. What is the difference in time at two places, one being situated in long. 44° 23′ west, and and the other in long. 37° 41′ east?

LESSON LV.

104. MULTIPLICATION OF COMPOUND NUMBERS.

If one acre of land cost £14 5s. 8d. 2 far. what will 9 acres cost?

MODEL OPERATION.

£14 5s. 8d. 2 far.
9
£128 11s. 4d. 2 far.

FORMULA.—If 1 acre cost £14 5s. 8d. 2 far., 9 acres will cost 9 times £14 5s. 8d. 2 far.

(*a.*) **Anal. Steps.—1.** 9 times 2 far. are 18 far. equal to 4 pence and 2 farthings; write the farthings in the place of farthings, and add the 4 pence to the product of pence.

2. 9 times 8d. are 72d., which with 4d. added make 76d.; 76d. equals 6s. 4d.; write the 4d. in the place of pence, and add the 6s. to the product of the shillings.

REVIEW.—What are measures of capacity? (96.) What is the use of liquid measure? (97.) Recite the table. (97., *a.*) What is the use of dry measure? (98.) Recite the table. (98., *a.*) Recite the comparative table of measures of capacity. (97., *b.*) What is time? (99.) Recite the table. (99., *a.*) How is the calender year divided? (99., *b.*) What is the exact length of the year? (99., *c.*)

3. 9 times 5s. are 45s., which with 6s. added make 51s.; 51s. equal £2 and 11s.; write the 11s. in the place of shillings, and add the £2 to the product of pounds.

4. 9 times £14 are £126, and £2 more make £128, to be written in the place of pounds.

Concl.—Therefore if 1 acre of land cost £14 5s. 8d. 2 far., 9 acres will cost £128 11s. 4d. 2 far.

(*b.*) **Rule.**—I. *Write the multiplier under the lowest denomination of the multiplicand.*

II. *Multiply as in simple numbers, and carry as in addition of compound numbers.*

(*c.*) **Proof.**—*Prove as in simple numbers.*

(430.)
6 hhd. 20 gal. 2 qt. 1 pt.
3

(431.)
9 bu. 2 pk. 6 qt. 1 pt.
4

(432.)
12 lb. 8 oz. 14 pwt. 16 gr.
5

(433.)
10 T. 15 cwt. 2 lb. 8 oz.
6

LESSON LVI.

434. Multiply 14 A. 2 R. 26 sq. rd. by 8.
435. Multiply 6 yd. 2 ft. 9 in. by 12.
436. Multiply 7℔ 8℥ 5ʒ 1℈ 10 gr. by 7.
437. Multiply 24 bu. 1 pk. 6 qt. by 10.
438. Multiply 9 cu. yd. 15 cu. ft. 520 cu. in. by 5.
439. Multiply £84 12s. 6d. 2 far. by 9.

440. If a pipe discharges 4 hhd. 20 gal. 3 qt. of water in 1 hour, how much, at the same rate, will it discharge in 5 hours?

441. If a load of coal, by the long or gross ton, weighs 1 T. 3 cwt. 3 qr. 18 lb., what will be the weight of 8 loads?

REVIEW.—What is angular measure? (100.) Recite the table. (100., *a.*) What is the distance around a circle called? (100., *b.*) What is the distance through it called? (100., *b.*) Recite the miscellaneous table. (101.) Give the rule for addition of compound numbers? (102., *c.*) Give the rule for subtraction of compound numbers. (103., *c.*)

442. If 1 acre of land produces 25 bu. 2pk. 7 qt. of corn, how much will 19 acres produce?

443. If a man travels 30 mi. 4 fur. 20 rd. in 1 day, how far will he travel in 12 days at the same rate?

LESSON LVII.

105. DIVISION OF COMPOUND NUMBERS.

If 5 acres of land produce 102 bu. 3 pk. 1 qt. of wheat, how much will 1 acre produce?

MODEL OPERATION.

$$\begin{array}{r|l} 5\text{ A.)} & 102\text{ bu. } 3\text{ pk. } 1\text{ qt. } 0\text{ pt.} \\ \hline & \;\;20\text{ " }\;\;2\text{ " }\;\;1\text{ " }\;\;1\tfrac{3}{5}\text{"} \end{array}$$

Formula.—If 5 acres of land produce 102 bu. 3 pk. 1 qt. of wheat, 1 acre will produce *one-fifth* of 102 bu. 3 pk. 1 qt.

(*a.*) **Anal. steps.—1.** $\frac{1}{5}$ of 102 bushels is 20 bushels, with a remainder of 2 bushels equal to 8 pecks; write the 20 bushels in the place of bushels and add the 8 pecks to the pecks.

2. 8 pecks and 3 pecks are 11 pecks, and $\frac{1}{5}$ of 11 pecks is 2 pecks, with a remainder of 1 peck equal to 8 qt.; write the 2 pecks in the place of pecks, and add the 8 qt. to the quarts.

3. 8 qt. and 1 qt. are 9 qt., and $\frac{1}{5}$ of 9 qt. is 1 qt. with a remainder of 4 qt. equal to 8 pints; write the 1 qt. in the place of quarts, and add the pints to the pints.

4. $\frac{1}{5}$ of 8 pints is $1\frac{3}{5}$ pt., to be written in the place of pints.

Concl.—Therefore one-fifth of 102 bu. 3 pk. 1 qt. is 20 bu. 2 pk. 1 qt. $1\frac{3}{5}$ pt.

(*b.*) **Rule.**—I. *Divide the highest denomination as in simple numbers, and each succeeding denomination in the same manner, if there is no remainder.*

II. *If there is a remainder after dividing any denomination, change it to the next lower denomination; add it to the number of that denomination, if any, and divide as before, until each denomination is divided.*

(*c.*) **Proof.**—*Prove as in simple numbers.*

LESSON LVIII.

444. Divide 33 wk. 5 da. 23 h. 45 m. 10 sec. by 7.
445. Divide 88 bu. 3 pk. 4 qt. by 6.
446. Divide 28℔ 9℥ 4ʒ 2℈ 5 gr. by 9.
447. Divide 376 gal. 3 qt. 1 pt. by 11.
448. Divide 9 hhd. 28 gal. 2 qt. by 12.
449. Divide 129 A. 2 R. 25 sq. rd. by 9.
450. Divide 217 mi. 5 fur. 19 rd. 12 ft. 6 in. by 7.
451. Divide 413 mi. 7 fur. 3 yd. 9 ft. 2 in. by 8.
452. Divide 185 lb. 1 oz. 19 pwt. 13 gr. by 11.
453. Divide £185 17s. 6d. by 8.
454. Divide 16 lb. 13 oz. 10 dr. by 6.
455. Divide 358 A. 1 R. 17 sq. rd. 6 sq. yd. 2 sq. ft. by 97
456. Divide 192 bu. 3 pk. 1 qt. 1 pt. by 139.
457. Divide 9 hhd. 28 gal. 2 qt. by 79.
458. Divide 328 yd. 1 ft. 3 in. by 330.

LESSON LIX.

MISCELLANEOUS PROBLEMS FOR REVIEW.

459. Add
4678 mills,
4130 cents,
6783 mills,
4367 mills,
5876 dimes.

460. Add
3786 mills,
4130 cents,
6473 dimes,
1000 mills,
1000 cents.

461. Add
6783 dollars,
6783 dimes,
6783 cents,
6783 mills,
$3.67.

462. Add.
6780 dimes,
6000 dollars,
6006 dimes,
6006 cents,
6006 mills.

463. From 36 dollars,
take 36 cents.

464. From 3 dollars 20 cents,
take 48 cents.

465. From 30 dollars 30 cents,
take 30 cents and 30 mills.

466. From 40 dollars 4 cents,
take 4 dollars 40 cents.

467. From 62 dollars 26 cents
take 26 dollars 62 cents.

468. From $3.87½,
take 347 mills.

469. Multiply 386 cents by 3.
470. Multiply 468 mills by 8.
471. Multiply 374 mills by 2.
472. Multiply 4 dollars 9 cts. by 18.
473. Multiply 1 dollar 1 cent 1 mill by 387.
474. Divide $3 by 7.
475. Divide $10 by 11.

REVIEW.—What is a unit? (1.) What is a number? (2.) What is the difference between a concrete and an abstract number? (3.) (4.)

476. Divide $3.08 by 18.
477. Divide $6.30 by 370.
478. Divide 3 dol. 4 cts. by 600.
479. Divide 5 dimes 8 cts. by 500.
480. Divide $113 by 786.
481. $3 are how many times 50 cts.?
482. $9 are how many times 40 cts.?
483. $3.80 are how many times 50 mills?
484. $8.82 are how many times $.882?
485. $401 are how many times $.401?
486. $20.2 are how many times $.202?
487. $4.04 × 3 ÷ 4 = what?
488. $30 ÷ 80 × 37 = what?
489. $27½ ÷ 37 × 25 = what?
490. $3.07 ÷ 41 × 36 = what?
491. $34 ÷ 4 × 3 − $3.84 = what?
492. $4.03 ÷ 3 × 7 + $3.67 = what?
493. $13 − $4.673 ÷ 80 × 2 = what?
494. 38 cts. ÷ 7 + $3.86 × 2 = what?
495. $29 ÷ 7 + $83 × 9 = what?
496. In 49183d. how many far.?
497. In 38786s. how many d.?
498. Change 39786 sov. to d.
499. Change 34786 sov. to far.
500. Change 91783 sov. to s.
501. Change 37186 half-sov. to s.
502. Change 1746 gr. to oz.
503. Change 5 lb. 7 pwt. to gr.
504. Change 38409 gr. to lb.
505. Change 7 ℥ 1 ʒ to gr.
506. Change 72 lb. 5 oz. to pwt.
507. Change 691 ℈ to lb.
508. Change 3127 lb. to pwt.
509. Change 115 ℔ 3 ʒ to scr.
510. Change 69125 gr. to lb.
511. Change 1 T. 3 qr. 7 lb. to oz.
512. Change 3 qr. 24 lb. 15 dr. to dr.
513. Change 27 T. 13 lb. to dr.

514. What is the cost of
8 gal. vinegar @ 43 cts. per gal.,
5 gal. molasses @ $.87 per gal.,
7 lb. sugar @ 22½ cts. per lb.,
9 yd. calico @ 37½ cts. per yd.,
15 yd. lace @ 12½ cts. per yd.?

515. What is the cost of
8 lb. 2 oz. rice @ 18½ cts. per lb.,
9 lb. 4 oz. sugar @ 19 cts. per lb.,
13 oz. indigo @ $5 per lb.,
7 oz. alum @ $1.75 per lb.,
11 oz. sulphur @ 87½ cts. per lb.?

516. What is the cost of
5 oz. gum camphor @ $5 per lb.,
1 lb. 3 oz. gum Arabic @ $1.60 per lb.,
5 lb. 2 oz. indigo @ $6 per lb.,
9 oz. white lead @ 60 cts. per lb.,
3 lb. 4 oz. antimony @ 87 cts. per lb.?

517. What is the cost of
3 f. ʒ aqua ammoniæ @ 30 cts. per f. ℥,
8 f. ℥ aqua rosæ @ 8 cts. per O.,
4 O. 3 f. ℥ tinct. cloves @ $1.80 per O.,
13 f. ℥ tinct. camphor @ 90 cts. per O.,
4 f. ℥ syrup rhubarb @ $1.80 per O.?

518. What is the cost of
8 lb. 3 oz. butter @ 60 cts. per lb.,
5 lb. 6 oz. beef @ 42 cts. per lb.,
8 lb. 3 oz. lamb @ 19 cts. per lb.,
3 lb. 5 oz. sugar @ 22 cts. per lb.?

Review.—Of what does arithmetic treat? (5.) What is notation? (6.) What is used in the Roman notation? (7.) Name the letters. (7., c.)

519. What is the cost of
3 qr. 26 lb. flour @ $3.23 per cwt.,
4 cwt. 3 qr. of coal @ $1.10 per cwt.,
3 cwt. 4 qr. 19 lb. coal @ 40 cts. per cwt.,
4 cwt. 76 lb. beef @ $46 per bbl.,
4 cwt. 39 lb. fish @ $.37½ per lb. ?

QUESTIONS.—What is a simple number? What is a compound number? What is the difference betweeen reduction ascending and reduction descending? Repeat the table of English money. Which is worth the more a £ or a s.? Which is the least denomination? How many gold coins are there in use? How many silver? How many bronze? What is the use of troy weight? Of avoirdupois weight? By which should butter be weighed? Sugar? Silver? Spice? Gold? Salt? A silver tea-set? Raisins? A silver cake basket? Dates? Silver spoons? A box of figs? What weight do physicians use for prescriptions? What do druggists use in preparing prescriptions? What in buying and selling drugs?

LESSON LX.

Problem.—What will 3 bu. 3 pk. 7 qt. 1 pt. clover seed cost at $5.37 per bushel?

MODEL OPERATION.

$$\begin{array}{rr}
4)\underline{\$5.370} \times 3 = & \$16.110 \\
8)\underline{1.343} \times 3 = & 4.029 \\
2)\underline{.168} \times 7 = & 1.176 \\
.084 \times 1 = & \underline{.084} \\
 & \$21.399
\end{array}$$

Anal. steps.—1. $5.37 *equals* 5370 *mills.**

2. *Find the cost of* 3 *bushels.*

FORMULA.—If 1 bushel costs 5370 mills, 3 bushels will cost 3 times 5370 mills, which is 16110 mills equal to $16.110.

3. *Find the cost of* 3 *pecks.*

FORM.—If 4 pecks or 1 bushel costs 5370 mills, 1 peck will cost 1 *fourth* of 5370 mills which are 1343 mills†, and 3 pecks will cost 3 times 1343 mills which are 4029 mills equal to $4.029.

4. *Find the cost of* 1 *quart and then of* 7 *quarts.*

5. *Find the cost of* 1 *pint.*

6. *Find the cost of* 3 *bu.* 3 *pk.* 7 *qt.* 1 *pt.*

520. How much will 6 yd. 3 qr. of calico cost at 37½ ct. a yard?

---

* Where great accuracy is required, change the cost to *tenths* of mills.

† When the remainder is less than a half-mill, reject it; if it equals or exceeds a half-mill, add *one* to the mills.

521. How much will 13 gal. 2 qt. 1 pt. 2 gi. of molasses cost at $1.15 per gallon?

522. What will 2 cwt. 3 qr. 18 lb. of hay be worth at $2.12 per hundred weight?

523. What will 3 T. 7 cwt. 2 qr. of hay cost at $20 per ton?

524. If $9.35 will buy a barrel of flour, what will 2 qr. 13 lb. cost at the same rate?

525. What should I pay for 3 A. 12 sq. rd. 8 sq. yd. of land at $35 per acre?

526. Joseph Doe cuts 13 cd. and 12 cd. ft. of wood at $2.25 per cord; how much should he receive?

527. What will 9 bu. 3 pk. 2 qt. of flaxseed cost at $4.37 per bushel?

528. If 360 tons of iron cost £6409, what is the cost of one ton?

529. A farmer raised 200 bu. 2 pk. of barley, 175 pk. of corn, 32 bu. 1 pk. of oats, and 22 bu. 2 pk. of rye; what is the whole quantity of grain raised?

530. Mr. Mills bought 12 cwt. 3 qr. 12 lb. of sugar at $8.36 per cwt.; what was the amount of his bill?

531. A grocer having purchased 98 cwt. 2 qr. of sugar sold 10 cwt. 1 qr. 20 lb.; how much remained unsold?

532. A merchant buys 3 hogsheads of molasses at 30 ct. a gallon, and sells it for 45 ct.; how much does he make?

533. If a man buys 10 bu. of chestnuts at $3 a bushel, and sells them at 10 ct. a pint, how much is his whole gain?

534. A person wishes to ship 720 bu. of potatoes in barrels holding 2 bu. 3 pk.; how many barrels must he use?

535. How many rods of fence will it take to enclose a farm a mile square?

536. A man owns 37 A. 2 R. 48 sq. rd. of land which he values at $250 per acre; he sells one-third, and divides the remainder into 36 building lots; what is the value of each building lot?

537. How many times will a wheel 13 ft. 4 in. in circumference turn round in going 12 miles?

538. If 8 horses eat 12 bu. 2 pk. of oats in 3 days, how many bushels will 20 horses eat in the same time?

539. How much sugar at 9 ct. a pound must be given for 2 cwt. 2 qr. 10 lb. of pork at $8 per cwt.?

540. A company of 144 persons purchased a tract of land containing 11067 A. 1 R. 8 sq. rd.; John Smith, who was one of the company and owned an equal share with the others, sold his part of the land at $35 per acre; what sum did he receive?

541. Joseph Randolph was 50 years old, April 6th, 1836, at 13 minutes past 11 o'clock, A. M.; how long had he lived Sept. 25, 1861, at 17 minutes past 3 o'clock, P. M.

542. A merchant purchased in London 17 bales of cloth at £17 18s. 10d. per bale; he disposed of the cloth at Havana for sugar at £1 17s. 6d. per cwt.; now if he purchased 144 cwt. of sugar, what balance did he receive?

543. A and B commenced traveling the same way, at the same time, around an island 30 miles in circumference; A travels 15 miles a day, and B travels 11 mi. and 2 fur. a day; how far will they be apart at the end of 8 days?

544. A and B commenced traveling from the same place, in the same direction, around an island 60 miles in circumference; A travels 19 mi. 3 fur. 7 rd., and B travels 24 mi. 28 rd. each day; how far will they be apart at the end of 12 days?

To the Teacher.—The subjects of U. S. money and compound numbers and their applications complete a two-years' course of study. Pupils should not be suffered to go on unless they

1. *Understand the principles studied,*
2. *Remember the facts that have been learned,*
3. *Are quick to find the steps necessary to the solution of each problem,*
4. *Are quick and accurate in their work.*

This alone will enable the pupil to understand and apply common and decimal fractions. The hasty and random judgment, the consequent discouragement, loss of manly spirit and pluck, and the final sinking into hopelessness and sullen indifference which are so common in all schools, may be prevented without fail, if this caution is properly observed; but not otherwise.

Pupils can be taught to do work mechanically, to *recite* explanations, and so may get on rapidly in the book; but this process dwarfs rather than develops the mind. It would be far better to keep the pupil back too long than to push him on too fast:—the former only makes his *growth slow;* the latter *deforms* him.

TEST QUESTIONS FOR EXAMINATION.

72. What is a simple number? How many denominations in a simple number?

73. What is a compound number? Is a compound number ever abstract? Of how many denominations may it consist? Name a simple number. Name a compound number. Name a simple denominate number. A simple abstract number. An abstract compound number.

74. What is reduction of compound numbers?

75. What is reduction descending? Why is it so called?

76. What is reduction ascending? Why called reduction ascending?

77. What is English money? What are the denominations? Name the denominations in order, beginning with the highest. Beginning at the lowest. Repeat the table. Repeat it backwards. What is meant by currency?

78. What are coins? Of what are they made? What is their use? Which of the English coins are made of gold? What is the value of each? What is the difference between a sovereign and a pound? Which are the silver coins? Give the value of each. Which are the bronze coins? Give the value of each. What is the difference between penny and cent? Which has the greater value? Ans.—*The penny.* What is a farthing? How is it usually written? What is the Canadian currency? How many steps are necessary to change pounds to farthings? What kind of reduction is it called? Will there be a greater or a less number of farthings than of pounds? A shilling is how many times as great as a penny? A penny is how many times as great as a farthing? A shilling is what part of a pound? A penny is what part of a shilling? A farthing is what part of a penny? A shilling is what part of a guinea? Is multiplication or division used in reduction descending? Which is the higher denomination, shillings or pence? Pounds or farthings? Guineas or pounds? Pence or pounds? Pence or shillings? In writing £9 13s. 11d. 3 far., what sign is understood between the denominations? What does £9 3s. mean? Ans.—£9+3s. Why cannot the 9 be added to the 3? What is the rule for reduction descending? Why are the abbreviations written with the denominations? Ans.—*To assist the memory.* How do we know which numbers are to be used as abstract? Ans.—*By the analysis.* How many steps are there in changing farthings to pounds? What kind of reduction is it? Why so called? What is reduction ascending? How can reduction descending be proved? How can reduction ascending be proved? Give the rule for reduction ascending.

79. What is weight? Which is heavier, a pound of lead or a pound of chalk? Which contains the most matter, a pound of chalk or a pound of lead? Which will weigh the most, a piece of lead or a piece of chalk of the same size? Which contains the most matter? What instrument is used in weighing?

80. What is the use of troy weight? What are the denominations of troy weight? Repeat the table. Say it backwards. What is the highest denomination of troy weight? What the lowest? How many steps are there in changing pounds to grains? A pennyweight is how many times as much as a grain? A pound is how many times as much as an ounce? An ounce is how many times as much as a pennyweight? What is the use of the carat? What is the weight called? For what else is the term carat used? What is meant by gold being 18 carats fine? By its being 14 carats fine? By its being 22 carats fine? What is the use of alloying gold, &c.? What alloy is generally used for gold? How many carats fine is pure gold? Repeat the steps in changing pounds to grains. In changing grains to pounds. Ounces to pennyweights. Pounds to pennyweights. Grains to ounces. Ounces to grains. Ounces to pennyweights. Ounces to pounds.

81. What is the use of avoirdupois weight? What articles are bought and sold by avoirdupois weight? Recite the table. Say it backwards. What are the denominations? Mention the analytical steps in reduction descending. In reduction ascending, which is the highest denomination? Which is the lowest? How many denominations in avoirdupois weight? What weight should be used in buying hay? Pork? Silver? Butter? Gold? Cheese? Iron? Jewelry? Fish? Beef? One pound is what part of a hundred weight? One ounce is what part of a pound? A pound is how many times as much as an ounce? An ounce is how many times as much as a dram? How many pounds in the English quarter? Where is the gross ton used? In what is butter usually packed? How many pounds make a barrel of flour? Of beef? Of pork? Of fish? Of salt? How many pounds make a bushel of oats? Of barley? Of corn? Of rye? Of wheat?

82. What is the use of apothecaries' weight? What weight do wholesale druggists use? What weight do apothecaries use in preparing prescriptions? Recite the table. Give the steps in reduction ascending. How many grains in a pound troy? How many in a pound apothecaries'? Which is heavier, a pound avoirdupois or a pound troy? How can pounds troy be changed to pounds avoirdupois? How can pounds avoirdupois be changed to pounds troy? What rule is used in reduction ascending? Reduction descending?

83. What is the use of apothecaries' fluid measure? Recite the table.

84. What dimensions has space? In how many ways can space extend? How much space is there? What things does space contain? What does a book occupy? How do we know how much space a book occupies?

85. What dimensions has a line? Can a line be seen? May it be represented on the blackboard? [The teacher, to give a clear conception of a line, may explain that a straight line is only the distance between two points.]

86. How many dimensions has a surface? Has surface any thickness?* How far can a line extend? How far can a surface extend?

87. What dimensions has a solid? Can any number of lines make a surface? Can any number of surfaces make a solid? How do we find out how much space a solid occupies?

88. By what measure do we measure lines? For what is it used? Is it ever used to measure curved or crooked lines? What are the denominations? Which denomination has the greatest length? Repeat the table. Give the steps in reduction ascending. In reduction descending. How many steps are required to change inches to feet? Inches to yards? Yards to half-yards? Half-yards to rods? Yards to rods? Rods to miles? Statute miles to degrees? Furlongs to yards? Rods to inches? Miles to quarter-miles? What is a knot? What is meant by "ten knots an hour"? How many feet in a statute mile? How many feet in a geographical mile? How many feet in a fathom? How many miles in a league?

89. What is the use of cloth measure? Into how many parts is a yard divided? What is one of the parts called? Into how many parts is the quarter divided? Is the eighth? How many sixteenths make an eighth? How many eighths a quarter? How many quarters a yard? Repeat the table. How is cloth sold? Sugar? Wool? Silver? Iron? Lead? Lace? Tape?

---

* The teacher should explain that if two flat bodies be laid one upon the other, that which *separates them* may be called a surface.

90. What instrument is used in Surveyor's Long Measure? How long is it? How long is a link? How many chains make a mile? Recite the table.

91. What is a square? How long is one side of a square foot? Repeat the table.

92. What is a square yard? How many feet long is one side of a square yard? Into what can a square yard be separated? ANS.—*Into sq. feet.* How many square feet in a square yard? What is the rule for finding the area of surface? What is the use of square measure? How are glazing and stone-cutting estimated? How are painting, plastering, paper-hanging, etc., estimated? How are flooring, roofing, etc., estimated? How bricklaying? How are square yards changed to square rods? How are square rods changed to square yards?

93. What is the use of surveyor's square measure? Recite the table.

94. What is a cube? What is the length of one edge of a cubic foot? Recite the table.

95. What is a cubic yard? What is the length of one edge of a cubic yard? Into what can every cubic yard be divided? Into what can every tier be separated? Into what can every row be separated? What is the rule for finding the solid contents of a cube? What is the use of cubic measure? What are the contents of a load of earth? How do railroad companies estimate freight? How large is a cord of wood? How many cubic feet in a perch of masonry? What is the usual size of a brick? How many bricks in a cubic foot? Do masons make any allowance for the corners of walls in buildings? Repeat the analytical steps of cubic measure in reduction ascending? In reduction descending? What are the steps for changing cubic feet to perches? Cubic yards to cords? Why is there said to be but 40 cubic feet in a ton of round timber?

96. What are measures of capacity? What is meant by capacity?

97. What is the use of liquid measure? Name the denominations. Repeat the table. What are hogsheads, pipes, butts, and tuns? Is ale or beer measure much used?

98. What is the use of dry measure? What things are measured by dry measure? What by liquid measure? How much does the English quarter contain? How many cubic inches in a gallon by wine measure? In a gallon by beer measure? In a gallon or 4 quarts dry measure? In 1 bushel dry measure? How can dry measure be reduced to liquid measure? How can gallons liquid measure be reduced to bushels? Which is more, a gallon liquid measure or a gallon dry measure? A gallon beer measure or a gallon wine measure?

99. What is time? By what instrument is time measured? Is a day a natural or an artificial division of time? Is a month a natural or an artificial division? Is the year a natural or an artificial division? What determines the year? How long is the year? Are minutes, seconds, and hours, natural or artificial divisions? What are the names of the months? How many days in each? From what date do we reckon time? How many years from creation to the birth of Christ? How long is it from that time to the present? What is meant by 3 o'clock, P. M., July 3, 1866? What is leap year? How can we tell a leap year? Why do we have leap years? What is the exact length of a year? Recite the table of time.

100. What is the use of circular and angular measure? Repeat the table. What is the distance around the earth called? What is the distance around a circle called? What is the distance across a circle called? Into how many parts is a circle divided? What is each part called? How many parts in a degree? What is each part called? What is the length of a degree on the circumference of the earth? Is the length of a degree of a large circle the same as that of a small circle? What part of a degree is one minute? What are minutes of the earth's surface called? Are they longer or shorter than the English mile? What is latitude? What is longitude? How long does it take the sun to pass around the earth in its apparent motion? [The teacher should explain this.] How much of the earth's surface does the sun appear to pass over in one hour of time? How long does it take the sun to pass over one minute of longitude? One second of longitude? Where does the sun seem to rise? On what is the sun said to be when it is just over head? What time of day is it then? Is the time later or earlier at all places east of this? How is it about the time at all places west of this? When it is noon at this place, is it before noon or after noon at all places east? At all places west? If it is 2 o'clock here, is it before or after 2 o'clock at all places east? At all places west? When is it 9 o'clock in New York, is it earlier or later in London? Is it earlier or later in San Francisco? How many degrees in a right angle? In a quadrant? Recite the table.

101. Repeat the table of particulars. What are bought by the gross? What by the dozen? How many sheets make a quire? Say the table of paper? What is meant by a folio, quarto, etc.

102. What is addition of compound numbers? How are the numbers written? Analyze an example at the blackboard. Give the rule.

103. What is subtraction of compound numbers? Analyze an example at the blackboard? Repeat the rule. When a number in the subtrahend is greater than the corresponding number in the minuend, what is to be done? How is subtraction proved?

104. Analyze at the blackboard an example of multiplication of compound numbers. Repeat the rule.

105. Analyze at the blackboard an example of division of compound numbers. Repeat the rule.

SECTION VIII.

FRACTIONS AND INTEREST.*

LESSON I.

COMMON MULTIPLE.

106. ***A Multiple*** of a number is one that can be divided by it without a remainder.

107. ***A Common Multiple*** of two or more numbers is one that can be divided by each of them without a remainder.

108. ***The Least Common Multiple*** of two or more numbers is the *least* number that can be divided by each of them without a remainder.

What is the least common multiple of 6, 9, and 12 ?

MODEL OPERATION.

(*a*.)

Prime factors.

$12=2\times2\times3=2^2\times\cancel{3}$

$9=3\times3=3^2$

$6=\cancel{2}\times\cancel{3}$

$3^2\times2^2=36$ l. c. m.

(*b*.)

$3\underline{)12,\ 9,\ \cancel{6}}$†

$4,\ 3$

Factors.

$3\times3\times4=36$ l. c. m.

FORMULA.—1. The least number divisible by 12 must contain only the highest powers of all the different factors of 12, which are 2^2, and 3.

2. The least number divisible by 12 and 9 must contain only the highest powers of all the different factors of 12 and 9, which are 2^2 and 3^2. We reject the factor 3 in the number 12, because a higher power of 3 is found in the number 9.

3. Since the least number divisible by any number is also divisible

---

* NOTE FOR THE TEACHER.—The following pages on the subject of Fractions and Interest, were appended to this work not as a part of the course, but for the accommodation of those pupils desirous of obtaining some knowledge of these subjects, who may leave school before entering the Second Book. Those pupils, however, who desire to obtain a thorough knowledge of Fractions may omit these pages and commence the subject in the Second Book.

† NOTE.—Reject the 6, for it is a factor of 12. Factors can be rejected before a formal division as well as by it.

by any factor of that number, therefore reject the factors of 6, for 6 is a factor of 12.

Therefore $2^2 \times 3^2 = 36$ is the least common multiple of 6, 9, and 12.

This abbreviated method of operation is generally preferred, and as the analysis is about the same as that of the other method, it will not be repeated.

(*a.*) **Rule.**—I. *Resolve the numbers into their prime factors, and the product of the highest powers of all the different factors will be the least common multiple.*

(*b.*) **Rule.**—II. *Write the numbers in a horizontal line, leaving out all numbers that are factors of any of the other numbers; divide by the least prime number that will divide more than one of them without a remainder, and write the quotients and the undivided numbers below. Reject factors and continue to divide as before, until there is no prime number greater than one that will divide two or more of them without a remainder. The product of the divisors and the undivided numbers is the least common multiple required.*

LESSON II.

EXERCISES FOR PRACTICE.

Find the least common multiples.

1. 4, 9, 12.
2. 16, 12, 24.
3. 15, 9, 6, 5.
4. 10, 6, 18, 15.
5. 24, 16, 15, 20.
6. 25, 60, 72, 35.
7. 63, 12, 84, 72.
8. 54, 81, 14, 63.
9. 12, 72, 36, 144.
10. 1, 2, 3, 4, 5, 6, 7, 8, 9.
11. 17, 29, 53, 37.
12. 8, 9, 55, 49.
13. 720, 336, 1736.
14. 8, 12, 16, 24, 36, 48, 72, 144.
15. 3, 7, 5, 9, 186.
16. 5, 3, 7, 9, 91.
17. 4, 3, 6, 7, 8, 16, 19.
18. 4, 8, 12, 24, 48, 84.
19. 4, 6, 7, 4, 5, 9, 13, 12.
20. 45, 78, 37, 42, 63.

LESSON III.

109. A Fraction is an expression denoting one or several of the equal parts of a number.* Fractions are of two kinds, Common and Decimal.

---

* NOTE.—When the number is not mentioned unity is understood. Thus $\frac{1}{4}$ signifies $\frac{1}{4}$ of one.

110. A Common Fraction is one whose denominator is not *ten* nor a power of *ten*; as, $\frac{6}{7}$, $\frac{3}{9}$ $\frac{4}{11}$, $\frac{3}{105}$.

(*a*.) A Common Fraction is expressed by a **Numerator** and a **Denominator**, which are called the *terms* of the fraction.

111. The Denominator of a fraction shows the number of equal parts into which the unit is divided, and gives the name to the fraction. It may be written at the right of the numerator in words; as, 4 *tenths*, 9 *sevenths*, 8-11*ths*; or under the numerator with a short horizontal line between them; as, $\frac{3}{9}\begin{array}{l}\text{numerator.}\\ \text{denominator.}\end{array}$

112. The Numerator of a fraction shows how many of the equal parts into which a unit is divided are expressed. It may be written either at the left of the denominator; as, 3 *tenths*, 4 *sevenths*, or over the denominator with a short horizontal line between them; as, $\frac{3}{10}$, $\frac{4}{7}$.

113. A Proper Fraction is less than a unit; as, $\frac{5}{7}$, $\frac{3}{8}$, $\frac{2}{9}$.

114. An Improper Fraction is equal to or greater than a unit; as, $\frac{15}{7}$, $\frac{13}{11}$, $\frac{8}{8}$, $\frac{16}{3}$.

115. A Mixed Number is an integer with a fractional number added to it; as, $3+\frac{4}{7}$, $7+\frac{8}{9}$. The sign of addition is usually omitted; as, $3\frac{4}{7}$, $7\frac{8}{9}$.

116. A Simple Fraction is one whose numerator and denominator are both integers.

117. The Denomination of a fraction is determined by the number of equal parts into which the unit is divided; as, *halves*, *fourths*, &c.

118. The Value of a fraction of any denomination depends upon the number of equal parts expressed by the numerator.

LESSON IV.

119. Principles of Reduction, Multiplication, and Division of Fractions.

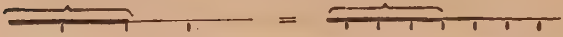
(*a.*) **The Numerator** of the fraction corresponds to the dividend, and the denominator to the divisor.

(*b.*) **Principles.**—I. If both numerator and denominator be multiplied by the same number, the value of the fraction will not be changed, because

The number of parts is increased in the same ratio as their size is diminished.

ILLUSTRATION.

$$\frac{2}{4}\,\begin{matrix}\times\\ \times\end{matrix}\,\begin{matrix}2\\2\end{matrix} = \frac{4}{8}$$



MODEL OPERATION.

(*a.*)

$$\begin{array}{l} \overset{\text{Dividend.}}{36} \div \overset{\text{Divisor.}}{12} = 3 \text{ Quotient.} \\ \;\,2 \qquad\;\; 2 \\ \hline 72 \div 24 = 3 \text{ Quotient.} \end{array}$$

(*b.*)

$$\frac{36\ \text{Numerator.}}{12\ \text{Denominator.}} = 3, \text{ Value.}$$

$$\frac{\text{Numerator. } 36\times 2}{\text{Denominator. } 12\times 2} = \frac{72}{24} = 3, \text{ Value.}$$

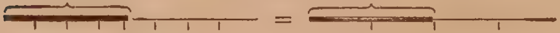
(*b.*) ILLUSTRATION.—36 is the numerator, 12 the denominator, and 3 the value. Multiplying both numerator and denominator by 2, we have 72 for the numerator, and 24 for the denominator, which gives 3 for the value of the fraction as before.

Princ.—II. If both numerator and denominator be divided by the same number, the value of the fraction will not be changed, because

The number of parts is diminished in the same ratio as their size is increased.

ILLUSTRATION.

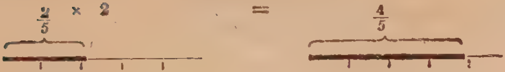
$$\frac{4}{8}\,\begin{matrix}\div\\ \div\end{matrix}\,\begin{matrix}2\\2\end{matrix} = \frac{2}{4}$$



III. If the numerator alone be *multiplied* by any number, the *value* will be multiplied by the same number, because

The number of parts is increased, while their size remains the same.

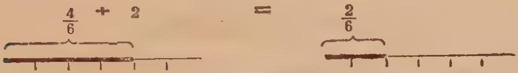
ILLUSTRATION.



IV. If the numerator alone be *divided* by any number, the *value* will be divided by the same number, because

The number of parts is diminished, while their size remains the same.

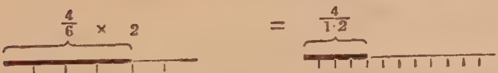
ILLUSTRATION.



V. If the denominator alone be *multiplied* by any number, the *value* will be be *divided* by the same number, because

The size of the parts is diminished, while the number remains the same.

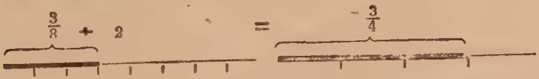
ILLUSTRATION.



VI. If the denominator alone be *divided* by any number, the *value* will be *multiplied* by the same number, because

The size of the parts is increased, while the number remains the same.

ILLUSTRATION.



(*c*.) **Law.**—Multiplying or dividing the numerator produces the *same* change in the value of the fraction. Multiplying or dividing the denominator produces the *opposite* change in the value.

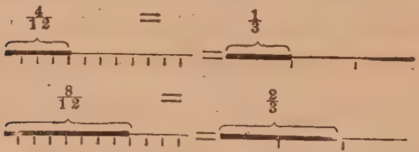
LESSON V.

120. ***To reduce a fraction*** to its *lowest terms,* or to a simple fraction of the *highest denomination.*[*]

MENTAL EXERCISES.

How many *thirds* in $\frac{8}{12}$?

ILLUSTRATION.

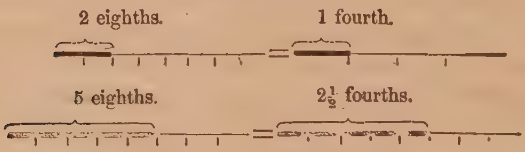


FORM.[†]—Since in 4 *twelfths* there is 1 third; in 8 twelfths there are as many *thirds* as 4 twelfths are contained times in 8 twelfths, which are 2 times; therefore in $\frac{8}{12}$ there are $\frac{2}{3}$.

1. How many 3ds in $\frac{8}{12}$? In $\frac{18}{27}$? In $\frac{14}{21}$? In $\frac{22}{33}$?
2. How many *fifths* in $\frac{9}{15}$? In $\frac{24}{40}$? In $\frac{21}{35}$? In $\frac{18}{30}$?
3. How many 7ths in $\frac{9}{21}$? In $\frac{15}{35}$? In $\frac{12}{28}$? In $\frac{24}{56}$?
4. How many 8ths in $\frac{14}{16}$? In $\frac{21}{24}$? In $\frac{42}{48}$? In $\frac{84}{96}$?

Change $\frac{5}{8}$ to *fourths.*

ILLUSTRATION.



---

[*] Changing a fraction to the *highest denomination* is equivalent to reducing it to its *lowest terms.* Much of the difficulty in the reduction of fractions arises from the fact that it is very confusing to the pupil to tell him that, in order to change a fraction from a *lower* to a higher denomination he must *reduce it to lower terms.* The teacher will also perceive that the reduction of fractions is analogous to the reduction of denominate numbers; for, as the individual parts of a unit or thing are decreased in size or *value*, the *number* of parts is increased in the same ratio. When these distinctions are properly presented, the subject of fractions will be divested of many difficulties.

[†] The teacher will find that it is difficult for the scholar to understand the *reason* of this formula unless it is illustrated by visible objects. If blocks are not at hand, an apple, potato, or turnip, cut into equal portions, will answer very well.

FORM.—Since in 2 *eighths* there is 1 *fourth*, in 5 *eighths* there are as many *fourths* as 2 *eighths* are contained times in 5 *eighths*, which are $2\frac{1}{2}$ times; therefore in 5 eighths there are $2\frac{1}{2}$ fourths.

LESSON VI.

Problem.—Change $\frac{48}{54}$ to its lowest terms.

MODEL OPERATION.

$$\frac{48\div3}{54\div3}=\frac{16\div2}{18\div2}=\frac{8}{9}\text{ Ans.}$$

(*a*.) **Anal. Steps.**—$\frac{48}{54}$ is not reduced to its *lowest terms*, because the numerator and denominator have the common factor 3; therefore:—

1. Change $\frac{48}{54}$ to a denomination 3 times as great as 54ths, which is 18ths.

FORM.—Since in 3-54ths there is 1 *eighteenth*, in 48-54ths there are as many *eighteenths* as 3-54ths are contained times in 48-54ths, which are 16 times; therefore 16 *eighteenths*. $\frac{16}{18}$ is not reduced to its *lowest terms*, since both numerator and denominator can be divided by the common factor 2; therefore,—

2. *Change* $\frac{16}{18}$ *to a denomination twice as great as* 18*ths, which is ninths.*

FORM.—Since in 2-18ths there is 1 *ninth*, in 16 eighteenths there are as many *ninths* as 2-18ths are contained times in 16-18ths, which are 8 times; therefore 8-*ninths*.

As there is no factor common to both terms of the fraction $\frac{8}{9}$, it is therefore expressed in its *lowest terms*.

(*b*.) **Rule.**—*Divide the numerator and the denominator of the fraction by any number that will divide them both without a remainder. Continue dividing in this way until no number greater than* ONE *will divide them.*

LESSON VII.

EXAMPLES FOR PRACTICE.

Reduce each of the following fractions to its lowest terms.

1. $\frac{5}{10}$, $\frac{6}{12}$.
2. $\frac{3}{6}$, $\frac{12}{16}$.
3. $\frac{5}{10}$, $\frac{8}{16}$.
4. $\frac{7}{14}$, $\frac{12}{18}$.
5. $\frac{27}{36}$, $\frac{21}{28}$.
6. $\frac{8}{10}$, $\frac{12}{15}$.
7. $\frac{16}{20}$, $\frac{20}{25}$.
8. $\frac{24}{30}$, $\frac{28}{35}$.
9. $\frac{32}{40}$, $\frac{36}{45}$.
10. $\frac{40}{50}$, $\frac{44}{55}$.
11. $\frac{10}{12}$, $\frac{63}{105}$.
12. $\frac{35}{55}$, $\frac{121}{132}$.
13. $\frac{30}{36}$, $\frac{132}{144}$.
14. $\frac{84}{132}$, $\frac{121}{220}$.
15. $\frac{60}{72}$, $\frac{132}{240}$.
16. $\frac{15}{18}$, $\frac{44}{246}$.
17. $\frac{42}{66}$, $\frac{38}{24}$.
18. $\frac{35}{42}$, $\frac{128}{443}$.
19. $\frac{81}{108}$, $\frac{678}{321}$.
20. $\frac{14}{22}$, $\frac{402}{804}$.
21. $\frac{20}{24}$, $\frac{3003}{5005}$.
22. $\frac{56}{88}$, $\frac{837}{534}$.
23. $\frac{40}{48}$, $\frac{400}{450}$.
24. $\frac{90}{120}$, $\frac{800}{700}$.
25. $\frac{21}{33}$, $\frac{238}{488}$.
26. $\frac{25}{30}$, $\frac{743}{608}$.
27. $\frac{77}{121}$, $\frac{9171}{9381}$.
28. $\frac{55}{66}$, $\frac{4004}{9005}$.
29. $\frac{45}{60}$, $\frac{602}{3913}$.
30. $\frac{28}{44}$, $\frac{840}{1155}$.

LESSON VIII.

121. ***To reduce an Improper Fraction*** to an integral or mixed number.

Prob.—Change $\frac{371}{6}$ to a mixed number.

MODEL OPERATION.

(*a.*)

$\frac{371}{6} = \frac{371}{6} \div \frac{6}{6} = 61\frac{5}{6}$. Ans.

(*b.*)

$$\begin{array}{r} 371 \\ \hline 6)371 \\ \hline 61\frac{5}{6} \end{array}$$ Ans.

Form.—Since in 6 sixths there is one unit, in 371 sixths there are as many units as 6 sixths are contained times in 371 sixths, which are 61 with a remainder of 5 sixths; therefore in $\frac{371}{6}$ there are $61\frac{5}{6}$.

Note.—Require the pupil to write a rule from the analysis.

EXERCISES FOR PRACTICE.

Change the following fractions to integral or mixed numbers.

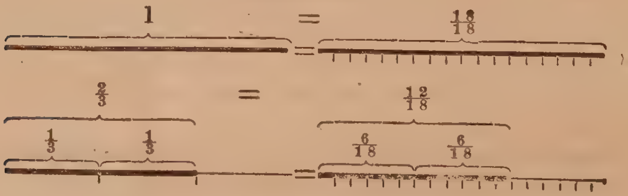
31. $\frac{341}{2}$, $\frac{112}{2}$.
32. $\frac{586}{2}$, $\frac{413}{2}$.
33. $\frac{371}{2}$, $\frac{672}{2}$.
34. $\frac{683}{3}$, $\frac{687}{3}$.
35. $\frac{596}{3}$, $\frac{913}{3}$.
36. $\frac{413}{3}$, $\frac{341}{3}$.
37. $\frac{683}{4}$, $\frac{213}{4}$.
38. $\frac{678}{4}$, $\frac{897}{4}$.
39. $\frac{896}{5}$, $\frac{371}{5}$.
40. $\frac{680}{5}$, $\frac{413}{5}$.
41. $\frac{6713}{6}$, $\frac{312}{4}$.
42. $\frac{4913}{7}$, $\frac{317}{5}$.
43. $\frac{9836}{8}$.
44. $\frac{4163}{8}$.
45. $\frac{7888}{11}$.
46. $\frac{83764}{10}$.
47. $\frac{4873}{16}$.
48. $\frac{2839}{22}$.
49. $\frac{61381}{37}$.
50. $\frac{2134}{67}$.
51. $\frac{4134}{382}$.
52. $\frac{6713}{42}$.
53. $\frac{789}{631}$.
54. $\frac{567}{567}$.
55. $\frac{397}{697}$.
56. $\frac{413}{413}$.
57. $\frac{8643}{241}$.
58. $\frac{6783}{874}$.
59. $\frac{637}{83}$.
60. $\frac{6913}{92}$.

LESSON IX.

122. ***To reduce an Integral or Mixed Number*** to an improper fraction.

Ment. Prob.—Change $\frac{2}{3}$ to 18ths.

ILLUSTRATION.



Formula.—Since in *one* unit there are 18 *eighteenths*, in one-*third* of a unit there is one-*third* of 18 *eighteenths*, which is 6 eighteenths, and in 2 *thirds* of a unit there are 2 times 6 eighteenths, which are 12 eighteenths; therefore, there are $\frac{12}{18}$ in $\frac{2}{3}$ of a unit.

(*a.*) Change $27\frac{3}{5}$ to an improper fraction.

(*b.*) Change $27\frac{3}{5}$ to fifteenths.

MODEL OPERATION.

(*a.*)

$$\begin{array}{ll} 27\frac{3}{5} & \\ \quad 5 & \text{fifths} \\ \hline 138 & \text{“} \quad = \frac{138}{5} \text{ Ans.} \end{array}$$

(*b.*)

$$\begin{array}{ll} 27\frac{3}{5} & \\ 15 & \text{fifteenths} \\ \hline 405 & \text{“} \\ 9 & \text{“} \\ \hline 414 & \text{“} \quad = \frac{414}{15} \text{ Ans.} \end{array}$$

Form.—(*a*) Since in one there are 5 *fifths*, in 27 there are 27 times 5 *fifths*, which, with three *fifths* added, make 138 *fifths*; therefore in $27\frac{3}{5}$ there are $\frac{138}{5}$.

(*b.*) **Anal. Steps.**—**1.** *Change 27 to 15ths.*
2. *Change $\frac{3}{5}$ to 15ths.*
3. *Find the sum.*

LESSON X.

Change each of the following numbers to improper fractions.

61. $3\frac{5}{8}$
62. $6\frac{7}{8}$
63. $11\frac{3}{5}$
64. $63\frac{7}{9}$
65. $46\frac{1}{7}$
66. $33\frac{2}{7}$
67. $67\frac{5}{9}$
68. $86\frac{3}{7}$
69. $9\frac{3}{4}$
70. $67\frac{5}{11}$
71. $8\frac{3}{7}$
72. $96\frac{4}{7}$
73. $39\frac{4}{9}$
74. $876\frac{4}{8}$
75. $86\frac{3}{9}$
76. $413\frac{1}{2}$
77. $617\frac{3}{7}$
78. $682\frac{5}{8}$
79. $83\frac{7}{9}$
80. $413\frac{3}{4}$
81. $1\frac{6}{7}$
82. $5\frac{8}{13}$
83. $416\frac{3}{4}$
84. $1\frac{6}{13}$
85. $9\frac{3}{7}$
86. $864\frac{1}{4}$
87. $87\frac{6}{7}$
88. $917\frac{3}{8}$
89. $56\frac{7}{8}$
90. $723\frac{4}{9}$

LESSON XI.

123. To Reduce Fractions to equivalent fractions having a common denominator.

124. Fractions are said to have a *common denominator* when they are of the same denomination.

Change $2\frac{3}{4}$, $\frac{2}{3}$, $\frac{5}{9}$, $\frac{7}{8}$, $3\frac{7}{12}$ to equivalent fractions having a common denomination.

MODEL OPERATIONS.

(*a.*)

$$\begin{array}{r} 2)\cancel{4},\ \cancel{3},\ 9,\ 8,\ 12 \\ \hline 3)\ 9,\ 4,\ \ 6 \\ \hline 3,\ 4,\ \ \cancel{2} \end{array}$$

$$2 \times 3 \times 3 \times 4 = 72, \text{ l. c. m.}$$

(*b.*)

(1.) $2\frac{3}{4} = 2\frac{54}{72}$.

(2.) $\frac{2}{3} = \frac{48}{72}$.

(3.) $\frac{5}{9} = \frac{40}{72}$.

(4.) $\frac{7}{8} = \frac{63}{72}$.

(5.) $3\frac{7}{12} = 3\frac{42}{72}$.

Anal. Steps.—The least common multiple of the denominators of the given fractions indicates the most convenient denomination to which they can all be reduced; * therefore,

1. *Change* $\frac{3}{4}$ *to an equal fraction of the denomination of* 72*ds.*

Form.—Since in one there are $\frac{72}{72}$, in $\frac{1}{4}$ of one there is $\frac{1}{4}$ of $\frac{72}{72}$, which is $\frac{18}{72}$, and in $\frac{3}{4}$ of one unit there are 3 times $\frac{18}{72}$, which are $\frac{54}{72}$, which added to two units make $2\frac{54}{72}$.

2. *Change* $\frac{2}{3}$ *to an equal fraction of the denomination of* 72*ds.*

Form.†—Since in one there are $\frac{72}{72}$, in $\frac{2}{3}$ of one there are 2 times $\frac{1}{3}$ of $\frac{72}{72}$ which are $\frac{48}{72}$.

3. *Change* $\frac{5}{9}$ *to an equal fraction of the denomination*, &c.

(*a.*) **Rule.**—*Multiply both terms of each fraction by such a number as will make them of the same denomination.*

(*b.*) Note.—The number by which the terms of the fraction should be multiplied, may be found by dividing the least common multiple by its denominator.

EXERCISES FOR PRACTICE.

Reduce each of the following expressions to equivalent fractions having a common denominator.

91. $\frac{3}{7}$, $\frac{5}{8}$, $\frac{6}{12}$, $\frac{5}{14}$, $\frac{3}{4}$.
92. $\frac{6}{7}$, $\frac{3}{14}$, $\frac{7}{21}$, $\frac{11}{12}$, $\frac{11}{18}$.
93. $\frac{8}{9}$, $\frac{3}{27}$, $\frac{6}{7}$, $\frac{5}{28}$, $\frac{3}{21}$.
94. $\frac{2}{11}$, $\frac{6}{121}$, $\frac{11}{14}$, $\frac{3}{28}$, $\frac{46}{12}$.
95. $4\frac{3}{7}$, $\frac{6}{8}$, $\frac{9}{14}$, $11\frac{6}{7}$.
96. $3\frac{5}{8}$, $3\frac{4}{7}$, $5\frac{3}{8}$, $9\frac{4}{14}$
97. $4\frac{3}{7}$, $5\frac{4}{28}$, $\frac{9}{12}$, $\frac{4}{15}$.
98. $6\frac{3}{5}$, $41\frac{2}{7}$, $6\frac{3}{7}$, $\frac{4}{28}$.
99. $4\frac{3}{4}$, $3\frac{11}{12}$, $\frac{6}{84}$, $\frac{3}{7}$.
100. $9\frac{4}{5}$, $3\frac{14}{28}$, $4\frac{8}{40}$, $\frac{3}{7}$.
101. $4\frac{3}{7}$, $8\frac{5}{14}$, $\frac{6}{7}$, $9\frac{2}{5}$.
102. $3\frac{6}{7}$, $5\frac{8}{9}$, $\frac{3}{27}$, $\frac{6}{28}$.
103. $3\frac{4}{7}$, $5\frac{9}{12}$, $\frac{1}{24}$, $3\frac{1}{28}$.
104. $\frac{6}{7}$, $5\frac{3}{21}$, $6\frac{6}{7}$, $3\frac{4}{7}$.
105. $\frac{8}{9}$, $6\frac{4}{7}$, $5\frac{3}{7}$, $6\frac{8}{14}$.

---

* Note.—Since the denominators of the several fractions must divide the least common denominator without a remainder, it is evident that the least common multiple of the denominators indicates the highest denomination to which they can be all reduced, each having an integral number of equal parts.

† When the pupil clearly understands the first formula this contraction is preferable.

LESSON XII.

ADDITION OF FRACTIONS.

125. To add Mixed Numbers with fractions of different denominations.

Prob.—What is the sum of $3\frac{3}{4}$, $\frac{2}{3}$, $5\frac{1}{8}$, $\frac{1}{6}$?

MODEL OPERATION.

(*a.*)

$$\begin{array}{r} 2)\cancel{4},\ \cancel{3},\ 8,\ 6 \\ \hline 4,\ 3 \end{array}$$

$$2 \times 4 \times 3 = 24 \text{ l. c. m.}$$

(*b.*)

$$\begin{array}{r} 3\frac{3}{4} = 3\frac{18}{24} \\ \frac{2}{3} = \frac{16}{24} \\ 5\frac{1}{8} = 5\frac{3}{24} \\ \frac{1}{6} = \frac{4}{24} \\ \hline 8\frac{41}{24} = 9\frac{17}{24} \text{ Ans.} \end{array}$$

(*a.*) **Anal. Steps.**—1. Find the least common multiple of the denominators 4, 3, 8, and 6, which is 24.

2. (*b.*) *Change the fractions* $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{8}$, *and* $\frac{1}{6}$, *to equivalent fractions having a common denominator.*

3. *Find the sum total.*

Form.—The sum of $3\frac{18}{24}$, $\frac{16}{24}$, $5\frac{4}{24}$, and $\frac{4}{24}$, equals $8\frac{41}{24}$; or $9\frac{17}{24}$; therefore, the sum of $3\frac{3}{4}$, $\frac{2}{3}$, &c.

PRACTICAL PROBLEMS IN REDUCTION AND ADDITION OF FRACTIONS.

A man bought a horse for \$$73\frac{3}{9}$, a carriage for $97\frac{5}{9}$, a set of harness for \$$37\frac{5}{12}$; what was the cost of the whole?

MODEL OPERATION.

$$\begin{array}{rl} \$73\frac{3}{9} = 73\frac{12}{36}, & \text{cost of horse.} \\ 97\frac{5}{9} = 97\frac{20}{36}, & \text{“ carriage.} \\ 37\frac{5}{12} = 37\frac{15}{36}, & \text{“ harness.} \\ \hline 207\frac{47}{36} = \$208\frac{11}{36}, & \text{total cost.} \end{array}$$

Anal. Steps.—1. *Reduce the several fractions to equivalent fractions having a common denominator.*

Form.—$\$73\frac{3}{9} = 73\frac{12}{36}$; $\$97\frac{5}{9} = 97\frac{20}{36}$; $\$37\frac{5}{12} = 37\frac{15}{36}$.

2. *Find the sum total.*

FORM.—The sum of $\$73\frac{12}{36}$, $\$97\frac{20}{36}$, and $\$37\frac{15}{36}$, is $207\frac{47}{36}$; $\$\frac{47}{36}$ equals $1\frac{11}{36}$, which added to 207, make $\$208\frac{11}{36}$.

CONCL.—Therefore, if a man bought a horse for $\$73\frac{3}{9}$, a carriage, &c.

106. A man bought 4 pieces of cloth; the first contained $37\frac{5}{9}$ yd., the second $43\frac{5}{7}$ yd., the third $27\frac{3}{11}$ yd., and the fourth $38\frac{1}{12}$ yd.; how many yards did he buy in all?

107. If a rubber cost $12\frac{5}{9}$ ct., a slate $10\frac{3}{8}$ ct., a pencil $\frac{11}{12}$ of a cent, and a sponge 5 ct.; how much do they all cost?

108. A merchant bought at one time $129\frac{3}{7}$ lb. of sugar, at another $439\frac{4}{11}$ lb., at another 372 lb., and at another $473\frac{4}{13}$ lb.; how many pounds did he buy in all?

109. A merchant bought $272\frac{2}{3}$ yd. of muslin of one man, $117\frac{3}{4}$ yd. of another, and $321\frac{5}{6}$ yd. of a third; how many yards did he buy in all?

110. A boy paid $\$4\frac{7}{8}$ for a pair of skates, $\$5\frac{3}{6}$ for a sled, and $\$1\frac{2}{3}$ for a cap; how much did he spend?

111. A grocer bought $26\frac{7}{8}$ bushels of potatoes, $3\frac{3}{4}$ bushels of turnips, and $6\frac{1}{3}$ bushels of beets; how many bushels did he purchase?

112. I paid $\$378\frac{4}{5}$ to one man, $\$450\frac{7}{10}$ to another, and $365\frac{13}{15}$ to a third, how much did I have at first?

113. My father gave me at one time $\$100\frac{1}{2}$, at another $\$75\frac{3}{4}$, to my sister he gave $\$175\frac{5}{8}$, and to my younger brother $\$80\frac{15}{16}$. How much did he give us?

114. A man paid for flour $\$655\frac{2}{3}$, for pork $\$478\frac{3}{4}$, for beef $\$508\frac{5}{6}$, for sugar $\$189\frac{17}{18}$. What did they all cost him?

115. A lady bought some calico for $98\frac{2}{3}$ ct., some lace for $85\frac{3}{4}$ ct., some pins for $25\frac{5}{6}$ ct. How much had she when she started?

LESSON XIII.

SUBTRACTION OF FRACTIONS.

126. To subtract mixed numbers containing fractions of different denominations.

MODEL OPERATIONS.

	(*a.*)	(*b.*)	(*c.*)	(*d.*)	(*e.*)
From	$15\frac{3}{4}=15\frac{6}{8}$	13	14	$13\frac{4}{5}=13\frac{24}{30}$	$47\frac{5}{11}$
Take	$6\frac{3}{8}=\ 6\frac{3}{8}$	$4\frac{4}{7}$	$9\frac{3}{11}$	$12\frac{5}{6}=12\frac{25}{30}$	13
Rem.	$9\frac{3}{8}$	$8\frac{3}{7}$	$4\frac{8}{11}$	$\frac{29}{30}$	$34\frac{5}{11}$

Form.—(*a.*) 3 *eighths* from 6 *eighths* leave 3 *eighths;* 6 units from 15 units leave 9 units, to which add $\frac{3}{8}$ making $9\frac{3}{8}$; therefore the difference, &c.

Form.—(*b.*) Since *sevenths* cannot be subtracted from units, take 1 unit from 3 units and change it to *sevenths.* 1 is equal to 7 *sevenths,* and 4 *sevenths* from 7 *sevenths,* leave 3 *sevenths;* 4 units from 12 units leave 8 units; therefore, &c.

Form.—(*d.*) Since 25 thirtieths cannot be taken from 24 thirtieths, take *one* unit from 3 units; 1 unit is equal to 30 thirtieths, and 30 thirtieths *plus* 24 thirtieths equal 54 thirtieths; 25 thirtieths from 54 thirtieths leave 29 thirtieths; 12 units from 12 units leave no units. Therefore, &c.

EXAMPLES FOR PRACTICE.

116. From $\frac{7}{12}$ take $\frac{2}{11}$.
117. From $\frac{11}{19}$ take $\frac{7}{19}$.
118. From $\frac{13}{18}$ take $\frac{6}{9}$.
119. From $\frac{15}{11}$ take $\frac{1}{3}$.
120. From $\frac{8}{13}$ take $\frac{11}{43}$.
121. From $\frac{47}{2}$ take $\frac{3}{4}$.

122. $\frac{4}{9}-\frac{3}{15}=$what?
123. $\frac{4}{7}-\frac{4}{8}=$what?
124. $\frac{16}{17}-\frac{17}{19}=$what?
125. $\frac{5}{11}-\frac{5}{12}=$what?
126. $\frac{6}{12}-\frac{5}{11}=$what?
127. $3\frac{1}{4}-\frac{4}{13}=$what?

128. $\frac{5}{7}-\frac{3}{11}$.
129. $\frac{5}{12}-\frac{2}{8}$.
130. $\frac{15}{18}-\frac{7}{9}$.
131. $\frac{15}{11}-\frac{3}{8}$.
132. $\frac{47}{2}-\frac{3}{4}$.

133. $4\frac{3}{7}-\frac{3}{8}+\frac{3}{4}$.
134. $41\frac{6}{11}-\frac{5}{7}+\frac{4}{5}$.
135. $31\frac{4}{5}-\frac{3}{7}+\frac{6}{11}$.
136. $41\frac{3}{4}+\frac{5}{9}+\frac{1}{2}$.
137. $83\frac{5}{7}-\frac{4}{14}+\frac{12}{4}$.

136. $63-\frac{4}{7}+\frac{1}{5}$.
139. $27-\frac{61}{12}+\frac{3}{8}$.
140. $24\frac{1}{2}-\frac{3}{92}+\frac{1}{2}$.
141. $18\frac{5}{8}-2\frac{1}{7}+\frac{1}{2}$.
142. $4\frac{3}{7}-\frac{2}{11}+4\frac{3}{7}$.

143. $5+\frac{3}{4}-\frac{6}{11}$.
144. $4+5\frac{3}{7}-2\frac{1}{7}$.
145. $8+5\frac{3}{4}-2\frac{1}{2}$.
146. $13+\frac{5}{8}-6\frac{3}{4}$.
147. $18-3\frac{2}{7}+5\frac{3}{14}$.

LESSON XIV.

127. ***To Multiply a Fraction,*** or mixed number, by an integral number.

(*a.*) What is the product of $\frac{4}{9}$ multiplied by 3.

(*b.*) What is the product of $3\frac{4}{9}$ multiplied by 3.

MODEL OPERATION.

(*a.*)

1. $\frac{4}{9}\times 3=\frac{4}{9\div 3}=\frac{4}{3}=1\frac{1}{3}$ Ans.

2. $\frac{4}{9}\times 3=\frac{4\times 3}{9}=\frac{12}{9}=1\frac{1}{3}$ Ans.

(*b.*)

$$\begin{array}{r} 3\frac{4}{9} \\ 3 \\ \hline 10\frac{1}{3} \end{array}$$ Ans.

(*a.*) Formula.—1. Since dividing the denominator of a fraction by any number, multiplies the value of the fraction by the same number, (138., Princ. VI.), therefore,

If the denominator of $\frac{4}{9}$ is divided by 3, the result $\frac{4}{3}$ is 3 times as great as $\frac{4}{9}$, which is equal to $1\frac{1}{3}$.

2. Or, 3 times 4 ninths are 12 ninths, equal to $1\frac{1}{3}$. (138., Princ. III.)

(*b.*) Form.—3 times $\frac{4}{9}$ are $\frac{4}{3}$, equal to $1\frac{1}{3}$; 3 times 3 units are 9 units, *plus* $1\frac{1}{3}$ units, equal to $10\frac{1}{3}$.

Rule.—(*a.*) I. Multiply the numerator **or** *divide the denominator, when it can be done without a remainder.*

(*b.*) II. *Multiply the integral number and the fraction separately, and add the products.*

Note.—It is *better to divide the denominator*, when it can be done without a remainder.

LESSON XV.

PROBLEMS FOR PRACTICE.

148. $\frac{3}{5}\times 8$; $\frac{6}{7}\times 9$; $\frac{9}{13}\times 60$; $\frac{4}{20}\times 5$; $\frac{6}{8}\times 37$.

149. $\frac{4}{7}\times 9$; $\frac{4}{11}\times 12$; $\frac{3}{19}\times 37$; $\frac{5}{11}\times 46$; $\frac{8}{7}\times 11$.

150. $\frac{4}{7}\times 21$; $\frac{8}{27}\times 8$; $\frac{62}{96}\times 40$; $\frac{43}{76}\times 20$; $\frac{4}{13}\times 133$.

151. $783\frac{2}{7}\times 8$; $6713\frac{6}{11}\times 5$; $9173\frac{4}{12}\times 8$; $6132\frac{3}{7}\times 6$.

152. $73\frac{4}{7}\times 12$; $136\frac{8}{13}\times 24$; $196\frac{5}{7}\times 47$; $37\frac{2}{7}\times 126$.

153. $13\frac{3}{9}\times 864$; $7\frac{3}{11}\times 8646$; $67\frac{2}{5}\times 834$; $16\frac{3}{9}\times 487$.

154. I bought 1 peck of apples for $\$\frac{8}{27}$; what will 8 pecks cost?

155. A man purchased a neck-tie for $\$\frac{62}{96}$; what would 40 neck-ties cost him?

156. A doctor sold a box of pills for $\$\frac{43}{76}$; what would he charge for 20 boxes?

157. A gentleman purchased a house for $\$1413\frac{3}{5}$; what would 6 of equal value cost?

MODEL OPERATION.

$$\begin{array}{rr} \$1413\times6= & \$8478 \\ \frac{3}{5}\times6= & 3\frac{3}{5} \\ \cline{2-2} & \$8481\frac{3}{5}. \end{array}\quad \text{Ans.}$$

Anal. steps.—**1.** *Find* 6 *times* \$1413.
2. *Find* 6 *times* $\$\frac{3}{5}$.
3. *Find the sum of these products.*

158. If one hat costs $\$7.83\frac{2}{7}$, what will 8 cost?

159. If one sewing machine is worth $\$67.13\frac{6}{11}$, what are 5 worth?

160. If I pay $\$917.03\frac{4}{12}$ for house rent per year, what shall I have paid in 8 years?

161. A gentleman receives a salary of $\$6132\frac{3}{7}$; how much will that be for 6 years?

162. If 1 farm costs $\$4136\frac{2}{9}$, what will 4 cost?

LESSON XVI.

EXAMPLES FOR PRACTICE.

163. $\frac{3}{4}\times6+\frac{1}{2}$.
164. $\frac{3}{7}\times5-\frac{1}{3}$.
165. $\frac{9}{11}\times8+\frac{1}{6}$.
166. $\frac{3}{4}\times9-\frac{3}{7}$.
167. $\frac{6}{7}\times8+\frac{3}{7}$.
168. $3\frac{4}{7}\times2$.
169. $6\frac{3}{7}\times5$.
170. $41\frac{3}{8}\times6$.
171. $67\frac{4}{7}\times9$.
172. $46\frac{5}{9}\times3$.
173. $72\frac{6}{7}\times12$.
174. $893\frac{1}{4}\times18$.
175. $416\frac{3}{7}\times67$.
176. $413\frac{5}{7}\times8$.
177. $4678\frac{3}{7}\times5$.
178. $\frac{3}{7}\times896$.
179. $\frac{4}{3}\times9186$.
180. $\frac{5}{8}\times871$.
181. $\frac{6}{9}\times467$.
182. $3\frac{7}{4}\times865$.
183. $2\frac{1}{2}\times637$.
184. $4\frac{3}{7}\times896$.
185. $3\frac{4}{7}\times913$.
186. $5\frac{3}{8}\times641$.

MISCELLANEOUS PRACTICAL PROBLEMS.

I had $\$117\frac{1}{2}$ and bought 7 hats at $\$7\frac{3}{5}$ each; how much had I left?

MODEL OPERATION.

(1.)

$$\begin{array}{rr} \$7\times7= & \$49 \\ \frac{3}{5}\times7= & 4\frac{1}{5} \\ \cline{2-2} & \$53\frac{1}{5} \end{array}$$

(2.)

$$\begin{array}{r} \$117\frac{1}{2} \\ 53\frac{1}{5} \\ \hline \$64\frac{3}{10} \end{array}\quad \text{Ans.}$$

Anal. Steps.—1. *Find cost of the hats.*
2. *Find amount left.*

187. A boy sold 6 knives at $\$\frac{3}{4}$ each; he then spent $\$\frac{1}{2}$; how much had he left?

188. A marketman sold 12 shad at $\frac{8}{9}$ of a dollar each, and 14 herrings at $\frac{3}{4}$ ct. each; how much did he receive for all?

189. A boy bought 7 kites at $2\frac{1}{3}$ cents each, and 2 tops at $11\frac{1}{4}$ cents each, and sold them so as to make $4\frac{1}{3}$ cents; how much did he receive for them?

190. I have 89 boxes of raisins for which I gave $\$3\frac{6}{7}$ per box; for how much must I sell them to make $\$11\frac{3}{4}$?

191. If the interest on $1 for one year is $6\frac{3}{7}$ cents, what is the interest on a note of $572?

LESSON XVII.

128. To Divide a Fraction or a Mixed Number by an integral number.

(*a.*) Divide $\frac{3}{4}$ by 3.

(*b.*) Divide $467\frac{3}{4}$ by 3.

MODEL OPERATION.

(*a.*)

1. $\frac{3}{4} \div 3 = \frac{3 \div 3}{4} = \frac{1}{4}$ Ans.

2 $\frac{3}{4} \div 3 = \frac{3}{4 \times 3} = \frac{3}{12} = \frac{1}{4}$ Ans.

(*b.*)

$$\begin{array}{r} 3)\underline{467\frac{3}{4}} \\ 155\frac{11}{12} \end{array}$$

Formula.—1. One-*third* of 3 fourths is 1 fourth. (138 Princ. IV.),

2. Or, since multiplying the denominator of a fraction by any number divides the value of the fraction by the same number, (138., Princ. V.), therefore,

If the denominator of $\frac{8}{4}$ be multiplied by 3, the result $\frac{3}{12}$ is 1 *third* as large as $\frac{3}{4}$, therefore $\frac{8}{4} \div 3 = \frac{3}{12} = \frac{1}{4}$, or $\frac{1}{3}$ of $\frac{8}{4}$ is $\frac{1}{4}$.*

---

* The expression "3/4 divided by 3" is equivalent to 1/3 of 3/4, but is not identical with it. The latter is an operation in multiplication, the former an operation in division. They are often used as synonymous; but their difference should be explained to the pupil, to prevent confusion.

Form.—1. One-*third* of 467 is 155 with a remainder of 2 units; which is to be added to the *fourths* of a unit.

2. Two units are equal to 8 fourths; 8 fourths and 3 fourths are 11 fourths, and 1 third of 11 fourths is 11 *twelfths;* therefore $467\frac{3}{4}\div 3=155\frac{11}{12}$.

Rule.—(*a.*) I. Divide the numerator *when it can be done without a remainder*, **or** *multiply the denominator.*

(*b.*) II. *First divide the integral number by the divisor, and add the remainder to the fraction; then divide the fraction, and annex the result to the integral quotient. If the mixed number is less than the divisor, change it to an improper fraction before dividing.*

(*c.*) Note.—It is better to divide the numerator, when it can be done without a remainder.

192. Divide $416\frac{3}{9}$ by 9, by 8, by 7, by 6.

193. Divide $347\frac{3}{19}$ by 5, by 6, by 3, by 12.

194. Divide $241\frac{3}{4}$ by 6, by 8, by 5, by 9.

195. Divide $4134\frac{6}{7}$ by 15, by 36, by 25, by 84.

LESSON XVIII.

PRACTICAL PROBLEMS.

196. What is the cost of one pound of tea, if 12 lb. cost $\$12.37\frac{3}{7}$?

197. What is the cost of 1 lb. of coffee, if 7 lb. cost $\$1.08\frac{3}{5}$?

198. What is the weight of 1 box of raisins, if 12 boxes weigh $84\frac{3}{7}$ lb.?

199. What do I give for 1 gallon of molasses, if 6 gallons cost $\$1.34\frac{3}{8}$?

200. How far do I walk in 1 hour, if I walk $16\frac{7}{8}$ miles in 8 hours?

201. If there are $117\frac{2}{5}$ yd. of silk in 9 dresses, how many in one dress?

202. What is the length of 1 board if 7 boards measure $56\frac{2}{3}$ feet in length?

203. If 12 men measure together $68\frac{3}{4}$ ft. in height, what is their average height?

204. What is the weight of a load of hay, if 5 loads weigh $10728\frac{2}{3}$ lb.?

205. If there are $926\frac{3}{7}$ lb. of flour in four barrels, how many lb. in one barrel?

Nine pounds of sugar cost $\$1.37\frac{1}{7}$; what will 4 pounds cost?

MODEL OPERATION.

$$9)\underline{137\frac{1}{7}}\text{ ct.}$$
$$15\frac{5}{21}\text{ cost 1 lb.}$$
$$\underline{4}$$
$$\$.60\frac{20}{21}\quad\text{“}\quad 4\text{ lb.}$$

Anal. Steps.—1. *Find cost of 1 lb.*
2. *Find cost of 4 lb.*

206. If 13 fish weigh $58\frac{1}{5}$ lb., how much will 3 fish weigh?

207. If 7 boys sell their marbles for $38\frac{2}{3}$ ct., and divide the money equally among 4 poor children, how much will each receive?

208. If a man give his son $\$5.74\frac{1}{6}$ per week how much will he give him in 5 weeks?

209. If a family burn $4\frac{1}{8}$ tons of coal a year, how much will they use in 5 years?

210. If 12 yd. of lace cost $\$13.82\frac{1}{8}$; what will 3 yd. cost?

211. If one man can eat $4\frac{1}{9}$ loaves in 8 meals, how many loaves can 9 men eat?

212. There are $18\frac{2}{8}$ qt. of vinegar in six jugs of equal size; how many quarts in 2 jugs?

213. If 7 men can drink $56\frac{1}{8}$ qt. of water in one week, how many quarts can 13 men drink?

214. If seventy and a fourth pounds of beef will last 13 men 4 days, how many pounds will 7 men require in the same time?

LESSON XIX.

EXAMPLES FOR PRACTICE.

215. $\frac{4}{7}\times 5+\frac{3}{7}$.
216. $\frac{3}{11}\times 8+4\frac{3}{7}$.
217. $\frac{5}{9}\times 9-\frac{3}{4}$.
218. $\frac{6}{7}\times 11-2\frac{4}{7}$.
219. $\frac{5}{9}\times 121+376$.
220. $\frac{4}{7}\times 8+\frac{5}{9}$.
221. $\frac{5}{7}\times 9+67\frac{3}{4}$.
222. $\frac{13}{12}\times 7-3\frac{4}{7}$.
223. $\frac{19}{20}\times 8-2\frac{3}{4}$.
224. $\frac{3}{13}\times 26+40\frac{1}{2}$.
225. $37\frac{2}{7}\div 4+\frac{3}{8}$.
226. $273\frac{1}{4}\div 9-1\frac{2}{3}$.
227. $4867\frac{1}{9}\div 7+4\frac{2}{3}$
228. $6713\frac{3}{7}\div 8-34$.
229. $463\frac{4}{9}\div 9+38$.
330. $463\frac{7}{8}\div 91$.
231. $837\frac{5}{11}\div 43$.
232. $6789\frac{3}{9}\div 786$.

LESSON XXI.

129. ***To Multiply an integral, fractional, or mixed number*** by a fraction or a mixed number.

1. Multiply 374 by $\frac{3}{4}$.
2. Multiply $\frac{3}{9}$ by $\frac{3}{4}$.
3. Multiply $37\frac{4}{7}$ by $4\frac{3}{4}$.
4. Multiply $\frac{3}{7}$ by $\frac{5}{8}$.

MODEL OPERATIONS.

1. $374 \times 3 \div 4 = 280\frac{1}{2}$. Ans.
2. $\frac{3}{9} \times 3 \div 4 = \frac{1}{4}$. Ans.
3. $37\frac{4}{7} \times 19 \div 4 = 178\frac{13}{28}$. Ans.

(4.)
1. $\frac{3}{7} \times \frac{8}{8}$ or $1 = \frac{3}{7}$.
2. $\frac{3}{7} \times \frac{1}{8} = \frac{3}{56}$.
3. $\frac{3}{7} \times \frac{5}{8} = \frac{15}{56}$. Ans.

Form.*—1. $\frac{3}{7}$ multiplied by $\frac{8}{8}$ or 1 unit equals $\frac{3}{7}$.

2. If $\frac{3}{7} \times \frac{8}{8} = \frac{3}{7}$, then $\frac{3}{7} \times \frac{1}{8}$ will equal one *eighth* of $\frac{3}{7}$, which is $\frac{3}{56}$.

3. If $\frac{3}{7} \times \frac{1}{8} = \frac{3}{56}$, then $\frac{3}{7} \times \frac{5}{8}$ will equal 5 times $\frac{3}{56}$, which is $\frac{15}{56}$; therefore, $\frac{3}{7} \times \frac{5}{8} = \frac{15}{56}$.

(*a.*) **Rule.**—I. Multiply by the numerator, *and divide the product by the denominator;* or

II. *Cancel all the common factors, and multiply the remaining numerators together for a new numerator, and the remaining denominators for a new denominator.*

When the multiplier is a mixed number

III. *Multiply by the integer and the fraction separately, and add the products.*

Note.—When the multiplier is smaller than the multiplicand, change it to an improper fraction and proceed according to the rule; if larger, change the multiplicand to an improper fraction and use it as a multiplier.

LESSON XXII.

233. $346 \times \frac{5}{8}$.
234. $413 \times \frac{6}{7}$.
235. $493 \times \frac{5}{8}$.
236. $6917 \times \frac{3}{7}$.
237. $467 \times 5\frac{3}{7}$.
238. $863\frac{4}{7} \times \frac{4}{7}$.
239. $683 \times 6\frac{7}{3}$.
240. $9183\frac{4}{11} \times \frac{3}{4}$.
241. $8673\frac{5}{9} \times \frac{8}{9}$.
242. $4783\frac{4}{7} \times 9\frac{3}{7}$.
243. $3476\frac{1}{7} \times 5\frac{3}{9}$.
244. $634\frac{3}{7} \times 6\frac{7}{8}$.
245. $413\frac{4}{11} \times 4\frac{2}{7}$.
246. $9163 \times 5\frac{3}{7}$.
247. $6\frac{7}{8} \times 891\frac{3}{4}$.
248. $4167\frac{3}{9} \times 89\frac{1}{2} + 6\frac{1}{4}$.
249. $374\frac{6}{7} \times 41\frac{2}{3} - 5\frac{3}{5}$.
250. $478\frac{5}{7} \times 89 - 37\frac{1}{2}$.

Questions.—What is meant by $^3/_4$? What is meant by $^3/_4$ of 11? How many units in $^3/_4$? How many units in $^3/_4$ of 11?

---

* Note.—Those who prefer it may use the following *analysis*:—$^3/_7 \times {}^5/_8 = {}^5/_8$ of $^3/_7$; $^5/_8$ of $^3/_7 = {}^1/_8$ of 5 times $^3/_7$; 5 times $^3/_7 = {}^{15}/_7$, and $^1/_8$ of $^{15}/_7 = {}^{15}/_{56}$. Ans.

LESSON XXIII.

PRACTICAL PROBLEMS.

251. What will $7\frac{1}{2}$ lb. of rice cost at $5\frac{3}{4}$ ct. a pound?

252. What will $9\frac{7}{8}$ qt. of beans cost at $7\frac{2}{3}$ ct. a qt.?

253. What will $6\frac{7}{8}$ yd. of ribbon cost at $12\frac{1}{2}$ ct. a yd.?

254. What will $7\frac{2}{3}$ yd. of lace cost at $9\frac{3}{4}$ ct. a yard?

255. How much will $12\frac{1}{2}$ bu. of oats come to at $62\frac{1}{2}$ ct. a bushel?

256. How much will $6\frac{1}{2}$ bu. of apples cost at $74\frac{1}{2}$ ct. a bu.?

257. What will $5\frac{1}{2}$ qt. of nuts cost at $9\frac{3}{4}$ ct. a qt.?

258. What will 15 lb. of starch cost at $9\frac{1}{2}$ ct. a lb.?

259. What will $2\frac{3}{5}$ yd. of cloth cost at $\frac{7}{8}$ of a dollar a yd.?

260. What will 15 lb. of honey cost at $26\frac{3}{5}$ ct. a lb.?

261. What will $7\frac{3}{4}$ lb. of coffee cost at $\frac{3}{5}$ of a dollar a lb.?

262. What will 42 bu. of apples cost at $63\frac{2}{7}$ ct. a bushel?

263. What will $7\frac{3}{4}$ yd. of calico come to at $12\frac{1}{2}$ ct. a yard?

264. How much will $13\frac{4}{5}$ bushels of potatoes cost at $37\frac{1}{2}$ ct. a bushel?

265. How much will $10\frac{3}{5}$ lb. of cheese cost at $15\frac{3}{2}$ ct. a lb.?

266. What will $75\frac{1}{2}$ lb. of meal cost at $7\frac{3}{4}$ ct. a lb.?

267. What will $3\frac{4}{5}$ tons of coal cost at \$$12\frac{3}{5}$ a ton?

268. A grocer bought $12\frac{3}{4}$ bushels of apples at $37\frac{1}{2}$ ct. a bushel; what was the total cost?

269. A butcher bought $13\frac{1}{2}$ lb. of veal at $9\frac{3}{4}$ ct. a lb,; what did he give for the whole?

270. A merchant bought 175 yd. of flannel at $39\frac{4}{5}$ ct. a yd.; what was the whole cost?

LESSON XXIV.

130. To Divide integral, fractional, or mixed numbers, by a fraction or mixed number.

1. Divide 324 by $\frac{3}{7}$.
2. Divide $\frac{5}{8}$ by $\frac{3}{7}$.
3. Divide $43\frac{1}{2}$ by $5\frac{4}{7}$.
4. Divide $\frac{4}{9}$ by $\frac{3}{7}$.

1. $324 \times 7 \div 3 = 756$. Ans.
2. $\frac{5}{8} \times 7 \div 3 = \frac{35}{24}$. Ans.
3. $43\frac{1}{2} \times 7 \div 39 = 7\frac{63}{78}$. Ans.

(4.)

1. $\frac{4}{9} \div \frac{7}{7} = \frac{4}{9}$.
2. $\frac{4}{9} \div \frac{1}{7} = \frac{28}{9}$.
3. $\frac{4}{9} \div \frac{3}{7} = \frac{28}{27} = 1\frac{1}{27}$. Ans.

FORM.—(4.) 1. $\frac{4}{9} \div \frac{7}{7}$ or 1 unit, equals $\frac{4}{9}$.

2. If $\frac{4}{9} \div \frac{7}{7}$ equals $\frac{4}{9}$, then $\frac{4}{9} \div \frac{1}{7}$ will equal 7 times $\frac{4}{9}$, which is $\frac{28}{9}$.

3. If $\frac{4}{9} \div \frac{1}{7}$ equals $\frac{28}{9}$, then $\frac{4}{9} \div \frac{3}{7}$ will equal one *third* of $\frac{28}{9}$, which is $\frac{28}{27}$, equal to $1\frac{1}{27}$; therefore, $\frac{4}{9} \div \frac{3}{7} = 1\frac{1}{27}$.

NOTE.—The teacher will observe that in the analysis the dividend is multiplied by a number equal to the denominator of the divisor, and that this product is divided by a numerator equal to the numerator of the divisor; hence the following rule.

(*a.*) **Rule.**—I. *Multiply by the denominator, and* DIVIDE *the product* BY THE NUMERATOR; OR,

II. *Invert the divisor, and proceed as in the multiplication of fractions.*

NOTE.—When the divisor is a mixed number change it to an improper fraction.

LESSON XXV.

271. $467 \div \frac{4}{7}$.
272. $596 \div \frac{3}{9}$.
273. $671\frac{3}{2} \div \frac{4}{8}$.
274. $573\frac{6}{7} \div \frac{5}{9}$.
275. $813\frac{5}{7} \div \frac{6}{7}$.
276. $3\frac{4}{7} \div 12$.
277. $46\frac{3}{7} \div 4\frac{3}{7}$.
278. $437\frac{4}{3} \div 6\frac{3}{7}$.
279. $913 \div 46\frac{1}{2}$.
220. $837\frac{4}{7} \div 3\frac{4}{7}$.
281. $673\frac{4}{7} \div \frac{8}{9}$.
282. $437\frac{3}{5} \div 2\frac{1}{7}$.
283. $613\frac{4}{7} \div 8\frac{1}{9}$.
284. $437\frac{4}{7} \div 6\frac{3}{7}$.
285. $493\frac{3}{11} \div 4\frac{3}{8}$.
286. $6\frac{3}{9} + 4\frac{3}{7} \div \frac{3}{7}$.
287. $41\frac{3}{2} + 6\frac{3}{4} \div \frac{4}{9}$.
288. $3\frac{3}{7} + 4\frac{3}{4} \div \frac{5}{8}$.

LESSON XXVI.

PRACTICAL PROBLEMS.

289. If $18\frac{4}{5}$ yd. of ribbon are worth $\$4.41\frac{4}{5}$, what is 1 yd. worth?

290. If 18 handkerchiefs cost $\$13\frac{1}{2}$, what will one cost?

291. If $22\frac{3}{5}$ yd. of cloth are worth $\$19.83\frac{3}{20}$, what is one yd. worth?

292. If $35\frac{4}{5}$ lb. of raisins are worth $\$6.71\frac{1}{4}$, what is 1 lb. worth?

293. If $75\frac{3}{4}$ bushels of wheat are worth $\$121\frac{1}{5}$, what is one bu. worth?

294. If $23\frac{4}{5}$ cd. of wood are worth $\$80.32\frac{1}{2}$, what is 1 cd. worth?

295. If $212\frac{2}{3}$ lb. of beef are worth $15.95, what is 1 lb. worth?

296. If $14\frac{4}{5}$ barrels of vinegar are worth $153.92, what is one barrel worth?

297. If $12\frac{3}{4}$ bbl. of sugar are worth \$201.45, what is one bbl. worth?

298. If $35\frac{1}{2}$ tons of coal are worth $\$246\frac{1}{8}$, what is one ton worth?

LESSON XXVII.

MISCELLANEOUS EXERCISES IN FRACTIONS.

299. $3\frac{1}{2}+2\frac{1}{4}$.
300. $5\frac{3}{8}+6\frac{1}{7}$.
301. $5\frac{7}{8}+9\frac{3}{7}$.
302. $64\frac{3}{7}+5\frac{9}{11}$.
303. $38\frac{6}{7}+\frac{3}{7}$.
304. $4\frac{3}{7}+25$.
305. $4\frac{4}{7}+38$.
306. $\frac{6}{7}+25$.
307. $\frac{4}{9}+38$.
308. $3+2\frac{3}{7}$.
309. $4-\frac{3}{7}$.
310. $6\frac{3}{7}-\frac{3}{7}$.
311. $8\frac{3}{4}-2\frac{1}{4}$.
312. $19\frac{5}{8}-11\frac{2}{3}$.
313. $47\frac{3}{11}-8\frac{9}{10}$.
314. $43\frac{4}{7}-\frac{4}{9}$.
315. $83\frac{2}{7}-11$.
316. $84\frac{3}{7}-13$.
317. $\frac{5}{11}-\frac{5}{13}$.
318. $\frac{8}{16}-\frac{8}{17}$.
319. $\frac{4}{5}-\frac{1}{6}$.
320. $\frac{1}{8}-\frac{1}{9}$.
321. $\frac{5}{7}-\frac{5}{8}$.
322. $\frac{2}{11}-\frac{1}{15}$.

LESSON XXVIII.

If 1 lb. of tea cost $37\frac{1}{2}$ ct., what will 63 lb. cost?

MODEL OPERATION.

(*a.*)

$$\begin{array}{r} 37\frac{1}{2}\text{ ct.} \\ 63 \\ \hline 31\frac{1}{2} \\ 111 \\ 222 \\ \hline \$23.62\frac{1}{2} \end{array}$$

ct. (*b.*)

$$\begin{array}{r} 37 \times 63 = 2331 \\ \frac{1}{2} \times 63 = \quad 31\frac{1}{2} \\ \hline \$23.62\frac{1}{2} \end{array}$$

FORMULA.—If 1 lb. of tea costs $37\frac{1}{2}$ ct., 63 lb. will cost 63 times $37\frac{1}{2}$ ct., which are $\$23.62\frac{1}{2}$.

Anal. Steps.—1. *Find* 63 *times* 37 *cts.*
2. *Find* 63 *times* $\frac{1}{2}$ *ct.*
3. *Find total amount.*

323. If 1 firkin of butter weighs $83\frac{1}{3}$ lb., what do 16 firkins weigh?

324. What must I pay for 75 lb. of butter at $38\frac{2}{3}$ ct. per pound?

325. If 1 lb. of beef costs $22\frac{3}{4}$ ct., what will 57 lb. cost?

326. What is the cost of 71 bu. of corn at $84\frac{2}{3}$ ct. per bu.?

327. If 1 bbl. of flour is worth \$7¾, what are 17 bbl. worth?

328. What must be given for 37 yd. of silk at \$2¾ per yd.?

329. One bag of meal is worth 59⅓ ct.; what are 27 bags worth?

330. If 1 qt. of vinegar cost 9⅔ ct., what will 23 qt. cost?

331. What is the cost of 11 reams of paper at \$8⅓ per ream?

332. If 1 ball of twine is worth 15½ ct., what are 33 balls worth?

333. The cost of 1 yd. of cloth is \$1.33⅓; what will 45 yd. cost?

What will ¾ of a pound of sugar cost at 19⅓ ct. per lb.?

(*a.*)	(*b.*)
$4\overline{)19\frac{1}{3}}$ ct.	$19\frac{1}{3}$ ct.
$4\frac{5}{6}$	3
3	$4\overline{)58}$
$14\frac{1}{2}$ Ans.	$14\frac{1}{2}$ Ans.

FORM.—(*a.*) Since 1 lb. costs 19⅓ ct., ¾ of a pound will cost 3 times 1 *fourth* of 19⅓ ct., which is 14½ ct.

FORM.—(*b.*) Since 1 lb. costs 19⅓ ct., ¾ of a pound will cost 1 *fourth* of 3 times 19⅓ ct., which is 14½ ct.

334. What will ⅙ of a bu. of corn weigh, if 1 bu. weighs 56 pounds?

335. What will ⅛ of a barrel of flour weigh, if 1 bbl. weighs 196 lb.?

336. If there are 60 lb. in 1 bu. of wheat, how many pounds in ¼ of a bu.?

337. If there are 200 lb. in 1 bbl. of beef, how many pounds in ⅕ of a barrel?

338. How many pounds in ⅔ of a box of starch, if 1 box contains 28⅔ lb.?

339. If 1 yd. muslin cost 42 ct., what will ⅜ yd. cost?

340. How many yd. in ⅗ of a piece of cloth, if a piece contains 176⅓ yd.?

341. If there are 48 lb. in 1 bu. of barley, how many in $\frac{3}{10}$ of a bushel?

342. What is the value of $\frac{4}{7}$ of a bu. of oats, if 1 bu. is worth $60\frac{1}{2}$ ct.?

LESSON XXIX.

343. $4\times3\frac{1}{7}$.
344. $6\times\frac{5}{8}$.
345. $9\times\frac{6}{7}$.
346. $4\times3\frac{1}{7}$.
347. $4\frac{6}{7}\times3\frac{5}{8}$.
348. $91\times3\frac{1}{7}$.
349. $98\times5\frac{1}{3}$.
350. $67\times6\frac{7}{8}$.
251. $34\times8\frac{2}{7}$.
352. $913\times5\frac{6}{7}$.
353. $86\frac{4}{7}\times\frac{3}{7}$.
354. $96\frac{4}{7}\times\frac{5}{9}$.
355. $41\frac{3}{8}\times\frac{8}{9}$.
356. $67\times34\frac{1}{4}$.
357. $59\times36\frac{3}{7}$.
358. $9\frac{3}{7}\times\frac{5}{7}$.
359. $8\frac{3}{9}\times6\frac{3}{7}$.
360. $28\div4\frac{3}{4}$.
361. $93\div6\frac{3}{7}$,
362. $4\frac{3}{7}\div4\frac{2}{5}$.
363. $413\times\frac{6}{7}$.
364. $3\frac{4}{4}\div9\frac{3}{7}$.
365. $864\frac{2}{7}\div4\frac{1}{2}$.
366. $673\frac{4}{7}\div\frac{5}{11}$.

367. What would $3\frac{1}{7}$ boxes of raisins weigh, if one box weighs $67\frac{1}{3}$ lb.?

368. If 1 bottle of mineral water is worth $47\frac{1}{3}$ ct., what are 15 bottles worth?

369. How many lb. in $3\frac{1}{3}$ bu. of corn, if 1 bu. weighs 56 lb.?

370. What will $26\frac{1}{3}$ qt. of molasses cost at $24\frac{1}{5}$ ct. per qt.?

371. How many sq. inches in $4\frac{3}{4}$ panes of glass, if in one pane there are $20\frac{1}{2}$ sq. inches?

372. If in one coat there are $3\frac{1}{5}$ yd., how many yd. in 17 coats?

373. How far can a man walk in $18\frac{1}{2}$ hours, if in one hour he can walk $2\frac{1}{4}$ miles?

374. If there are $90\frac{1}{4}$ qt. of milk in 1 can, how many qt. in $\frac{1}{7}$ of a can?

375. What is the value of $69\frac{2}{3}$ yd. of carpet, if 1 yd. is worth \$$1.85\frac{1}{2}$?

LESSON XXX.

If 87 cows are worth \$$3870\frac{1}{5}$, what is the value of 1 cow?

MODEL OPERATION.

$$\begin{array}{r} 87)\$3870.20 \\ \hline \$44.48 \end{array}$$

$44.48 Ans.

376. If 26 lb. of cheese cost \$$3.91\frac{1}{3}$, what will 1 lb. cost?

377. If there are $421\frac{1}{5}$ lb. of salt in 7 boxes, how many lb. in 1 box?

378. How many pounds of cotton in 1 bale, if in 66 bales there are $9720\frac{2}{3}$ lb.?

379. If 600 bales of cotton are worth $\$63000.37\frac{1}{3}$, what is 1 bale worth?

380. If 12 men together weigh $1680\frac{2}{5}$ lb., what is their average weight?

381. How many qt. in 1 keg of beer, if in 8 kegs there are $96\frac{3}{4}$ qt.?

382. How many barrels of apples in 1 load, if in 10 there are $176\frac{3}{5}$ barrels?

383. If there are $19652\frac{1}{7}$ tons of coal in 4 equal heaps, how many tons in each heap?

384. If in 13 sticks of tape there are $69\frac{1}{8}$ yd., how many yd. in 1 stick?

385. What will 1 yd. of calico cost, if 65 yd. cost $\$14.75\frac{1}{2}$?

LESSON XXXI.

386. $413\frac{1}{7} \times 4 - 3\frac{4}{7}$.
387. $687 + 43\frac{1}{2} + 6\frac{7}{2}$.
388. $413 + 6\frac{1}{2} - 5\frac{2}{7}$.
389. $937 + 5\frac{3}{8} - 5\frac{5}{8}$.
390. $918 \times 3\frac{1}{7} - 6\frac{1}{7}$.
391. $3\frac{4}{7} + 2\frac{1}{2} \times \frac{1}{2}$.
392. $4\frac{3}{7} - \frac{2}{7} \times \frac{5}{7}$.
393. $6\frac{3}{8} - \frac{2}{9} \times 8$.
394. $84\frac{3}{7} - 37 \times 2$.
395. $\frac{3}{4} \times \frac{2}{6} + \frac{2}{3}$.
396. $8\frac{3}{7} + 4\frac{3}{8}$.
397. $6\frac{4}{9} + 4\frac{1}{3}$.
398. $9\frac{3}{5} + \frac{6}{7}$.
399. $8\frac{3}{11} - 5\frac{2}{5}$.
400. $67 \times 4\frac{3}{5}$.
401. $8\frac{6}{7} + 4\frac{3}{7}$.
402. $8\frac{6}{7} \times 4\frac{3}{7}$.
403. $8\frac{6}{7} - 4\frac{3}{7}$.
404. $2 + 8\frac{3}{4}$.
405. $3 - 2\frac{3}{4}$.

406. If 3 lb. of flour cost $28\frac{1}{3}$ ct., what will $\frac{1}{4}$ lb. cost?

407. What is the value of $\frac{1}{8}$ of a yd. of silk, if 12 yd. cost $\$16.46\frac{2}{3}$?

408. If there are 126 lb. of paint in 6 kegs, how many pounds in $\frac{2}{3}$ of a keg?

409. If there is an average of 1250 clusters of grapes to every 5 vines, how many clusters on $\frac{1}{5}$ of a vine?

410. If it takes 2 millions 235 grains of barley to make a bushel, how many gr. in $\frac{1}{5}$ of a bushel?

411. If a house can be built in 25 weeks, how long will it take to do $\frac{1}{3}$ of the job?

412. If there are 40 bags of corn in one wagon load, how many in $\frac{4}{5}$ of a load?

413. If 175 bags of corn cost $\$266$, what will $\frac{2}{5}$ of a bag cost?

414. If a locomotive runs $180\frac{1}{2}$ miles in 6 hours, how far is that for $\frac{2}{7}$ of an hour?

415. How many pounds of tea in $\frac{3}{10}$ of a chest, if in 11 chests there are 485 lb.?

416. What is the value of $2\frac{4}{3}$ baskets of strawberries, if 17 baskets cost $110\frac{1}{2}$ cts.?

417. What is the number of inches around 1 orange, if 6 oranges measure $45\frac{1}{4}$ inches in circumference?

418. If there are 326 bundles of wood in 1 wagon load, how many in $\frac{1}{6}$ of a wagon load?

419. If there are 2126 lemons in 4 boxes, how many in $\frac{1}{8}$ of a box?

420. What will $1\frac{1}{2}$ bushels of wheat cost, if $46\frac{1}{3}$ bu. cost $\$82.30\frac{1}{4}$?

421. If $25\frac{2}{5}$ yd. of flannel are worth $\$12.87\frac{2}{3}$, what is the value of $\frac{1}{2}$ yd.?

422. How much will 1 oz. indigo cost, if $11\frac{2}{5}$ oz. cost $\$1.88\frac{1}{4}$ ct.?

423. What will $\frac{1}{8}$ lb. tea cost, if $7\frac{2}{7}$ lb. are worth $\$8\frac{3}{5}$?

424. If there are $440\frac{1}{5}$ lb. rice in $8\frac{3}{4}$ bags, how many in 1 bag?

425. If $36\frac{2}{3}$ acres of wheat yield 637 bu., at that rate what will $5\frac{4}{5}$ acres yield?

LESSON XXXII.

426. $36+47\frac{2}{3}\div 83$.
427. $46+83\frac{4}{7}\div 47$.
428. $38\frac{3}{4}+67\frac{2}{3}\div 96$.
429. $1\frac{1}{2}+12\div 300$.
430. $4\frac{3}{7}+2\div 674$.
431. $25-11\frac{8}{12}\div 14\frac{7}{8}$.
432. $46-27\frac{3}{4}\div 1864$.
433. $2374\frac{3}{4}-287\div 9173$.
434. $21+347\frac{5}{9}\div 8641$.
435. $\frac{3}{4}\div 1+41\frac{3}{7}$.
436. $\frac{3}{4}+\frac{1}{4}\div 1+\frac{5}{7}\times 2$.
437. $\frac{5}{9}+\frac{1}{3}\div\frac{1}{2}+\frac{6}{7}-\frac{1}{12}$.
438. $\frac{8}{3}+\frac{5}{7}+\frac{6}{7}\div 8+\frac{1}{9}$.
439. $1\frac{4}{7}+\frac{3}{7}+6\frac{3}{7}\div 438$.
440. $\frac{3}{7}-\frac{3}{8}\div 46734$.

441. If I can buy 22 lemons for $\$1.12\frac{1}{2}$, what will 56 lemons cost?

442. If 27 yd. of ribbon cost $\$4.90\frac{1}{2}$, what will $58\frac{3}{7}$ yd. cost?

443. How much must be given for 19 umbrellas, if 5 cost $\$14.37\frac{1}{2}$.

444. How many cords in 15 piles of wood, if there are $79\frac{3}{5}$ cords in 11 piles?

445. How many words can a man write in $15\frac{1}{2}$ minutes if he can write 159 words in 3 minutes?

446. If there are on the average 9 damaged pens in a gross, how many in $112\frac{2}{3}$ gross?

447. What will be the cost of 12 pianos, if 7 cost $\$2620\frac{3}{4}$?

448. If 9 horses cost $\$1305\frac{2}{5}$, what will 13 cost?

449. If 11 pineapples cost $\$2.23\frac{1}{3}$, what will 25 cost?

450. If 17 pecks of turnips are worth $\$3.12\frac{1}{4}$, what are 27 pecks worth?

LESSON XXXIII.

MISCELLANEOUS EXAMPLES IN FRACTIONS.

451. $34 \div 4 + 3\frac{1}{7} \times 5\frac{7}{8}$; $59 - 37\frac{1}{2} \times \frac{4}{7}$.
452. $38\frac{6}{7} \times \frac{3}{7} \div 4\frac{3}{7}$; $63\frac{5}{9} - 20\frac{5}{9} \times 8$.
453. $41\frac{3}{7} \times 2 \div 467$; $9 \times 5\frac{3}{7} \div 413$.
454. $46\frac{3}{7} + \frac{3}{4} + \frac{5}{7}$; $37\frac{2}{3} + \frac{5}{9} \div \frac{3}{8}$.
455. $14\frac{3}{5} + \frac{6}{7} \div \frac{5}{9}$; $3\frac{1}{4} + \frac{6}{8} \times \frac{3}{5}$.

If a man can saw $13\frac{1}{3}$ cords of wood in $\frac{1}{6}$ of a month, how many can he saw in $\frac{3}{4}$ of a month?

MODEL OPERATION.

$$13\tfrac{1}{3} \div \tfrac{1}{6} \times \tfrac{3}{4} = 60. \quad \text{Ans.}$$

Anal. Steps.—1. *Find what he can saw in* 1 *mo.*
2. *Find what he can saw in* $\frac{3}{4}$ *mo.*

456. If a bird can fly $10\frac{3}{4}$ miles in $\frac{1}{3}$ of an hour, at that rate how far can it fly in $2\frac{1}{2}$ hours?

457. If $\frac{2}{5}$ yd. of velvet cost $74\frac{1}{4}$ ct., at that rate how much must be given for $13\frac{2}{5}$ yd.?

458. In $\frac{2}{3}$ of the wall of a house there are 2200 bricks, how many bricks in a wall twice as great?

459. If in $\frac{2}{7}$ of a ream of paper there are $\frac{3}{4}$ of a pound, how many pounds in 6 reams?

460. If a man's postage amounts to $14 for $\frac{1}{5}$ of a year, to how much will it amount for $2\frac{3}{4}$ years?

SECTION IX.

LESSON I.

INTEREST.

131. *Interest* is an amount paid for the use of money.

132. *The Principal* is the sum on which interest is paid.

133. *Rate per cent.* is the per cent. paid annually.

134. *Amount* is the sum of the principal and the interest.

135. *To find the Interest or Amount* of any principal, for any time, at any rate per ct.

What is the interest on $325 for 8 yr. 3 mo. 11 da. at 7 per cent.?

MODEL OPERATION.*

$325.00

.07 int. on $1

12)$22.7500 × 8 years = $182.0000

30)1.8958 × 3 mo. = 5.6874

.0632 × 11 days = .6952

$188.3826 Interest.

325.0000

$513.3826 Amount.

FORM.—1. If the interest on $1 for one year is 7 cts., the interest on $325 for the same time is 325 times 7 cts., which are $22.75; and if the interest is $22.75 for one year, for 8 years it is 8 times $22.75, which are $182.

2. If the interest on the given principal for one year is $22.75, the interest for one month is one *twelfth* of $22.75, which is $1.8958; and for 3 months the interest is 3 times $1.8958, or $5.6874.

3. If the interest on the given principal for one month is $1.8958, for one day it is one *thirtieth* of $1.8958, which is $1.0632; and for 11 days it is 11 times $0.0632, which is $0.6952.

---

* The teacher should show the necessity of retaining the *tenth* of a mill in operations of this kind when great accuracy is required.

4. If the interest on the given principal for 8 years is $182, for 3 months $5.6874, and for 11 days $0.6952, the interest for the entire time is the sum of these quantities, which is $188.3826.

5. The amount is the sum of the principal and interest, which is $513.3326. Therefore, &c.

(*a*). **Rule.**—I. *Multiply the given principal by the rate per cent., divide the product by* 100, *and the result will be the interest for one year.*

II. *Divide the interest for one year by* 12, *and the result will be the interest for one month; divide the interest for one month by* 30, *and the result will be the interest for one day.*

III. *Multiply the interest for one year by the number of years the interest of one month by the number of months, and the interest for one day by the number of days, and the sum of the products will be the interest required, to which add the principal for the amount.*

LESSON II.

What is the interest of $341.25 for each of the following periods?

1. 3 yr. 2 mo. 12 da. at 6 per ct.
2. 4 yr. 3 mo. 11 da. at 6 per ct.
3. 3 yr. 11 mo. 22 da. at 7 per ct.
4. 13 yr. 10 mo. at 8 per ct.
5. 4 yr. 25 da. at 8 per ct.
6. 13 mo. 27 da. at 6 per ct.
7. 5 mo. 15 da. at 6 per ct.
8. 14 yr. 8 mo. at 1 per ct.
9. 11 yr. 25 da. at 1 per ct.
10. 3 yr. 4 mo. 16 da. at 1 per ct.
11. 4 yr. 8 mo. 11 da. at 1 per ct.
12. 3 yr. 5 mo. 12 da. at 7 per ct.
13. 6 yr. 12 mo. 11 da., at 6 per ct.
14. 4 yr. 5 mo. 13 da. at 8 per ct.
15. 5 yr. 9 mo. 11 da. at 6 per ct.
16. 4 yr. 11 mo. 9 da. at 7 per ct.
17. 5 yr. 8 mo. 19 da. at 8 per ct.
18. 8 yr. 3 mo. 2 da. at 1 per ct.
19. 1 yr. 1 mo. 1 da. at 1 per ct.
20. 4 yr. 4 mo. 4 da. at 6 per ct.

LESSON III.

21. $342 for 3 yr. 12mo. 9 da. at 7 per cent.
22. $241 for 2 yr. 3 mo. 11 da. at 6 per ct.
23. $241.31 for 5 yr. 8 mo. 12 da. at 8 per ct.
24. $11.24 for 8 yr. 9 mo. 26 da. at 6 per ct.
25. $41.38 for 9 yr. 11 mo. 15 da. at 6 per ct.
26. $11.43 for 8 yr. 11 mo. 3 da. at 1 per ct.

27. $137.18 for 11 yr. 5 mo. 4 da. at 6 per ct.
28. $100 for 11 mo. 23 da. at 8 per ct.
29. $600.41 for 19 mo. at 1 per ct.
30. $5.36 for 1 yr. 1 mo. 1 da. at 1 per ct.

31. What is the amount of a note for $378, for 3 yr. 9 mo. 7 da. at 6 per ct.?

32. What is the amount of a note for $275, for 3 yr. 7 mo. 8 da. at 7 per ct.?

33. What is the amount of a note for $360.30, for 2 yr. 9 mo. 11 da. at 5 per ct.?

34. I gave a note Jan. 1, 1864, for $275; what will be the amount April 1, 1865, at 7 per cent. int.?

35. I gave a note Oct. 11, 1863, for $576; what will be its amount Feb. 11, 1865, at 6 per ct.?

36. I hold a note for $127, dated Aug. 13, 1837; what should I receive Sept. 13, 1841, interest being at 7 per ct.?

37. I loaned $373 July 11, 1863 at 6%; how much interest should I receive April 13, 1868?

LESSON IV.

38. What was the interest on the following note, at 7 per ct. when paid June 12, 1864?

300\frac{75}{100}$. BROOKLYN, N. Y., June 30, 1858.

One day from date, I promise to pay Delos Curtiss or order, three hundred dollars and seventy-five cents, value received.

E. S. BUNKER.

39. What is the interest on the following note, when paid April 11, 1863, at 6 per cent.?

$730. BROOKLYN, N. Y., June 11, 1859.

Nine months from date, we promise to pay J. Legget or order, seven hundred and thirty dollars, with interest from date, value received.

BEACH, BRADISH, GODWIN & CO.

40.

$575. ALBION, N. Y., April 12, 1863.

Two years seven months from date, I promise to pay Orlow C.

Gager or order, five hundred and seventy-five dollars, with interest from date at 7 *per cent., value received.*

SEYMOUR OLMSTEAD.

What is the amount of the above note when due?

41.

$438. BROOKLYN, N. Y., April 9, 1864.

One day from date, I promise to pay Foster Potter or order, four hundred and thirty-eight dollars, with interest at 8 *per cent., value received.* WILLIAM HOLMES.

What is the amount of the above note July 13, 1866?

42. The principal is $463.38, the time 4 yr. 8 mo. 2 da., the rate per cent. 7; what is the interest?

43. The principal is 374.80; the time is 2 yr. 11 mo. 11 da.; the rate per cent. is 8; what is the interest?

44. The principal is $486.70; the time is 4 yr. 2 mo. 2 da.; the rate per cent. is 6; what is the interest?

45. The principal is $963; the time 2 yr. 11 da.; the rate per cent. is 6; what is the amount?

46. The principal is $113; the time is 4 mo. 11 da.; the rate per cent. is 7; what is the amount?

47. The principal is $413.14; the time is 8 mo. 12 da.; the rate per cent. is 7; what is the amount?

48. The date of a note is Apr. 11, 1864, the face is $370, the rate is 5 per cent.; what is the interest July 9, 1866?

49. The date of a note is Aug. 13, 1863, the face is $530 the rate is 7 per cent.; what is the interest Apr. 11, 1864?

50. The date of a note is Aug. 11, 1862, the face is $378.25, the rate is 7 per cent.; what is the interest Feb. 13, 1863?

51. If a person borrows $368.76 at 7 per cent., how much will be due at the end of 2 yr. 7 mo. 19 da.?

52. If a person borrows $437 at 6 per cent., how much will be due at the end of 3 yr. 7 mo. 18 da.?

53. What is the interest paid on a loan of $1374.68 at 7 per cent., made Jan. 1, 1858, and paid July 7, 1863?

54. A note of $374.26 was given May 23, 1864, bearing interest at 8 per cent.; what will be the amount due Aug. 3, 1866?

55. What is the interest paid on a loan of $374.20 at 6 per cent., made Jan. 11, 1862, and paid Apr. 12, 1866?

56. What is the interest paid on a loan of $860 at 7 per cent., made Jan. 13, 1836, and paid July 27, 1839?

57. I gave a note of $647.37 Apr. 13, 1863; what was the amount due Feb. 12, 1864, at 7 per cent. interest?

58. I gave a note of $46.37 July 30, 1860; what was the amount due Dec. 13, 1864, at 6 per cent. interest?

59. What is the interest on $387.41 for 8 mo. 27 da. at 7 %?

60. What is the amount of $48.37 for 3 yr. 3 mo. 3 da. at 6 %?

61. A man bought property for $2873.27, and agreed to pay for it in 2 yr. 7 mo. 11 da., with 7 per cent. interest; at the end of the time, what was the amount due?

62. I settled with a creditor and gave my note for $317.42 to be paid in 1 yr. 2 mo. 5 da., with 7 per cent. interest; at the end of that time, what will be the amount due?

LESSON V.

136. ***To find the Interest on any Principal*** for any time at 6 per cent.

What is the interest on $372.12 for 8 yr. 9 mo. 13 da.?

MODEL OPERATION.

(*a.*)

Interest on $1 for 8 yr. = $0.48.
" " 1 " 9 mo. = .045.
" " 1 " 13 da. = $.002\frac{1}{6}$.

$\$0.527\frac{1}{6} \times 372.12 = \$196.169+$ Ans.

Anal. Steps.—1. *Find the interest on* $1 *for the given time.*
2. *Find the interest of* $372.12 *for the same time.*

(*a.*) **Rule.***—I. *Calculate the interest for every year at* 6 *cts.,*

---

* For other methods of computing interest, see Commercial Arithmetic.

for every 2 *months at* 1 *ct., for every* 6 *days at* 1 *mill, and for every day at* $\frac{1}{6}$ *of a mill; the sum will be the interest on* $1 *for the given time.*

II. *Multiply the interest on* $1 *for the given time by the principal, and the product will be the interest required.*

EXAMPLES FOR PRACTICE.

NOTE.—When no rate per cent. is mentioned, 6 per cent. is always understood.

63. What is the interest on $397.46 for 8 yr. 3 mo. 9 da.?

64. What is the interest on $371.46 for 9 yr. 3 mo. 19 da.?

65. What is the interest on $572.86 for 18 yr. 11 mo. 29 da.?

66. What is the amount of $37.26 for 13 yr. 2 mo. 11 da.?

67. What is the amount of $33.46 for 2 yr. 2 mo. 23 da.?

08. What is the interest on $6.47 for 3 yr. 9 mo. 11 da.?

69. What is the interest on $62.35 for 2 yr. 8 mo. 29 da.?

70. What is the interest on $12.42 for 2 yr. 8 mo.?

71. What is the interest on $37.85 for 1 yr. 9 mo. 28 da.?

72. What is the interest on $12.80 for 3 yr. 3 mo. 3 da.?

LESSON VI.

PARTIAL PAYMENTS.

137. *A Partial Payment* is the payment of a part of the amount due upon a note, or bond, &c.

138. *An Indorsement* is a writing on the back of a note or bond stating the amount paid, the date of payment, and the name of the person receiving the money.

(*a.*) NOTES.—1. A *joint* note is one signed by two or more persons, who are together bound for its payment.

2. A *joint* and *several* note is one signed by two or more persons, who are together and separately bound for its payment.

3. A negotiable note is one which is written payable to *order* or to *bearer*, and which can be transferred from one to another.

4. When a note is written *payable to order*, the *payee* of the note must write his name on its back, when he transfers it to another person; in so doing, he becomes liable for its payment, if the *maker* fails to pay it and the payee is duly notified of the fact.

5. The *maker* of a note is the person who signs it. The *payee* is the person to whom it is made payable.

6. The *indorser* is the person who writes his name on the back to transfer it, or to guarantee its payment. The *face* of a note is the amount for which it is given.

7. The *indorsee* is the person to whom the note is transferred by indorsement.

8. The *holder* is the person who *holds* or owns the note; he may be either the *payee* or any *indorsee.*

(*b.*) **Supreme Court Rule.**—The following rule for computing interest on bonds, notes, &c., when partial payments have been made, is used by the Supreme Court of the United States, and by all the States of the Union, with the exception of Connecticut and New Hampshire.

Rule.—I. *Compute the interest on the principal to the time of the first payment; if the payment equals or exceeds this interest, add the interest to the principal, and subtract the payment from the sum: the remainder forms a new principal.*

II. *If the payment is less than this interest, compute the interest on the principal to the time of that payment which with all preceding payments will exceed the interest on the principal to the time of that payment; then add the interest to the principal, and subtract the sum of the payments: the remainder will form a new principal, with which proceed as before.*

$500. SALISBURY, CT., August 9, 1860.

Two years from date I promise to pay Henry Vandeusen or order, five hundred dollars, with interest from date at 6 per cent., value received.

WILLIAM GAGER.

On this note are the following indorsements:—

Aug. 11, 1861, $40. July 12, 1862, $15.13.

What was the amount when the above note became due?

MODEL OPERATIONS.

Rule I.—Principal, or face of the note, Aug. 9, 1860,	$500.000
Interest on $500 from Aug. 9, 1860, to Aug. 11, 1861, time of first payment (1 yr. 2 days),	30.166
Amount,	$530.166
First payment, Aug. 11, 1861 (greater than the interest, therefore to be deducted),	40.000
Balance for a new principal,	$490.166
Rule II.—Interest on $490.166 from Aug. 11, 1861, to July 12, 1862, time of *second payment** (11 mo. 1 day,)	$27.040
Interest on $490.166 from July 12, 1862, to Aug. 9, 1862, time of settlement (27 days),	2.205
Amount,	$519.411
Second payment, July 12, 1862,	15.130
Amount due Aug. 9, 1862, time of settlement,	$504.281

73.

$585. Binghamton, N. Y., Aug. 11, 1860.

One day from date I promise to pay William S. Lawyer or order five hundred and eighty-five dollars with interest at 7 per cent., value received.

Charles Bardsley.

Received on the within note :—

May 11, 1861, $122.13.
Sept. 19, 1862, $11.12.
Nov. 27, 1863, $146.23.

What will be the amount, July 13, 1865 ?

---

* Note.—The second payment was only $15.13, while the interest was $27.040 consequently, no notice is taken of the payment until the time of settlement.

LESSON VII.

74. Demand Note.

$\$300\frac{75}{100}$. Nottingham, Sept. 20th, 1865.

On demand, I promise to pay E. D. Crofts three hundred and $\frac{75}{100}$ dollars, with interest at $6\frac{1}{2}$ %. Value received.

K. H. Nash.

What is due Jan. 12, 1867?

75. Negotiable Note.

$\$175\frac{36}{100}$. Pentwater, Mich., June 13, 1866.

One year after date, I promise to pay to the order of J. Marson one hundred and seventy-five and $\frac{36}{100}$ dollars, with interest at 7 %. Value received. Amos Dresser, Jr.

Received on the within note,

July 1st, 1866, \$25;
Nov. 12th, 1866, \$5;
Jan. 12th, 1867, \$50;

What was due Jan. 1st, 1868?

76. Note Payable to Bearer.

$\$358\frac{95}{100}$. New York, Jan. 27th, 1866.

Six months after date, I promise to pay W. A. Kipp, or bearer, three hundred fifty-eight and $\frac{95}{100}$ dollars with interest, for value received.

N. M. Shafer.

77. Note in Bank.

$\$50\frac{25}{100}$. New York, May 21st, 1866.

Thirty days after date, I promise to pay to the order of P. J. Pelz, at the Tradesmen's Bank, N. Y., fifty and $\frac{25}{100}$ dollars, value received.

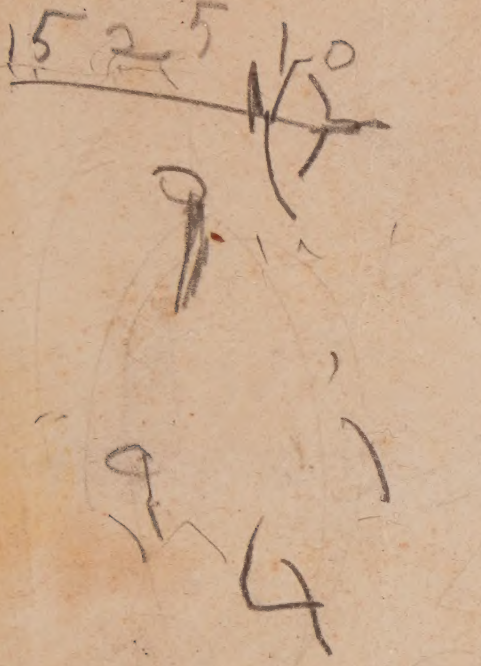
Charles W. Fuller.

78. $\$150\frac{75}{100}$. Philadelphia, Pa., May 3d, 1865.

Ninety days after date, I promise to pay E. S. Pierce or bearer one hundred and fifty and $\frac{75}{100}$ dollars with interest at $5\frac{1}{4}$ % value received.

Geo. W. Hatch.

2 15



45

6 14

POPULAR ARITHMETICAL SERIES.

I.
FELTER'S
FIRST LESSONS IN NUMBERS.
Illustrated.

II.
FELTER'S PRIMARY ARITHMETIC.
Illustrated.

III.
FELTER'S
INTERMEDIATE ARITHMETIC.

IV.
FELTER'S
GRAMMAR-SCHOOL ARITHMETIC.

V.
FELTER'S
INTELLECTUAL ARITHMETIC.

VI.
FELTER'S
TEACHER'S MANUAL OF ARITHMETIC.

VII.
FELTER'S
HIGH-SCHOOL ARITHMETIC.
(In preparation.)

N. B.—For those who prefer it, the Intermediate and Grammar-School Arithmetics are bound in one volume and called the

PRACTICAL.

CHARLES SCRIBNER & CO.,
NEW-YORK, 654 BROADWAY.